COMPARATIVE STUDY OF SURFACE MATCHING ALGORITHMS

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ABSTRACT

Airborne laser ranging systems provide a new source for determining surfaces. There is an increasing need to compare surfaces and to combine and merge surface data obtained with different sensors. This paper addresses the problem of determining the 3D transformation parameters between two sets of points describing the same physical surface. However, the two sets are in different reference systems and the points are not identical. We describe two mathematical models that are suitable to find optimal transformation parameters such that the differences between the two sets of surface descriptions become minimal. The first approach minimizes the remaining differences along the $z-$axis of one of the coordinate systems while the second approach minimizes the distances between points of one surface description to surface patches of the other surface, parallel to the corresponding surface normals. After deriving the mathematical models, the two methods are tested with synthetic data. The performance of both methods are investigated with different topographic configurations The experiments confirm the theoretical expectations that the transformation parameters can only be determined well if the surface has reasonable surface slopes with surface normals pointing in all different directions. If these conditions are not satisfied, the transformation problem becomes ill-posed and only a subset of the parameters can be determined. For surfaces with large slopes, the second method performs better because it minimizes distances along surface normals.

1 INTRODUCTION

There is an increasing demand for the rapid generation of surface models. For example, orthophotos require a known surface; city modeling poses new challenges to surface generation; surface change detection, such as monitoring ice sheets, requires a comparison of surfaces, captured at different times.

Airborne laser ranging systems are quickly emerging as a viable alternative to more traditional methods of acquiring data suitable for describing surfaces. Laser ranging and automatic matching methods of digital photogrammetry may be considered sampling processes, resulting in a set of points that constitute a discrete description of the visible surface. The measured points on the surface (by laser ranging or stereoscopy) are usually irregularly distributed. Moreover, characteristic surface properties, such as breaklines and surface roughness, are not explicitly available.

Matching surfaces is a fundamental task that must be solved whenever we want to compare or merge data sets describing surfaces. In the most general scenario, the two sets are obtained with different systems (e.g. laser ranging and photogrammetry, or newly acquired data and existing surfaces) where the density, distribution, and accuracy of points is different as well as their reference system—that is, the transformation parameters between the two coordinate system are not known. There are two distinctly different problems involved in solving this general task. For one, a relationship of surface features between the two surfaces must be established. We call this the *correspondence problem* in this paper. Once the correspondence is established, a set of transformation parameters must be determined such that differences between the two surface descriptions are minimized. We refer to this problem as *transformation.* Thus, matching (or comparing) surfaces entails the correspondence and transformation problem.

In this paper we use points and surface patches as features for the correspondence and transformation problem. This is motivated by the desire to work with entities that are as close to the observed quantities as possible. Since both sets of surface descriptions consist of observed 3D points, a point to point correspondence would be most desirable but it does not exist because the points are differently distributed. As a consequence, we determine surface patches in one surface, for example triangles through a TIN model. Then, the correspondence problem is to identify the triangle in the first surface to which a point in the second surface belongs. We do not elaborate further on the correspondence problem in this paper; the interested reader is referred to *Habib and Schenk* (1999), for example.

Another approach to the surface matching problem would be to extract features in both surfaces that are invariant to transformation, and establish a correspondence between features, followed by the transformation. This approach resembles image matching and has, in fact, been proposed by researchers. To employ straight-forward image processing and image matching algorithms, the range data must be first converted to an image, however. Considering the irregularly distributed points, the inevitable interpolation may introduce unacceptable errors. Additional complications arise from the different reference systems the two sets of surface points are recorded in.

Since we assume that the two surface descriptions are in different coordinate systems with unknown transformation parameters, we cannot simply interpolate the irregularly distributed points to a grid and compare the $z-$values at the grid posts. Originally, the idea of transforming the points of the second surface and comparing the elevations at known points was suggested by *Ebner and Strunz* (1988). Here, the application was to determine the absolute orientation of a model by using a known DEM, instead of control points. The approach is based on minimizing the $Z-$differences between model points—subject to an unknown transformation to object space—and points in object space found by DEM interpolation. Subsequent studies demonstrated the applicability of the method (see, e.g. *Ebner et al.* (1991)).

In this paper we examine two different mathematical models to establish the transformation parameters between the two representations of the same physical surface. The models differ in the target function. The first approach minimizes differences in elevations between the two surfaces. The second method minimizes distances along surface normals. This is a more general approach since the quantity to be minimized is independent of the coordinate system. Thus, surfaces containing vertical sub-surfaces do not cause a problem.

To examine the influence of the surface topography on the solution of the 3D transformation, we present experimental results obtained with synthetic surfaces. These experiments confirm the theoretically motivated expectation that the surface must consist of surface patches with surface normals in all directions. Imagine the surface normals expressed by longitude and latitude; the values should be distributed over the entire hemisphere.

In the interest of brevity we do not include in this paper results of practical applications obtained with laser data sets, but refer to *Postolov et al.* (1999) and *Schenk* (1999a) for example.

2 MATHEMATICAL MODELS

In this section we briefly present two mathematical models that can be used to establish a correspondence between two data sets describing the same surface. The derivations and the experiments performed in this paper are based on the assumption that the two data sets are related by a 3-D similarity transformation. It is very easy to employ other transformation types, however.

2.1 Problem Statement and Solution

Let $S_1 = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ be a surface described by n discrete points $\mathbf{p}$ that are randomly distributed. Let $S_2 = \{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_m\}$ be a second surface described by m randomly distributed points $\mathbf{q}$. Suppose that the two sets are, in fact, describing the same surface, however, they are in different reference systems. In the example with the absolute orientation, set S_2 may be the model system and set S_1 is the object space reference system. Another example is two laser data sets that have been obtained over the same surface but with different systems. After proper transformation we have $S_1 = S_2$, except for differences due to random errors of the observed points $\mathbf{p}$ and $\mathbf{q}$. Yet another difference may arise from the discrete representation of the surfaces, for example, $n \neq m$. Even in cases where $n = m$, the different distribution may cause a differently interpolated surface. Suppose further that no points in the two sets are known to be identical (same surface point).

The problem is now to establish a transformation between the two sets such that the two surfaces S_1 and S_2 become as similar as possible in terms of closeness and shape. The problem is cast as an adjustment problem where the second set of points $\mathbf{q}$ is transformed to the first set in a way that minimizes the differences between the two surfaces. Additionally, the orientation of surface normals between S_1 and S_2 can also be minimized. Minimizing the distances assures the best positional fit while minimizing differences in surface normals assures the best shape fit.

2.2 Target Function: Minimize ΔZ-Differences

The approach of minimizing the $Z-$differences between two surfaces—subject to an unknown transformation—was originally proposed by *Ebner and Strunz* (1988). Recently, *Postolov et al.* (1999) suggested to employ the well-known decomposition of the 3-D transformation into a sequence of planimetry and elevation transformations that are both linear. Moreover, the authors showed that the matching, that is the correspondence of a point in one surface to the appropriate surface patch in the other surface, is not explicitly required, but established simultaneously together with the transformation parameters.

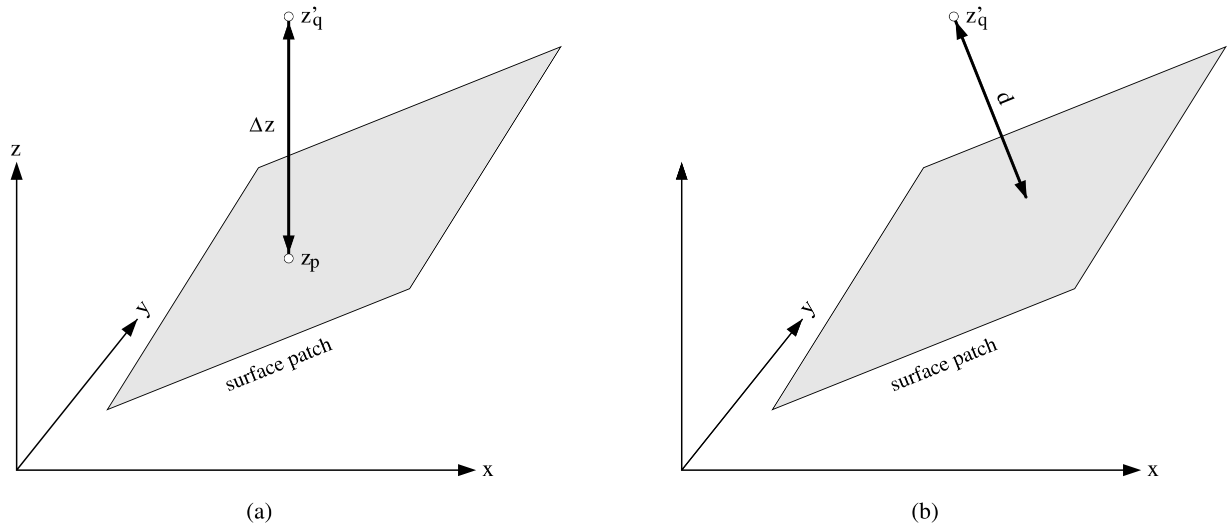

Figure 1: Point **q** is a point of surface S_2 that is transformed to surface S_1. In the vicinity of **q**, S_1 is represented by a planar surface patch. In (a), Δz is used as a target function to obtain transformation parameters. In (b), the shortest distance from **q** to the surface is used for determining the transformation.

The basic idea of the mathematical model is illustrated in Fig. 1(a). Suppose point $\mathbf{q}_i$ of the second surface is transformed by a 3-D similarity transformation as follows

$$\mathbf{q}_i' = s\mathbf{R}\mathbf{q}_i - \mathbf{t} \tag{1}$$

where $\mathbf{R}$ is a 3-D rotation matrix, $\mathbf{t}$ a translation vector, and s a scale factor. Assuming small rotation angles $d\omega, d\varphi, d\kappa$ and small scale factor ds, we obtain for the components (see, e.g. *Kraus* (1993) for detailed derivation)

$$x_q' = ds \cdot x_q - d\kappa \cdot y_q + d\varphi \cdot z_q - x_t \tag{2}$$
$$y_q' = d\kappa \cdot x_q + ds \cdot y_q - d\omega \cdot z_q - y_t \tag{3}$$
$$z_q' = -d\varphi \cdot x_q + d\omega \cdot y_q + ds \cdot z_q - z_t \tag{4}$$

Suppose further that we have approximated the first surface in the neighborhood of $\mathbf{q}_i$ by an analytical function, for example a plane. Then, this surface patch is expressed by

$$z = a \cdot x + b \cdot y + c \tag{5}$$

with a, b, c the coefficients of a plane fitting through a few points $\mathbf{p}$ of surface S_1. Now, the offset of point $\mathbf{q}'_i$ to the surface patch along the $z-$axis is $\Delta z = z'_q - z_p$ (see also Fig. 1). We obtain z_p by substituting x and y in Eq. 5 with x'_q and y'_q of Eqs. 2 and 3. Considering Δz an observation, we obtain the following design matrix coefficients for determining the seven transformation parameters

$$
\begin{aligned}
d\omega &= y'_q + b \cdot z'_q \\
d\varphi &= -x'_q - a \cdot z'_q \\
d\kappa &= a \cdot y'_q - b \cdot x'_q \\
ds &= z'_q - a \cdot x'_q - b \cdot y'_q \\
x_t &= a \\
y_t &= b \\
z_t &= -1
\end{aligned}
$$

with a, b, c the parameters of a plane (Eq. 5). We can interpret this procedure as finding transformation parameters in such a fashion that the remaining differences between the two surfaces are minimized along the $z-$axis.

2.3 Target Function: Minimize Distance along Surface Normal

The second approach to find optimal transformation parameters between two surfaces S_1 and S_2 is based in minimizing the distance between a point of the second surface parallel to the normal of a surface patch in S_1. Fig. 1(b) illustrates the concept.

If the surface patch is expressed in Hessian normal form, using the three directional cosines and the distance p from the the origin, then the shortest distance d from $\mathbf{q}'$ to the surface patch is

$$d = \mathbf{q}' \cdot \mathbf{h} - p \tag{6}$$

with $\mathbf{h} = [\cos\alpha, \cos\beta, \cos\gamma]^T$. Substituting $\mathbf{q}'$ with the right hand side of Eq. 1 yields the following observation equation:

$$r = (s\mathbf{R}\mathbf{q} - \mathbf{t}) \cdot \mathbf{h} - p - d \tag{7}$$

We skip the details of linearizing this equation and refer the interested reader to *Schenk* (1999b), p. 410. Using the same notation as in the previous model, the coefficients of the design matrix are

$$
\begin{aligned}
d\omega &= y'_q \cos\gamma - z'_q \cos\beta \\
d\varphi &= z'_q \cos\alpha - x'_q \cos\gamma \\
d\kappa &= x'_q \cos\beta - y'_q \cos\alpha \\
ds &= (x'_q - y'_q \cdot d\kappa + z'_q \cdot d\varphi) \cos\alpha + \\
&\quad (x'_q \cdot d\kappa + y'_q - z'_q \cdot d\omega) \cos\beta + \\
&\quad (-x'_q \cdot d\varphi + y'_q \cdot d\omega + z'_q) \cos\gamma \\
x_t &= -\cos\alpha \\
y_t &= -\cos\beta \\
z_t &= -\cos\gamma
\end{aligned}
$$

3 EXPERIMENTS WITH SYNTHETIC DATA

We performed several experiments with synthetic data in order to examine the geometric conditions that are necessary to determine the seven transformation parameters between surface S_1 and S_2. The following considerations elucidate the problem.

Suppose S_1 is a horizontal plane. With one point of set $\mathbf{q}$ we can solve for z_t. With two additional points, the two rotation angles ω and φ are determined. Obviously, the farther apart the points, the better the solution for the rotation angles. Regardless of how many more points we use, the remaining four parameters cannot be determined because we can shift, rotate and scale the two planes with respect to each other without changing their closeness. To solve for the two translation parameters x_t, y_t, two planes parallel to the coordinate planes are needed. To determine the scale, a fourth plane is needed, ideally parallel to one of the other planes. Note that points in vertical planes also solve for the κ-rotation.

In conclusion, the ideal configuration is a surface that consists of one horizontal plane and three vertical planes. Moreover, one pair of vertical planes should be perpendicular while the other pair should be parallel. The distances to be minimized between the two planes should be taken along surface normals. It follows that minimizing elevation differences does not work satisfactorily.

Fig. 2 depicts the synthetic surfaces that we have used in the experiments. Surface S_1 consists of the four surface patches $SP_1, \ldots, SP_4$. Surface S_2 is represented by 12 points, $\mathbf{q}_1, \ldots, \mathbf{q}_{12}$. As indicated in Fig. 2, in every surface patch of S_1, three points are selected which are then transformed into the reference system of S_2. Additionally, noise is added to these points. The task is to determine the transformation parameters by minimizing surface discrepancies between S_1 and S_2. In order to study the effect of the topography on the solution of the transformation parameters, surface patches SP_2, SP_3, SP_4 are tilted about different angles.

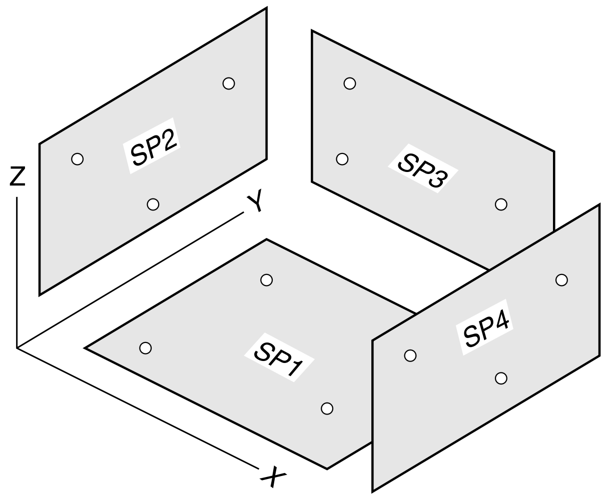

Figure 2: Configuration of the synthetically generated test surfaces. Surface S_1 consists of four surface patches $SP_1, \ldots, SP_4$, each represented by three points, resulting in 12 points $\mathbf{p}_1, \ldots, \mathbf{p}_{12}$. Different points were used for surface S_2. The points $\mathbf{q}_1, \ldots, \mathbf{q}_{12}$ were obtained by transforming them with varying transformation parameters.

Table 1 contains some of the results obtained from the experiments. The first column contains angle α about which surface patches SP_2 to SP_4 are rotated. The condition number (ratio of largest to smallest eigenvalue of the normalized normal equation matrix) in column 2 and 5 is an indicator for the geometric stability of the surface matching problem. Minimizing the surface differences along the $z-$axis is not an adequate mathematical model in situations with large tilts. In fact, if surface slopes of more than 60^o are reached, the first model is unable to recover the parameters any longer. In contrast, the second model gets stronger with increasing slope angles. The lowest condition number is reached for 90^o slopes, confirming our initial analysis presented in the beginning of this section.

The third and sixth column represent the average distance of the points $\mathbf{q}$ to the first surface. After the adjustment, the distance along the surface normal of all transformed points was computed and averaged (geometric mean). The distances in column 6 are just around the noise level. The numbers in column

Table 1: Results obtained with synthetic surface patches depicted in Fig. 2. Surface patches $SP2$, $SP3$ and $SP4$ were rotated about the principle coordinate axis by the angle α.

	math. model 1			math. model 2		
tilt angle α	condition number	distance	$\frac{\sigma_z}{\sigma_{x,y}}$	condition number	distance	$\frac{\sigma_z}{\sigma_{x,y}}$
0.5	155	0.07	$3.0 \cdot 10^{-5}$	119	0.046	2.6
1	152	0.08	$1.3 \cdot 10^{-4}$	116	0.042	2.4
5	141	0.09	$3.5 \cdot 10^{-3}$	114	0.041	2.1
10	141	0.09	$1.4 \cdot 10^{-2}$	113	0.035	2.1
20	148	0.09	$6.1 \cdot 10^{-2}$	108	0.032	2.0
30	165	0.08	0.14	101	0.033	1.9
50	278	0.11	0.31	80	0.035	1.5
70		divergence		61	0.037	0.8
90		singular		50	0.044	0.5

3 are slightly higher and increase with larger sloped surfaces. This is no surprise since the first model begins to deteriate with slope angles larger than 50^o.

The ratio $\sigma_z/\sigma_{x,y}$ is computed from the cofactors of the variance-covariance matrix. The numbers express the relationship in precision between the $z-$component of the translation vector and its planimetric component. It is interesting to note that the precision of the components is only balanced in situations with large slope angles.

4 CONCLUDING REMARKS

Comparing surfaces is a frequently occurring task and a prerequisite for merging data sets. In a general scenario the two sets of discrete points describe the same physical surface but they may differ in the density, distribution, and accuracy of points. Moreover, the reference systems for the two sets may also be different. We have presented a solution to the latter problem that is based on the assumption that there are no identical points in the two sets for establishing the transformation parameters. Two mathematical models were examined; the first model minimizes differences between the two surface descriptions along the $z-$axis while the second approach minimizes the differences along surface normals.

The experiments conducted with synthetic data revealed that the transformation parameters can be recovered even in cases of moderately sloped surfaces. To recover all 7 parameters, the surface must have slopes in all directions, however. With increasing slope angles, the second method performs better. Such situations may occur in urban areas where man-made objects are added to the topographic surface.

We have performed the experiments with a simplified adjustment model. The distance of a point in the second surface to the corresponding patch of the first surface is considered a pseudo observation. In this approach, errors are only modeled in the direction of the distance. We have also compared this approach with an adjustment of condition equations with parameters. This is a more realistic stochastic model because it considers the surface points as random observations. *Jaw, J.* (1999) treats all points in both surface sets as observations and includes also the effect of interpolating surface patches into the stochastic model.

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