THE ACCURACY OF USING THEODOLITE IN CLOSE-RANGE ENGINEERING MEASUREMENTS

Hüseyin Gazi BAŞ
University of Sakarya
Engineering Faculty
Department of Civil Engineering
Sakarya- TÜRKİYE

Working Group V

ABSTRACT

Digital theodolites have been increasingly used in engineering measurements and industrial applications recently. In these applications the accuracy of object space coordinates is a function of the object distance, the base, and the measured angles with theodolites. In this study, a mathematical formulation has been developed to estimate the accuracy of object space coordinates when a theodolite as a measuring tool and the parallax equations of normal case of photogrammetry are used to compute the object space coordinates. Some tables related to data acquisition have been given depending on the main parameters which affect the accuracy of coordinates.

1   INTRODUCTION

In the current measurement technology, digital theodolites have been increasingly used in close range engineering measurements and theodolite measurements heve become a part of industrial application. This technique, called "combine method", (Baş, 1993) is preferred to the close range photogrammetric technique used by Abdel-Aziz (1979). The basic features of the method consists of portability, non-contact, veriable geometry and automation. The simplest components of the system are a theodolite (traditional or digital) and calculator. A digital theodolite, a computer and suitable software are the best and most suitable components of the system.

The engineering and industrial measuring areas in which this method has been applied require results with positional accuracies to within fractions of a millemetre. In addition to the instrumentation and software features of the system, the other components of the system are very important for achieving this high level of accuracy. Briefly the accuracy of the object space coordinates is a function of the elements of the system which are *the theodolite used, pointing and centering the theodolite telescope to a targeted object point and the configuration of the control network.*

 The most important characteristics of the theodolite in achieving the desired accuracy are reading precision and sensitivity of the level bubbles. The resolution of the theodolite angular measuring units is the initial limiting factor and currently available theodolites have resolutions of 0.3 seconds of arc (0.1 mgon). The factor affecting the accuracy in the photogrammetric procedure such as limited format, image distorsion, and focal length are not present when a theodolite is used.

Tests have indicated that under good conditions it is possible to achieve sighting precision of 1 second of arc (0.3 mgon) with a single pointing of the theodolite (Grist, 1991). The configuration of the control network controls the geometry and the data acquisition of the method. The main geometrical factors which affect the accuracy are directions or angles measured with theodolite, object distance and the base. In addition to these factors, other secondary sources of errors such

as targets, stabilitiy of the theodolite stations, lighting, environmental conditions, the redundancy of the measurements, the skill of the operator, and the computing method also affect the accuracy of the object space coordinates.

In this study, a mathematical formulation is developed for estimating the acuracy of object space coordinates which can be achieved using a theodolite as a measuring tool and the parallax equations of the normal case of photogrammetry as the mathematical model.

2   GEOMETRY

The basic geometry of measuring with a theodolite is the same as that of an intersection. On the other hand such a theodolite can be assumed as a ficititious camera positioned on a station to take a photograph. The relationships between such a fictitious camera and a theodolite are as follows:

- The perspective center of the fictitious camera coincides with the points of intersection of the theodolite   horizontal and vertical axes.
- The optical axis of that camera is parallel to the plane of the theodolite horizontal circle and perpendicular to the base line between the two theodolite stations.
- The principal distance, f, of fictitious camera can be assumed equal to any value (Abdel-Aziz, 1979).

The respective x axes of theodolite images, which are parallel to the base line, are parallel, and the respective y axes of the theodolite images, which are parallel to the theodolite vertical axes at the theodolite stations, are parallel in this system. The angle between the optical axis f and the ray to object point i is the horizontal angle α_{ij}. The angle between the horizontal plane and the ray to object point i is the vertical angle of inclination β_{ij} . The zero direction of the theodolite horizontal circle is any direction when the theodolite is set up at the station. The relationship between theodolite image coordinates x_{ij}, y_{ij} and the horizontal direction α_{ij} and vertical direction β_{ij} is shown in Figure 1. These relationship can be expressed mathematically according to Figure 1, in general terms, in the following way:

$$x_{ij} = f.\tan\alpha_{ij}$$
(1a)

$$y_{ij} = f.\tan\beta_{ij} / \cos\alpha_{ij}$$
(1b)

where $\alpha_{ij} = (100 - \alpha_{0j} + \bar{\alpha}_{ij})$, α_{0j} and $\bar{\alpha}_{ij}$ are the horizontal direction of other theodolite station and horizontal direction to image i, and the subscripts i and j represent point number and theodolite station number, respectively. As shown in equations (1), the acuracy of theodolite image coordinates x_{ij} and y_{ij} is a function of horizontal and vertical angles measured with a theodolite. Applying the principles of error propagation to the above equations and assuming an errorless focal length, one gets estimated standart deviations, m_{xij} ,m_{yij} as a function of observed angles. In this error propagation x_i and y_i are assumed uncorrelated.

$$m_{xij} = f (1/\cos^2\alpha_{ij}) m_{\alpha ij}$$
(2a)

$$m_{yij} = f \{(1/\cos^2\beta_{ij})(1/\cos^2\alpha_{ij}) m_{\beta ij} + \tan\beta_{ij} (\sin \alpha_{ij} / \cos^2\alpha_{ij}) m_{\alpha ij} \}$$
(2b)

in which $m^2_{\alpha ij} = (m^2_{\bar{\alpha} ij} + m^2_{\alpha oj})^{1/2}$ where $m_{\bar{\alpha} ij}$ and $m_{\alpha oj}$ are estimates for standart deviations in directions ij and oj, and $m_{\beta ij}$ is an estimate for the standart deviation in the vertical angle β. Further, if one assumes that observations are generally carried out under the same conditions, with similiar instruments and by the same operator, $m_{\alpha ij}$ can be assumed equal to $m_{\beta ij}$, that is $m_{\alpha ij} = m_{\beta ij} = m_{ij}$. This assumpsion will be used all subsequent developments.
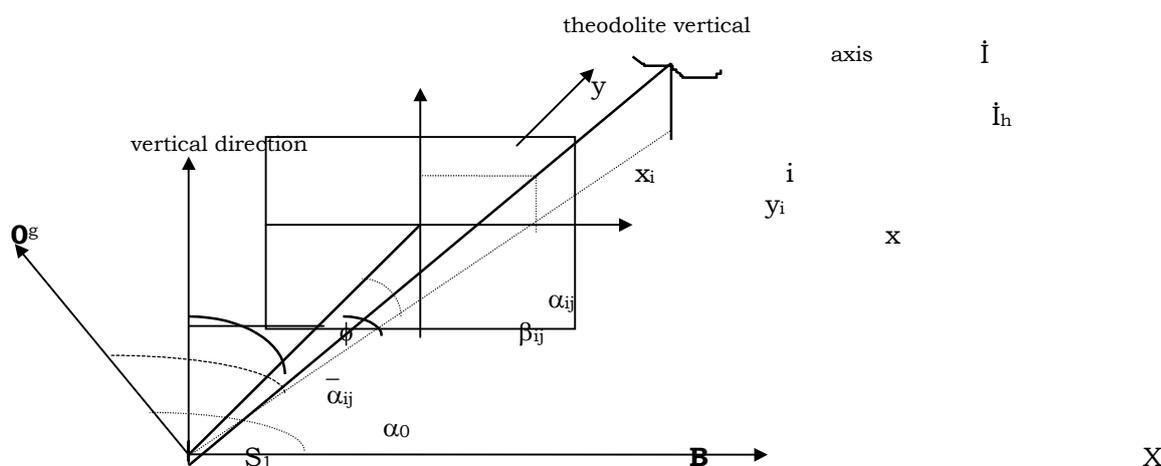

Figure 1. The relationship between the theodolite image coordinates and the horizontal and the vertical angles

3 MATHEMATICAL PRINCIPLES

In this measuring method, the left hand theodolite is assumed as fixed and the right hand theodolite is oriented with respect to the left one. The origin of the coordinate system is the station of the left theodolite. The X axis is pararllel to the direction of B, and the Y axis used in this study is a vertical line to the plumb bob direction at the origin. The Z axis is parallel to the optical axis of the fictitious camera. In this coordinate system, the X, Y, Z values of the origin are held fixed at zero and the rotations around axes are fixed at zero during the set up procedure. Object space coordinates are computed using the following equations obtained from the normal case of photogrammetry.

$$X_i = x_{i1} \cdot B / (x_{i1} - x_{i2}) \qquad Y_i = y_{i1} \cdot B / (x_{i1} - x_{i2}) \qquad Z_i = f \cdot B / (x_{i1} - x_{i2})$$

(3)

where B is the horizontal distance between the two theodolite stations and is assumed as an errorless value.

4 THE ACCURACY OF OBJECT SPACE COORDINATES

As shown in the above equations (3), object space coordinates are a function of the theodolite image coordinates x_i, y_i. To obtain the standart deviations of the object space coordinates m_X, m_Y, m_Z as a function of the image coordinates m_{xij}, m_{yij}, the error is propageted through equations (3);

$$m_{Xi} = B /(x_{i1} - x_{i2}) \cdot (-x_{i2} \cdot m_{xi1} + x_{i1} \cdot m_{xi2})$$

$$m_{Xi} = Z_i /f \cdot 1/(x_{i1} - x_{i2}) \cdot (x_{i1} \cdot m_{xi2} - x_{i2} \cdot m_{xi1})$$

substituting $(x_{i1} - x_{i2}) = b_i$, $x_{i2} = x_{i1} - b_i$, $x_i/b_i = X_i / B$ and $m_{xi1} = m_{xi2} = m_{xi}$ the above equation takes the form of

$$m_{Xi} = Z_i /f \cdot \{2(X_i / B)^2 - 2(X_i / B) + 1\}^{1/2} \cdot m_{xi}$$
(4)

m_{Yi} and m_{Zi} are also obtained in the same manner :

$$m_{Yi} = Z_i /f \{1/2 + 2(Y_i/B)^2 \}^{1/2} \cdot m_{yi}$$
(5)

$$m_{Zi} = Z_i^2 /(B \cdot f) \cdot \sqrt{2} \cdot m_{xi}$$
(6)

For the central point in the object space, assuming $X_i = B/2$ and $Y_i = 0$ (Marzan, 1976), equations (4), (5) and (6) become

$$m_{Xi} = 1/ \sqrt{2} \cdot (Z_i /f) \cdot m_{xi}$$
(7a)

$$m_{Yi} = 1/ \sqrt{2} \cdot (Z_i /f) \cdot m_{yi}$$
(7b)

$$m_{Zi} = \sqrt{2} \{ \cdot (Z_i /f) \cdot (Z_i /B) \} \cdot m_{xi}$$
(7c)

These equations are commonly used for normal case of photogrammetry to express the relationship between the accuracy of the object space coordinates given the base, object distance and the principals distance and can be found in most textbooks on elementary photogrammetry. Substituting (2) into (7) and assuming $m_{xij} = m_x$ and $m_{yij} = m_y$ and errorless f and B we obtain

$$m_X = 1/ \sqrt{2} \cdot Z \cdot (1/\cos^2\alpha) \cdot m$$
(8a)

$$m_Y = 1/ \sqrt{2} \cdot Z \cdot \{(1/\cos^2\beta) \cdot (1/\cos\alpha) + \tan\beta \cdot (\sin\alpha / \cos^2\alpha)\} \cdot m$$
(8b)

$$m_Z = \sqrt{2} \cdot (Z^2 /B) \cdot (1 / \cos^2\alpha) \cdot m$$
(8c)

Thus, the accuracies of the object space coordinates are obtained as a function of the basic parameters of the system such as object distance, base distance and the error of theodolite circle readings. In equations (8) Z represents the object distance. Analizing equations (8), one can note the following:

- The error in X changes by ratio $1 / \cos^2\alpha$. The normal case is obtained when angle α is equal to zero. The error in X increases with the increasing of α. The increase in α depends two factors : object distance Z and the base between two theodolite stations. If it is taken into account that the object distance does not change from point to point in close range measurements and that it is a generally small distance, the amount of α can be assumed to be directly related to the base.

- The error in Y is related to both horizontal and vertical angles. The error increases with the increase in these angles. Therefore small values of vertical angles should be preferred to reduce their effect on the error.

- The error in Z increases as the object distance and horizontal angle increase. This error decreases as the base increases.

The positional error M_P obtained from equations (8) is $M_P = (m^2_X + m^2_Y + m^2_Z)^{1/2}$ that is ,

$$M_P = Z / \cos\alpha\{(\tfrac{1}{2} \cos^2\alpha) + (1/(\sqrt{2} \cos^2\beta) + \tan\beta \tan\alpha)^2 + (\sqrt{2}\cdot Z/B (1/ \cos\alpha))^2\}^{1/2} \cdot m$$
(9)

Furthermore, if the geometry of intersection is taken into account and if $S_1I = S_2I$ (I is the intersected object point); $B/Z = 2\tan(\gamma/2)$ and $Z/B = \tfrac{1}{2}\cot((\gamma/2)$ are obtained (see Ghosh, 1979). Substituting (Z/B) into equation (9)

$$M_P = Z\cdot[1/\cos\alpha \{\tfrac{1}{2} \cos^2\alpha (1 + \tfrac{1}{2}\cot^2(\gamma/2)) + \tfrac{1}{2} \cos^2\beta (1 + \tfrac{1}{2} (\tan\beta \tan\alpha) + \tfrac{1}{2}(\tan^2\beta \cdot \tan^2\alpha)\}^{1/2}] \cdot m$$
(10)

is obtained. Equations (8), (9) and (10) can be considered as the general formulas expressing the object space coordinates errors as a function of the theodolite reading error (m), horizontal and vertical angles (α,β), errorless base (B) and the object distance (Z).

5   OPTIMUM CONFIGURATION

As mentioned earlier, the least coordinate and the least positional errors can be obtained by minimizing the object distance Z, maximizing the base (B) and minimizing α and β angles. However, as the base increases, the horizontal angle α also increases. Therefore a base length which can keep the values of α between 0–45 degrees should be preferred in close range measurements with theodolite. Furthermore α and β are not a single parameter for measuring system, the value of these angles change from point to point. Therefore to get an optimum configuration by using the partial derivations of function ($df/d\alpha=0$, $df/d\beta=0$) in close range measurement with theodolite is a difficult operation.

In this study equations (8), and (9) have been solved for every two meters of distances of object and the base, and for every five degrees of horizontal and vertical angles, and the theodolite reading error m have been assumed as 2 mgon. Some of the results are listed on Table 1.

Table 1.   Object Space Coordinates Errors and the Positional Error for Some Parameters

α^g	$\beta^{\,g}$	Z	B	m_X	m_Y	m_Z	m_P
0	5	2	2	4.44×10^{-6}	4.44×10^{-6}	8.89×10^{-6}	1.09×10^{-5}

5	5	4	4	8.94×10^{-6}	9.02×10^{-6}	1.79×10^{-6}	2.19×10^{-5}
10	15	6	6	1.37×10^{-5}	1.42×10^{-5}	2.73×10^{-5}	3.37×10^{-5}
15	25	10	10	2.35×10^{-5}	2.70×10^{-5}	4.70×10^{-5}	5.91×10^{-5}
20	35	14	14	3.44×10^{-5}	4.66×10^{-5}	6.88×10^{-5}	8.99×10^{-5}
30	30	12	12	3.36×10^{-5}	4.14×10^{-5}	6.72×10^{-5}	8.57×10^{-5}
40	25	10	10	3.39×10^{-5}	3.68×10^{-5}	6.79×10^{-5}	8.43×10^{-5}
50	40	16	16	7.11×10^{-5}	9.99×10^{-5}	1.42×10^{-4}	1.88×10^{-4}
60	20	8	8	5.14×10^{-5}	4.20×10^{-5}	1.03×10^{-4}	1.22×10^{-4}
70	30	12	12	1.29×10^{-4}	1.17×10^{-4}	2.59×10^{-4}	3.13×10^{-4}
85	25	10	10	4.08×10^{-4}	2.34×10^{-4}	8.15×10^{-4}	9.41×10^{-4}

The results of the calculations have been shown that while the increase in m_X, m_Y and m_Z is very slow for the different values of α and β between 0-45 degrees, this increase is fast for the values between 45-90 degrees (see Table 2). If it is taken into account that the change in the object distance from point to point is negligible in close range measurement, for the same system, the errors in the object space coordinates depend on angles α and β. Base-Object distance ratio is an important criterion in terrestrial photogrammetry and the convergent case is preferred to the normal case of photogrammetry to increase the base length in the analytical method. Because the horizontal angle will increase with a longer base length and a shorter object distance, an optimal base-object distance ratio should be considered (for example B/Z=2/1) in close range measurements with theodolite.

Table 2. Object Space Coordinates Errors for the same system (Z=6.0 meter, constant)

α	Z	m_X	m_Y	m_Z
0	6	1.33×10^{-5}	1.37×10^{-5}	2.67×10^{-5}

10	6	1.37×10^{-5}	1.42×10^{-5}	2.73×10^{-5}
20	6	1.47×10^{-5}	1.51×10^{-5}	2.95×10^{-5}
30	6	1.68×10^{-5}	1.65×10^{-5}	3.36×10^{-5}
40	6	2.04×10^{-5}	1.88×10^{-5}	4.07×10^{-5}
50	6	2.67×10^{-5}	2.23×10^{-5}	5.33×10^{-5}
60	6	3.86×10^{-5}	2.82×10^{-5}	7.72×10^{-5}
70	6	6.47×10^{-5}	3.92×10^{-5}	1.29×10^{-5}
80	6	1.40×10^{-5}	6.52×10^{-5}	2.79×10^{-5}
90	6	5.45×10^{-5}	1.72×10^{-5}	1.09×10^{-5}

6   CONCLUSION

In close range engineering measurements with a theodolite the accuracy of object space coordinates is a function of the object distance, the base, the measured angles with theodolite and the theodolite reading error. Equations (8), (9) and (11) can be used to calculat the estimated object space coordinate errors for any configuration when the object distance, the base, the approximate angles of the points and theodolite reading error are given. In addition, the equations can be used to determine the base and object distance and to estimate the maximum magnitudes of horizontal and vertical angles that should be used in data acquisition in order to attain a certain required accuracy. Table 1 can also be used with the same aims.

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