
THE CARTOGRAPHIC PROJECTIONS FOR THE REPRESENTATION OF DOUBLE CURVED SURFACES

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ABSTRACT

As is well known, the projection on the plane of a double curved surface is possible only by introducing warping techniques, be these either angular or superficial: the simple execution operation on the plane is not possible. However, these representations are of great interest, not only in the sphere of cartography but also in architecture, for the survey of vaults and domes.

This article presents the testing done by applying the usual cartographic projections with the use of specific software, such as Matlab, usually used for applications of a cartographic nature. The intent is to understand which representations to choose, in other words, which warping to minimise, in relation to the use that must then be made of these representations.

The survey of such surfaces has changed significantly in the recent years, with the advent of new motorised topographical instruments equipped with laser distance measuring equipment (diastimeters), which allow for the automatic survey for polar coordinates of inaccessible points; in addition, the opportunity of using digital images, or those transformed into digital, facilitates the processing and the transformation of these.

Here we have a possible survey methodology which has the objective of representing double curved surfaces such as vaults and domes on the plane. This includes the creation of a software for the automatic management of topographic instruments such as the MDL through the use of a lap top computer, the handling of the measurements, the realisation of a geometric model of the dome beginning with the set of points measured and the projection of the photogrammetric settings on the three dimensional model.

THE APPLICATION

The application executed concerns the survey of the niche (semi-dome) found on the door of S.Alipio, inside the St. Mark's Basilica in Venice; the intrados is decorated with a mosaic representing the body of Saint Marco brought into the basilica, which is a representation of significant importance since it also includes a design of the façade of the basilica itself in the background (*fig.1*).

THE SURVEY PHASE

The calculation of the coordinates of the points which serve to orient the photograms and those which will be utilised as a basis for the calculation of the approximate surface have been realised with the new generation of topographic instruments.

The topographic methods have undergone great development in this period, associated with the introduction of diastimeters using waves which do not require a reflecting station. These diastimeters are always combined with electronic theodolites so as to allow for the survey of polar co-ordinates of inaccessible points. The survey for polar co-ordinates, while on the one hand do not allow for any control over the surveyed points (there are three observations for

three unknowns and it is necessary to resolve an isodeterminate system), on the other hand, they do allow for a much easier execution of the survey.



Fig.1. Mosaic of the dome of S.Alipio's door

For this application, the MDL was utilised, a motorised instrument with laser diastimeter capable of measuring 3600 points for 1 hour, a sort of three-dimensional scanner; usually this instrument is employed in the survey of caves, where the measurement of a large number of points positioned, in the large part of the cases, on planes which along general lines can be defined as vertical. In these cases, the definition of the scanning plane is simple since it is sufficient to measure the two opposing vertices (top left and bottom right) of the plane itself.

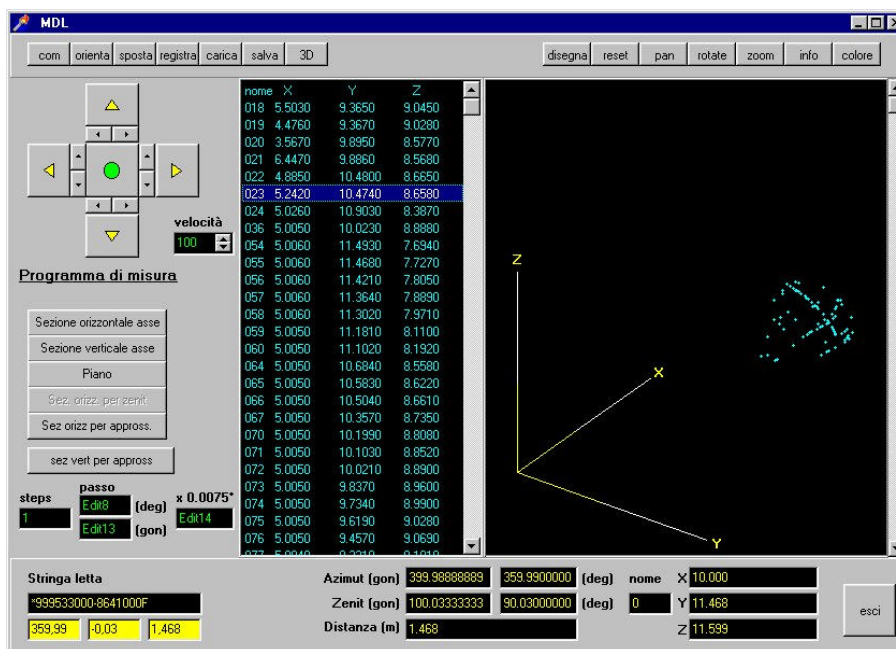


Fig. 2. Instrument's control software

In the case of a geometrically complex surface such as the one in a dome, the definition of the scanning plane is more complicated, unless it the survey accepts that he must measure a number of points much greater than those strictly necessary. For this reason, we have proceeded with the processing of a software which, through the use of a lap top computer, handles the instrument with more versatility (*fig.2*). Through the use of this software, it is now possible to define a series of scanning planes which pass either through the principle axes of the instrument or which are defined by the operator/surveyor, who can also determine the scanning frequency and the angular range up within which the measurements can be taken. On the screen of the lap top, the coordinates of the points, inside a table, as well as their position, inside a 3D visualisation window, can be visualised simultaneously during the measurement phase. These functions are greatly useful, allowing as early as the “campaign phase” to understand if the measurements done are sufficient for the successive laboratory processing or if integration procedures are needed in order not to leave any area of the object under examination undefined.

But what appears to be truly innovative is the opportunity that these instruments, which on their own are endowed with great potential, offer when combined with a computer. A very elevated level of automation and control in the survey can be reached.

This use of the MDL has allowed for the facilitated survey of a large number of points, uniformly distributed on the entire surface of the dome; this distribution has a determined influence on the definition of the geometric model of the surfaces.

DETERMINATION OF THE GEOMETRY OF THE OBJECT

To create the geometric model of the domes, it is possible to:

- create a model of finished elements, constructing a DEM of the dome with a triangular or quadrangular mesh, that is, keeping the surveyed points fixed, and interpolating them according to a regular mesh;
- search for a mathematical surface, which interpolates the set of points, in other words, acts as in a geodesy where the sphere and the ellipsoid are taken as reference surfaces;
- construct the spline, a solution which is halfway between the previous two; the surface is defined, beginning with the set of surveyed points, by equations without having first made hypotheses on the form of the object .

Very often, the domes have the circumference or the ellipse as the generatrix and this is why the surface which interpolates the points in the best way is that of the sphere or the ellipsoid. This hypothesis is usually supported by an understanding of the construction techniques used in the time period and by the treatment of the architecture; beginning with these hypotheses, the second research method for the geometric model was followed; it is necessary however to underline that as in this case, we don't have the possibility of describing the possible anomalies and the shifting from the ideal hypothesised geometry (*fig.3, 4*).

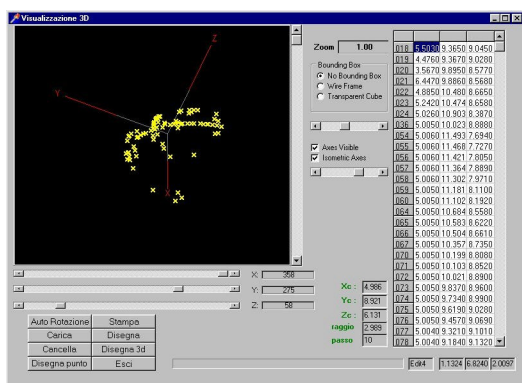


Fig. 3. Set point on the dome

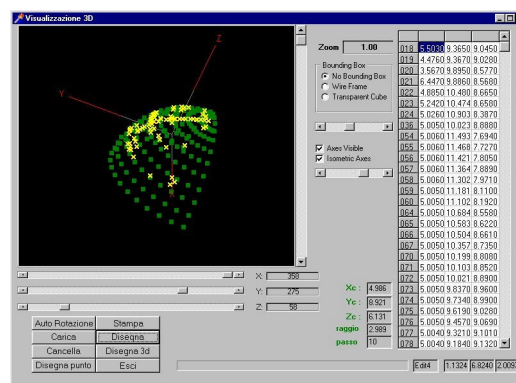


Fig. 4. Superimposition of the points on the geometric model

The realisation of a specific software has allowed us to evaluate how much the surface of the dome at S.Alipio distances itself from that of a sphere (more precisely, a quarter of a sphere); the calculation uses the method of compensation of minimum squares, beginning with the surveyed points and the equation $x^2 + y^2 + z^2 = R^2$ of the sphere; the analysis of the remainders has effectively highlighted how the surface of the dome is similar to the surface of a quarter of a sphere. The software provides the value of the R radius as an output and the Xc,Yc,Zc coordinates of the centre of the sphere (*fig.5*).

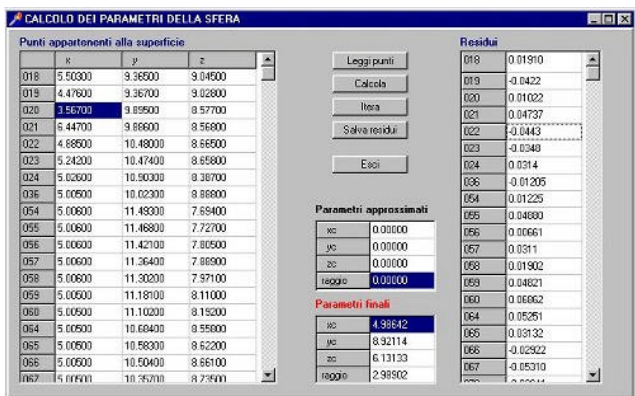


Fig. 5 Sphere's parameters

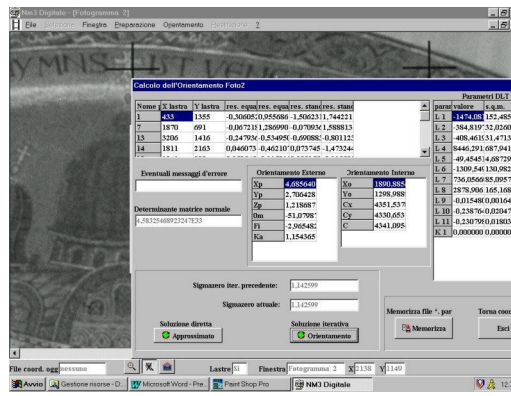


Fig. 6. DLT parameters

THE SETTINGS

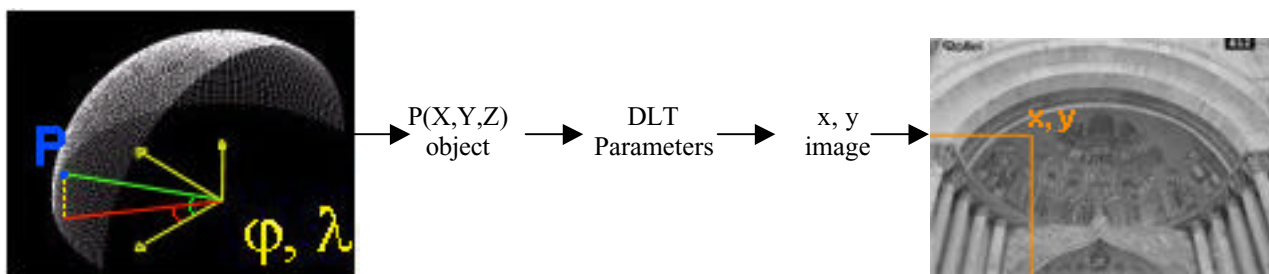
For the settings, a semi-metric Rollei 35 mm photographic camera was used; the photograms have been made digital and the orientation was carried out with a program (NM3) which uses the equations of the DLT (fig. 6). For the orientation of the single photograms, a series of support points have been surveyed, which correspond to recognisable points of the mosaic which entirely covers the surface under examination. It is important that these points belong to the same reference system in which the model is defined.

REPRESENTATION ON THE PLANE

The representation on a plane of the surfaces of the domes, on which there may be frescoes or mosaics present, cannot be carried out unless one resorts of projective operations or analytical transformations. The domes are objects whose surface is a double curve and for which it is not possible to carry out the simple operation on a plane. To do this operation, it is natural and logical to make reference to the cartography that has traditionally dealt with the representation on the plane of double curved surfaces: spheres and ellipsoids. Traditionally, the cartographic representation methods are distinguished in pure projections and analytical transformations; today, considering that the use of the computer requires analytical formulation even for pure projections, the two cases are uniformly treated, defining them commonly with the name "representations". In the representation of the dome, the procedure almost always resorted to perspective projections, avoiding the polycentric projections which would break the continuity of the surfaces. In particular, the polar stereographic projections have been adopted, which are conform, and the azimuthal scenography.

- The approach proposed begins with the following assumptions:
- it is necessary to obtain a digital image which is in a given cartographic projection;
 - the digital photographic setting is transformed directly in the desired projection, without "passing" through the orthographic projection.

For this application as well, a software has been created which associates the spherical coordinates individuating the points of the vault with the pixels of the oriented digital image. The algorithm creates an empty image (matrix) whose coordinates φ, λ are the spherical coordinates of the model. For every φ, λ , the corresponding Cartesian X,Y,Z triad is calculated, keeping in mind the translation of the sphere (X_c, Y_c, X_c). Note that the parameters L1,...,L11 of the DLT which come from the corresponding coordinates of the image x_i, y_i to which a radiometric value of the pixel corresponds : this value goes to fill the empty matrix.



In the projection on the plane phase, the “Mapping Toolbox” module by MatLab was used, which allows for the executing beginning with the raster image created by a desired cartographic projection. This image is none other than a matrix having $\varphi \times \lambda$ dimensions whose contents is given by the value of the radiance of the pixels. Beginning with the matrix, it is therefore easy to project the surface of the dome on the plane, choosing the cartographic projection which best is adapted to this purpose (fig.7-8).

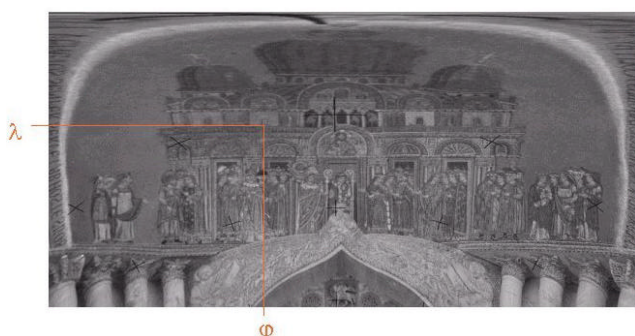
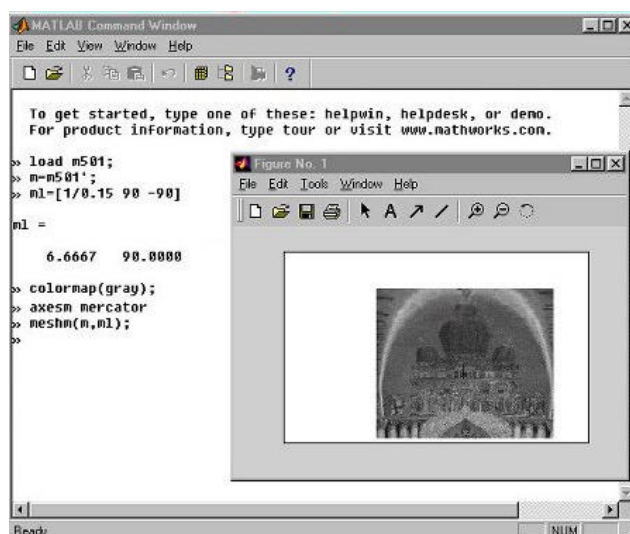


Fig.7. Resampled image: to each couple of coordinates φ, λ corresponds a pixel in the source image

Fig.8. Develop of the surface in a cartographic projection using MatLab.



The possibility of choosing a given projection is motivated by the use that must be made of the survey: for a study of representational methods, the projections are chosen which emphasise the generatrix characteristics of the figures represented on the dome, for a material restoration, the most suited projections are those which allow for an evaluation of the phenomena associated with the quantification of the areas. In fact, the development on the plane of a double curved surface is obtained only by admitting any deformation: it is up to operator to decide if this deformation must involve the areas or the corners, or both, while always striving to minimise as much as possible the deformations themselves (fig. 9-10-11-12-13-14).

For this application, a series of projections has been chosen, usually used in cartography and which each have the characteristic to minimise one of the deformations described; in this case it was opportune to emphasise how, once the matrix containing the matrix containing the “referenced” radiances of the pixels of the beginning image was defined, with minimal effort, it was possible to obtain a wide series of representations on the plane each of which has different characteristics.

The results obtained are of significant importance in the field of restoration where the need arises to graph the results of many analysis which are carried out within sight of the project.

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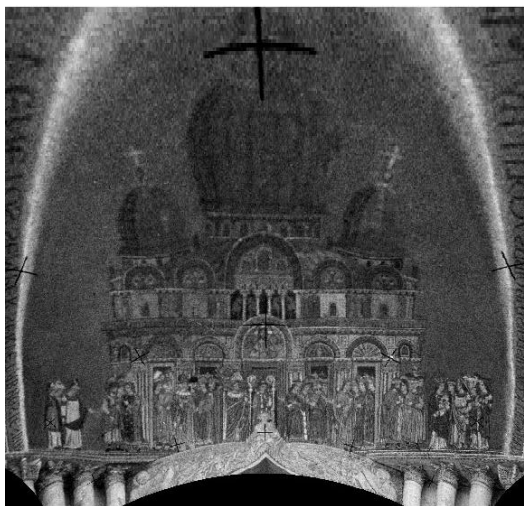


Fig.9. Mercators projection, conformal cylindrical



Fig.10. Stereographic projection, conformal azimuthal



Fig.11. Equidistant conic projection



Fig.12. Werner projection, pseudoconic, equal-area

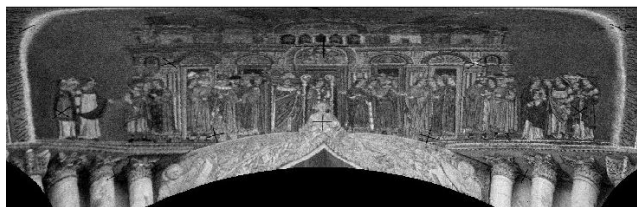


Fig.13. Lamber projection, cylindrical equal-area



Fig.14. Orthographic projection, azimuthal