Linear Feature Based Aerial Triangulation

A. Akav, G. H. Zalmanson and Y. Doytsher

Department of Transportation and Geo-Information Engineering
Faculty of Civil and Environmental Engineering
Technion – Israel institute of technology
Technion City, Haifa 32000, Israel
(akav, garry, doytsher)@tx.technion.ac.il

KEY WORDS: feature based photogrammetry, orientation parameter estimation, registration, homography, fundamental matrix

ABSTRACT:

For the past fifteen years line photogrammetry has been an extremely active area of research in the photogrammetry and computer vision communities. It differs from traditional analytical photogrammetry in the nature of the primitives employed in a variety of its fundamental tasks. While in traditional photogrammetry zero-dimensional entities, i.e., points are exclusively used as a driving power in its various orientation and exploitation procedures – in line photogrammetry as the name suggests linear, that is one-dimensional features often corresponding to elongated man-made features in the object space are employed. Of course, that means that no prior correspondence between distinct point in object space and its projection in the image is required and the entire linear feature (with arbitrary geometry) is accommodated in the appropriate mathematical model. However, despite a great effort in that field, only the resection problem, i.e. the solution of the exterior orientation from linear features' correspondences has been thoroughly investigated so far. Two additional fundamental photogrammetric problems - space intersection and relative orientation, completing a triple of the most basic photogrammetic procedures needed to support feature-based triangulation have not been adequately addressed in the literature. This paper provides that missing link by presenting a procedure for relative orientation parameters estimation from linear features. We restrict our attention in this paper to planer curves only. We start with the simple idea of optimization procedure using ICP algorithm and proceed to the recovery of the homography matrix induced by the plane of the curve in space.

1. INTRODUCTION

Line Photogrammetry (LP) has been a tremendously active field of research for almost two decades. Over these years many researchers have argued in favor of accommodating linear features instead of points for different photogrammetric tasks. Some of their central arguments are set forth as follows:
1. In many typical scenarios, linear features can be detected more reliably than point features (Mikhail, 1993). 2. Images of urban and man made environment are rich of linear features (Habib, 2001). 3. Close range applications employed in industrial metrology often lack an adequate amount of natural point features, thus requiring a costly use of artificial marks for automating the involved mensuration tasks. (Kubik, 1989) 4. Matching linear features is easier and more reliable than matching point features (Zalmanson, 2000).
This paper presents possible solutions for the classic problem of determining the relative orientation parameters. The procedures proposed here are based on using free form 3-D planar curves instead of conjugate points.
In the resent years we are witnessing the entrance of more and more digital photogrammetry workstations. Developing automatic processes for photogrammetric applications has attracted a large body of research in the photogrammetry and computer vision communities. The natural step towards automatic aerial triangulation would be to adopt higher level entities for determining orientation parameters. Autonomous solutions for relative orientation with linear features employing Hough search techniques have been proposed by Habib (2003, 2001). Solutions for relative orientation using a subclass of linear features, namely, planar curves and conic sections have been introduced by Shashua (2000) who dealt with 3^{rd} degree algebraic planar curves, Ma (1993) who used planar conics and Petsa (2000) who worked with straight coplanar lines.

2. USING PLANAR FREE FORM CURVES

We represent free form curves in image space by a sequence of 2-D points. Trying to represent such curves in polynomial or parametric form would yield a more simplified mathematical modeling but at the same time would result in some loss of information due to inherent generalization process being involved.
The procedures shown here are valid for planar curves. We start with the simple idea of recovering the relative orientation parameters from free form planar curves. Every planar curve adds 3 parameters to the overall solution. The redundancy and the minimum number of planar curves needed for recovering the R.O.P will be discussed later.

2.1 Simplest idea

First we have to determine initial parameters. Since we try to recover the relative orientation, we can refer to the model space as the object space in exterior orientation. Initial parameters for the relative orientation can be determined in the classical way, assuming aerial photos, most likely near-vertical and highly correlated. Dependent relative orientation model had been chosen, which defines the model coordinate system parallel to the first (left) image's coordinate system. As for the plane in the model space, horizontal plane can be used to determine initial parameters.
After determining all initial parameters needed, one can project the curve from both images to the plane in the object/model space and get the intersections of the surfaces created by the

correspondences curves from the images with the plane. Those intersections must be identical to get the full overall solution. When applying this procedure with initial parameters we will get two separated curves. Now in order to bring the two curves closer ICP algorithm is proposed.

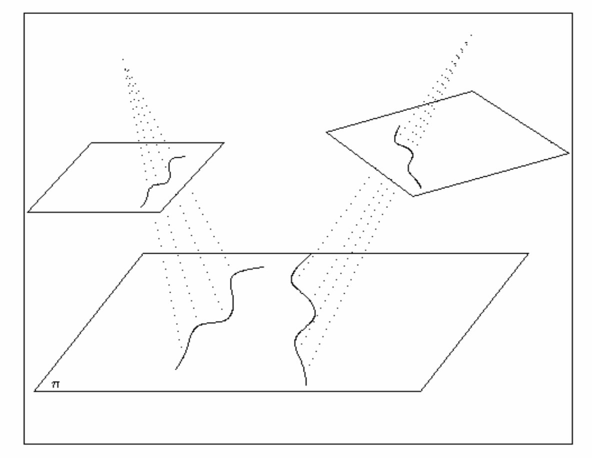

Figure 1. Projection of correspondences curves

2.2 ICP – iterative closest point

The ICP algorithm, first introduced in (Besl and McKay, 1992) can be used with several representations of geometric datasets, such as point sets, line segments set, implicit curves, surfaces etc. The geometric datasets used in this paper are point sets, representing the free form curves. The datasets we deal with are the projected curve from the first (left) image and the second (right) image of the photogrammetric model. As mentioned before the data sets are point sequences, Xl and Xr for left and right projected curves respectively.

ICP steps:

1. Compute the closest points: Yk=C(Pk,X) where C is an operator for finding the closest point between P and X.
2. Compute the registration to minimize the sum of square distance between the closest points found. (qk,dk)=Q(Po,Yk).
3. Apply the registration: P_{k+1}=qk (Po).
4. Stop the iteration when the change mean squares error small then threshold.

with X being the model shape, P the data shape and Y representing the closest points found. In our case there is no model shape and data shape, both shapes are data changing with the refinement of the parameters (R.O.P + plane parameters).

Point sequences are obtained by computing the planar curves as intersections of cones having the perspective center of the images as their origin with image space curves and the plane.

The plane equation is represented by 3 parameters for instance: ax+by+cz=1. Any point from the point sequence could be computed by multiplying the vector, starting with the perspective center through the point from image plane, by scale factor. The scale factor can easily found using plane parameters. Full description of ICP algorithm can be seen in Besl and McKay work(1992).

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \frac{1}{\begin{bmatrix} a & b & c \end{bmatrix} \cdot R \cdot \begin{Bmatrix} x_i \\ y_i \\ -f \end{Bmatrix}} \cdot R \cdot \begin{Bmatrix} x_i \\ y_i \\ -f \end{Bmatrix} + T \qquad (1)$$

where R = rotation matrix
T= displacement vector
f = focal length

Projecting all the points in both images through equation (1) yields the planar curves shown in figure 1. The Euclidean distance d(xp,l) between the point xp and the line segment l is computed using equation (2) where x1 and x2 are points determining the line segment l.

$$d = \frac{\left|(x_2 - x_1) \times (x_1 - x_p)\right|}{\left|x_2 - x_1\right|}$$

$$d = \frac{\left|(x_2 - x_1)(y_1 - y_p) - (x_1 - x_p)(y_2 - y_1)\right|}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \qquad (2)$$

with the first equation being a 3-D vector equation with 3-D point vectors x1, x2 and xp, and with the second equation corresponding to 2D image coordinates.

The closest point xo is the point satisfies the equality d (xp, xo) =min d (xp, li) {i=1...n}. With the resultant corresponding point sets the registration is computed using least squares optimization.

2.3 Experiments with synthetic curves

Synthetic planar curves were projected from model/object space to the images using relative orientation parameters as exterior ones. ICP algorithm was performed for the recovery of the relative parameters of the photogrammetric model and the plane parameters. High sensitivity has been observed to initial values of the 3D plane parameters.

The primary reason for these unsatisfying results and the numerical problems confronted is a possible high correlation between the plane parameters and the bending angles of the model images. The plane representation could lead to numerical problems because the multipliers of X and Y are nearly negligible compared to Z multiplier. We therefore suggest a different representation for the plane.

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix}$$

$$n_1(X - Xo) + n_2(Y - Yo) + n_3(Z - Zo) = 0 \qquad (3)$$

$$n_1 X + n_2 Y + n_3 Z = D$$

where : θ-angle from XY plane
φ -turning angle around Z axis
n – unit vector of plane normal
D – the distance of the plane from origin

The last representation changes the sought parameters to θ, φ and D, but leads to another problem. If the plane is horizontal so θ = 0, the derivative according to φ is infinite, because it does not change the normal vector when multiplying with 0. Therefore, when dealing with planes close to horizontal, numeric problems are expected.

3. FINDING FUNDAMENTAL MATRIX USING HOMOGRAPHY MAPPING

3.1 Homography mapping

Homography mapping transfer points from one image to the second image as if they were on the plane in the object space (Hartley 2000). As seen in figure 2. points on a plane are related to correspondences point on the images of the photogrammetric model. In fact this is a projective, having 8 free parameters. The 8 parameters can be obtained from the 5 relative orientation parameters and the 3 planar parameters.

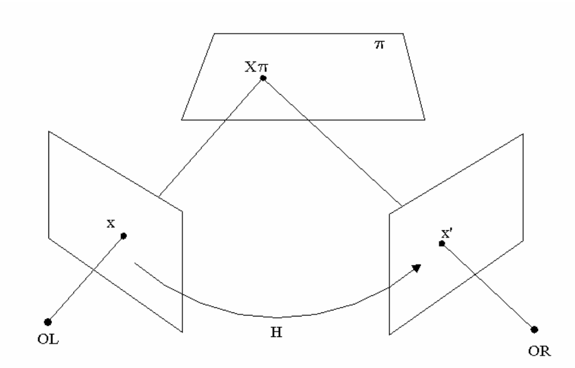

Figure 2 . homography mapping

The homography induced by the plane is unique (see Tsai (1982)), meaning that every planar curve can contribute one homography. The homography transfer operator is linear for homogenous coordinates and the mapping from one image to the other is unique up to a scale factor.
The homography matrix:

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{bmatrix} \qquad (4)$$

and the mapping from right to left image vectors are readily given by

$$U_l = H \cdot U_r \qquad (5)$$

where Ur, and Ul are homogenous coordinates in right and left images respectively.

One should notice that the mapping can be from the right image to the left image and vice versa. In this paper we have chosen the one from right image to the left image.

The homography matrix can be computed directly from the relative orientation parameters and the plane parameters. The common way to compute the homography matrix is by determining the coordinate system of the model parallel to the coordinate system of the left image. By choosing this coordinate system the rotation matrix of the left image is the identity matrix and the translation vector is zero. The rotation matrix and the translation vector of the right image can be obtained from the relative orientation parameters.
The homography matrix can be computed with rotation matrix R and the displacement vector T according to equation (6).

$$H = [R + T \cdot n' / D] \quad (6)$$

where : R – rotation matrix
T - displacement vector
n – unit vector of the plane normal
D – distance from the origin

The homogenous coordinate are obtained by dividing the image coordinate by focal length as follows:

$$U = \frac{1}{-f} \cdot \begin{bmatrix} x \\ y \\ -f \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad (7)$$

Using homogenous coordinate makes the homography mapping correct up to scale. Multiplying the homogenous coordinate with the homography matrix H is simply linear procedure, but getting the right scale requires a determination of a scale factor. Dividing the outcome vector by the 3rd coordinate can answer this question so the transformation from one image to the other can be written as follows:

$$\begin{aligned} x' &= \frac{h_1 \cdot x'' + h_2 \cdot y'' + h_3}{h_7 \cdot x'' + h_8 \cdot y'' + 1} \\ y' &= \frac{h_4 \cdot x'' + h_5 \cdot y'' + h_6}{h_7 \cdot x'' + h_8 \cdot y'' + 1} \end{aligned} \qquad (8)$$

where: x", y" – right image coordinate
x' y' – left image coordinate
h1 . . h8 component of the homography matrix

Equations (8) remind the collinear equations, but one should remember that the collinear equations transform from 3D object space to 2D image space while those equations transform from 2D image space to another 2D image space.

3.2 Fundamental matrix

Yet another well-known relation between two images is the epipolar geometry. One point from first image determines line in the second image. The fundamental matrix defines this relation with the constrain $x''^t F x'' = 0$, obeying the coplanar condition. Using the homography matrix we can write the constrain $(Hx'')^t F x'' = x^t H^t F x = 0$ for any point on the plane that induced the specific homography. Hence, the matrix (Ht F) must be skew-symmetric, namely

$$H^t F + F^t H = 0 \qquad (9)$$

Note that equation (9) shows that in fact two homography matrices provide sufficient set of linear equations for the fundamental matrix.

3.3 Finding the homography matrix using ICP

In this section we describe procedure to find the homography matrices of free form planar curves using ICP algorithm. This time the ICP algorithm is carried out in image plane. The chosen image is the first (left) image of the photogrammetric model. Every curve in the right image that was digitized and matched to curve from the left image is transformed to the left image using initial homography matrix. The initial component of the homography can be computed using regular assumption for initial relative orientation parameter and applying equation (6). One possible value set for initial components of H is simply:

Ho = {1,0,0,0,1,0,0,0,1}. After we determine the initial component of the homography matrix we transform the curve from right image plane to the left image plane and compute the closest points between the two corresponds curves. The registration is computed using least square adjustment and Levenberg-Marquardt (Fitzgibbon 2001) algorithm is tested as well.

The proposed algorithm is sketched as follows:

- find the corresponding curves in both images
- set the initial homography matrix to H={1,0,0,0,1,0,0,0,1}.
- Transform the curve from right image to left image using the initial matrix.
- Find the closest points between the curves.
- Compute the design matrix A and the error vector **e** for the current components of the homography matrix.
- Compute the update vector **x** for the current components.
- Update the homography matrix H = H + x
- Continue to transfer the curve and update the homography matrix until maximum distance between closest points smaller than tolerance.

Compared to the algorithm presented in the previous section, the current procedure reduces the dimension of the problem from 3D to 2D. In addition, here, only one transformation from right image plane to left image plane is required.

3.4 Design matrix for registration

The error is actually the distance computed between the closest points obtained. The optimal situation is that all the distances are zero. The design matrix is computed by the taking derivatives of the distance function with respect to any component of the homography matrix. The distance is computed between every point that was transformed from right to left image and the closest point to it from the correspond curve. When the curve is a free form curve then the distance is actually computed between the point and the closest segment to the point. The segment from the left image curve is for now considered as fixed.

The derivative of equation (2) for xp and yp (the transformed point from right image) is:

$$\begin{aligned}\frac{\partial d}{\partial xp} &= \frac{(y_2-y_1)}{L_{12}}\\ \frac{\partial d}{\partial yp} &= -\frac{(x_2-x_1)}{L_{12}}\end{aligned} \qquad (10)$$

where: L_{12} is the distance between points 1,2 of the closest segment from left image curve.

Now for the derivatives of equation (8) for h1..h8 we rewrite equation (8) as:

$$\begin{aligned}xp &= \frac{Nx}{D}\\ yp &= \frac{Ny}{D}\end{aligned} \qquad (11)$$

and:

$$\begin{aligned}\frac{\partial xp}{\partial h_i} &= \frac{\partial xp}{\partial Ny}\cdot\frac{\partial Ny}{\partial h_i}+\frac{\partial xp}{\partial D}\cdot\frac{\partial D}{\partial h_i}\\ \frac{\partial yp}{\partial h_i} &= \frac{\partial yp}{\partial Ny}\cdot\frac{\partial Ny}{\partial h_i}+\frac{\partial yp}{\partial D}\cdot\frac{\partial D}{\partial h_i}\end{aligned} \qquad (12)$$

so :

$$\begin{aligned}\frac{\partial d}{\partial h_1} &= \frac{y_2-y_1}{L_{12}}\cdot\frac{-f}{D}\cdot xr\\ \frac{\partial d}{\partial h_2} &= \frac{y_2-y_1}{L_{12}}\cdot\frac{-f}{D}\cdot yr\\ \frac{\partial d}{\partial h_3} &= \frac{y_2-y_1}{L_{12}}\cdot\frac{-f}{D}\cdot(-f)\\ \frac{\partial d}{\partial h_4} &= -\frac{x_2-x_1}{L_{12}}\cdot\frac{-f}{D}\cdot x\\ \frac{\partial d}{\partial h_5} &= -\frac{x_2-x_1}{L_{12}}\cdot\frac{-f}{D}\cdot yr\\ \frac{\partial d}{\partial h_6} &= -\frac{x_2-x_1}{L_{12}}\cdot\frac{-f}{D}\cdot(-f)\\ \frac{\partial d}{\partial h_7} &= -\frac{y_2-y_1}{L_{12}}\cdot\frac{-f\cdot Nx}{D^2}\cdot xr+\frac{x_2-x_1}{L_{12}}\cdot\frac{-f\cdot Ny}{D^2}\cdot xr\\ \frac{\partial d}{\partial h_8} &= -\frac{y_2-y_1}{L_{12}}\cdot\frac{-f\cdot Nx}{D^2}\cdot yr+\frac{x_2-x_1}{L_{12}}\cdot\frac{-f\cdot Ny}{D^2}\cdot yr\end{aligned} \qquad (13)$$

where: xr, yr are the right image coordinates of the specific point.

3.5 Experiments

The proposed procedure has been tested on synthetic data showing good converging between the curves as shown in figure 6. The initial homography matrix component for this experiment was H={1,0,90/f,0,1,0,0,0,1} where f is the focal length and the value 90 is given to keep the model scale close to the image scale. Figure 4 describes the synthetic images of the curves. For example, one of the curves is transformed from right image to the left one using initial values for the homography matrix. After two iteration of ICP algorithm we get close to the original curve as can be seen in figure 6. Figure 6 shows enlargement of a part from the specific curve where the points indicated by 'o' are the points transferred from right image using the computed homography matrix.

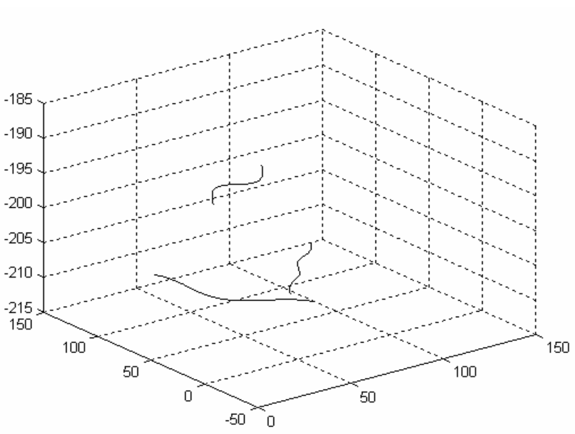

Figure 3. 3D model space

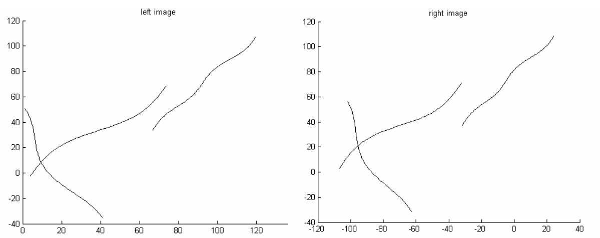

Figure 4. left and right images

In figure 5 we can see the curve transformed from the right image using initial homography matrix.

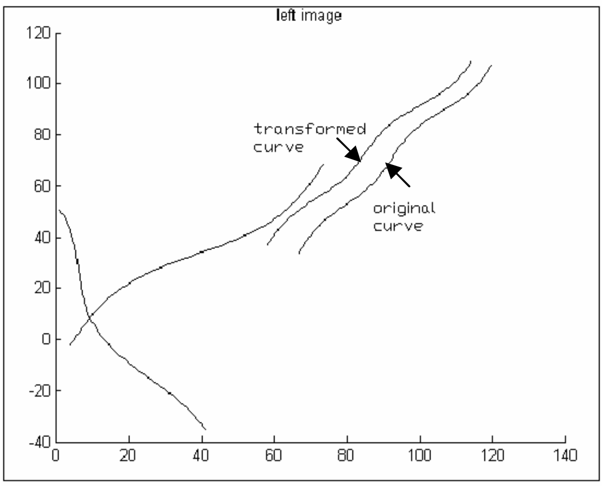

Figure 5. Curve transformed with initial values

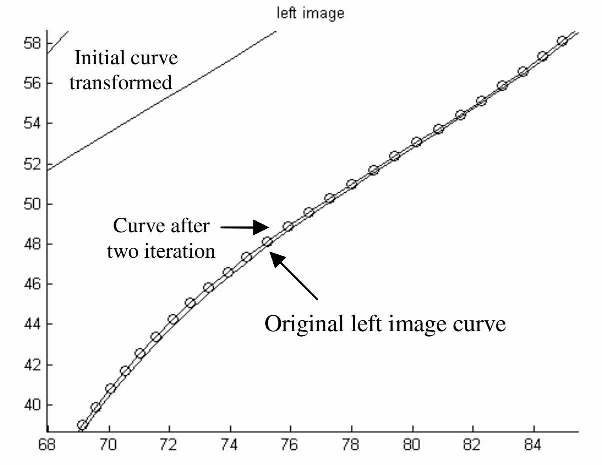

Figure 6. Enlargement of the converged curve

Further to the advantages of the homography-based algorithm mentioned above, i.e., fewer transformations required and a reduction of the problem dimension from 3-D to 2-D, probably the most significant one pertains to its insensitivity to the plane parameters associated with our planar curves. This unlike the algorithm presented in section 2 being subject to singularities associated with nearly horizontal planar curves.

3.6. Recovering rotation matrix and translation vector directly from homography matrix.

From equation (9), we need 2 homography matrices to recover the fundamental matrix. As shown in Tsai(1982) the homography matrix can be decomposed using SVD and the rotation matrix, the translation vector and the plane parameters can be recovered. Two possible solutions for the recovering of the relative orientation parameters are obtained using Tsai recovering. The possibility of two different solutions using direct recovering from the homography matrix is well suited the need of two curves providing two homography matrices for the recovery of the fundamental matrix (Shashua 2000).

Computing the rotation matrix and the translation vector from the homography matrix is as follows:

$$
\begin{aligned}
&[UDV^t] = H \\
&s = \det(U)\det(V) \\
&\delta = \pm\left(\frac{\lambda_1^{\,2} - \lambda_2^{\,2}}{\lambda_2^{\,2} - \lambda_2^{\,2}}\right)^{\frac{1}{2}} \\
&\alpha = \frac{\lambda_1 + s\lambda_3\delta^2}{\lambda_2(1+\delta^2)} \\
&\beta = \pm\sqrt{1-\alpha^2} \\
&R = U\cdot\begin{bmatrix} \alpha & 0 & \beta \\ 0 & 1 & 0 \\ -s\beta & 0 & s\alpha \end{bmatrix}\cdot V^t \\
&T = [-\beta U_1 + \left(\frac{\lambda_3}{\lambda_2} - s\alpha\right)U_3]
\end{aligned}
\tag{14}
$$

where: U_3 U_1 are the 1st and 3rd vector of U

$l_1..l_3$ are the singular values of H

Two options to obtain the epipolar geometry of the stereo model have been shown. While the first one requires two homography matrices for the recovery of the fundamental matrix and the last require only one, but provide two different solutions. Both options lead to the need of at least two planar curves to get unique solution. Equation (9) provides 6 linear equations for each homography matrix so Least squares adjustment has to be performed having two or more homography matrices. Equations (14) provide two solutions for R and T for each homography matrix. When having more than one homography matrix we need some elimination procedures to get the right solution.

4. SUMMARY

Two methods for recovering the relative orientation parameters from free-form planar curves have been presented and tested. While both perform quite well in most cases, the homography-based algorithm exhibited more robust behavior in terms of not being sensitive to some singular configurations observed for the alternative method. A forthcoming paper on the subject will present a more thorough analysis of the aforementioned methods including experiments with real data.

REFERENCES

Besl, P. J., and McKay, A Method for registration of 3D shapes. *IEEE transaction on pattern analysis and machine intelligence*. Vol. 14, no. 2, Feb. 1992.

Fitzgibbon, A.W., Robust registration of 2D and 3D point sets. *In proc. British machine vision conference*. Manchester, sep. 2001. pp. 411-420.

Habib, A., and Kelley D., Single-Photo Resection Using the Modified Hough Transform. *Photogrammetry Engineering and Remote Sensing*. Vol. 67, No. 8 August 2001, pp 909-914.

Habib, A., and Kelley, D., Automatic Relative Orientation of Large Scale Imagery Over Urban Areas Using Modified Iterated Hough Transform. 2001. *ISPRS Journal of Photogrammetry and Remote Sensing* Vol. 56, pp. 29-41.

Habib, A., and Lee, Y., and Morgan, M., Automatic Matching and Three-Dimensional Reconstruction of Free-Form Linear Features from Stereo Images. *Photogrammetry Engineering and Remote Sensing*. Vol. 69, No. 2, February 2003, pp. 189-197.

Hartley, R., and Zisserman, A., *Multiple view geometry in computer vision*. Cambridge university press. 2000.

Kraus, K., *Photogrammetry* vol. 2, Ferd. Dummler Verlag. Bonn 1997.

Kubik, K., Relative and Absolute Orientation Based on Linear Features. 1988. *ISPRS Journal of Photogrammetry and Remote Sensing* Vol. 46, pp. 199-204.

Ma, S.D., Conics-Based Stereo, Motion Estimation, and Pose Determination. 1993. *International Journal of computer vision* 10:1 pp. 7-25.

Mikhail, E., Linear features for photogrammetric restitution and object completion. Integrating Photogrammetric Techniques with Scene Analysis and Machine Vision, SPIE Proc. No. 1944, pages 16-30, Orlando, FL, 1993.

Petsa, E., and Karras, G.E., Constrained Line-Photogrammetric 3D-Reconstruction from Stereopairs. IAPRS, vol. XXXIII, Amsterdam, 2000.

Shashua, A., and Kaminski, J.Y., On calibration and reconstruction from planar curves. *Proc. of the European Conference on Computer Vision* (ECCV), June 2000, Dublin, Ireland.

Tsai, R.Y., and Thomas, S.H., Estimation Three-Dimensional Motion Parameters of a Rigid Planar Patch. 1981. *IEEE Trans. Acoust., Speech, Sign. Process*., 29:1147-1152.

Tsai, R.Y., Thomas, S.H., and Zhu, W.L., Estimation Tree-Dimensional Motion Parameters of a Rigid Planar Patch II: Singular value decomposition. 1982. *IEEE Trans. Acoust., Speech, Sign. Process*., 30:525-534.

Zalmanson, H.G., Hierarchical Recovery of Exterior Orientation from Parametric and Natural 3-D Curves. 2000. IAPRS Vol. XXXIII, Amsterdam.