

## Adjustment of Airborne Laser Altimetry Strips

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### ABSTRACT

Whereas the objectives of a strip adjustment procedure are simple to define, namely improving the accuracy of laser data and creating a seamless dataset, achieving them is difficult. Complex data acquisition systems and data that carry only little information are two aspects that make the formulation of a strip adjustment model difficult. Furthermore, difficulties in processing the data manually and the existence of partial data that usually contain only the laser points but not the system measurements, are other aspects that should be handled by a strip adjustment algorithm to make it useful. This paper presents a 3D system-driven strip adjustment algorithm. Based on the properties of the data the error recovery model is surface based. To eliminate manual processing of the data, handling tie and control information is autonomous, and to have the model applicable, the input data are the laser points. The application of the procedure is demonstrated here for the estimation of GPS biases to analyze the magnitude of positional offsets in the data. Results show the existence of significant errors in position.

### 1 Introduction

Adjustment of airborne laser scanning data has received growing attention in recent years. The aim of improving the accuracy of the data and the existence of offsets in the overlapping areas of the laser swaths has turned wider attention to the need for adjustment. The effect of the offsets is not limited to degraded accuracy but pose great difficulties in forming a seamless dataset out of the individual laser strips and complicate significantly further processing of the data. Indeed, there is still a debate whether laboratory and in-flight calibration can eliminate those errors, but reality shows that noticeable errors still exist in the laser data. Their removal requires the derivation of adjustment procedures for airborne laser scanning data.

The elimination of the errors from the laser data presents several challenges, among which the error model is the central one. Errors in the systems can be attributed to the individual components of the data acquisition system (GPS, INS, and range-finder systems) as well as to their integration; some errors might be constant for a whole mission while others may vary over space and time. A thorough error modeling does not guarantee yet their estimation as some errors may have similar effect as others and result in rank-deficient matrices or in weak estimation of the parameters. The error model is therefore not limited to the error analysis but should be followed by a recoverability analysis of the modeled errors. In addition, the nature of laser data requires the development of adequate algorithms to recover the systematic errors. In contrast to traditional reflectance data that are used in photogrammetry laser points sample the shape of the overflown surface. Point correspondence is practically impossible to establish under such conditions, and therefore shape based rather than traditional point based algorithms are needed.

While there is a growing interest in the elimination of systematic errors from the laser strips the work that has been

carried out so far is rather limited. The majority of the reported algorithms consider the offsets only along the height direction and can be termed as one-dimensional adjustment procedures (Vaughn et al., 1996; Ridgway et al., 1997; Crombaghs et al., 2000; Kager and Kraus, 2001; Kornus and Ruiz, 2003). The level of complexity of the models vary among implementations but their concern with minimizing the height offsets while ignoring in large positional offsets, is common. Reference features (control or tie objects) are usually chosen over flat horizontal surfaces where the height offsets are noticeable, well defined, and easy to measure. Existence of planimetric offsets in the data (see e.g., Bretar et al., 2003) will hardly be compensated for with this adjustment and are likely to remain. Another noticeable shortcoming of common adjustment procedures is the error recovery model that is usually chosen. In most cases, the applied procedure models the biases as transformation parameters on the laser point coordinates; they can therefore be regarded as data-driven solutions. As was demonstrated in Schenk (2001); Filin (2003a) some of the errors cannot be compensated for this way and some offsets will be left in the data untreated. An alternative solution in which the system errors are modeled and solved for allows for the actual errors to be modeled properly and therefore eliminated. However, as a system driven solution it requires the system observations as an input (see e.g., Burman, 2002). These observations are usually not available to the end-user so with no access to them this procedure is rather limited in its applicability.

An optimal solution for the adjustment should opt at modeling the actual errors in the system but at the same time be practical and assume the existence of the type of data that is usually available to the user. The work presented here concerns the implementation of a system-driven strip adjustment model. The strip adjustment model is three-dimensional and accounts for positional as well as height offsets. The implementation assumes that only laser points are given as an input. Analysis of the model and implementation concerns are discussed in greater details on Filin (2003b). To assess the significance of a 3D strip adjustment model the paper an-

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analyzes the positional and height offsets as error sources and evaluates their magnitude and their contribution to the removal of the offsets between strips. The paper is organized as follows; first the error recovery model is outlined, where GPS offsets are modeled here as systematic errors. Following a description of the extraction of tie information for the adjustment results are presented and are followed by conclusions.

## 2 Error recovery model

The error recovery model is based on modeling the system errors and their effect on the geolocation of the laser point. Based on the integration of the observations from the three individual components of the laser system the geolocation of the laser points, when transformed into a local reference frame, is given by equation 1

$$\begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + R_{INS} \left( \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + R_M R_S \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} \right) + \begin{bmatrix} \bar{e}_x \\ \bar{e}_y \\ \bar{e}_z \end{bmatrix} \quad (1)$$

with  $x_l, y_l, z_l$  the footprint location;  $X_0, Y_0, Z_0$  location of the phase center of the GPS receiver in the local frame;  $R_{INS}$  rotation from body reference frame to reference frame defined by local vertical;  $\delta_x, \delta_y, \delta_z$  offset vector between the phase center of the GPS antenna and laser firing point;  $R_M$  the mounting bias, which designate rotation between the altimeter and the body frame;  $\rho$  range vector measured by laser system;  $\bar{e}_x, \bar{e}_y, \bar{e}_z$  random error components.

Introducing GPS biases that contribute offsets into the position of the laser point, equation 1 becomes

$$\begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} + R_{INS} \left( \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} + R_M R_S \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} \right) + \begin{bmatrix} \bar{e}_x \\ \bar{e}_y \\ \bar{e}_z \end{bmatrix} \quad (2)$$

with  $\delta X_0, \delta Y_0, \delta Z_0$  as the GPS biases in the  $x, y,$  and  $z$  directions respectively. In general, one cannot assume that the offsets will remain constant throughout the mission as offsets in position may vary over time. As a result the offsets are assigned to the laser strips themselves. An error model that covers a wider variety of error sources in the laser system is given in Filin (2003b).

The model for solving the errors is based on utilizing natural and man-made surfaces to recover the systematic errors. The approach is based on minimizing the distance between the laser point coordinates and the actual surface. By locally approximating the surface as a plane and constraining the laser points to lie on the plane, the following relation can be written down.

$$s_1 x_l + s_2 y_l + s_3 z_l + s_4 = 0 \quad (3)$$

where  $s_1, s_2, s_3, s_4$  are the surface parameters. Using the relation in equation 2 to substitute laser point coordinates by the observations and the systematic and random errors, the error recovery model is obtained. The formula for solving the parameters is given in equation 4.

$$w_i = s_1 \delta X_0 + s_2 \delta Y_0 + s_3 \delta Z_0 + s_1 \bar{e}_x + s_2 \bar{e}_y + s_1 \bar{e}_z \quad (4)$$

With  $w_i$ , the transformed observation. The form provides one row in the Gauss-Helmert model

$$\mathbf{w}_n = \mathbf{A}_{n \times m} \boldsymbol{\xi}_m + \mathbf{B}_{n \times 3n} \mathbf{e}_{3n} \quad , \quad \mathbf{e} \sim \{0, \sigma_0^2 \mathbf{P}^{-1}\} \quad (5)$$

with  $\mathbf{w}$ , the transformed observation vector;  $\mathbf{A}$ , the coefficient matrix;  $\mathbf{B}$ , the conditions matrix;  $\boldsymbol{\xi}$ , the vector of unknowns;  $\mathbf{e}$ , the observational noise vector;  $\mathbf{P}$ , the weight matrix;  $\sigma_0^2$ , the variance component;  $n$ , the number of laser points; and  $m$ , the number of unknowns.

Following the least-squares criterion the parameters are solved by equation 6.

$$\hat{\boldsymbol{\xi}} = (\mathbf{A}^T (\mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T)^{-1} \mathbf{A})^{-1} \mathbf{A}^T (\mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T)^{-1} \mathbf{w} \quad (6)$$

## 3 Control and tie information

Similar to photogrammetric strip adjustment errors are recovered by using tie and control information that register the strips to one another and the laser block to the ground. The model is surface based and therefore the natural control entities are control surfaces. Since not too many, if any, control surfaces will be available, most of the control information will be provided in the form of control points. These entities are introduced into the model as additional surface constraints. Control points constrain the surface to the given point position, namely

$$s_1(X + e_X) + s_2(Y + e_Y) + s_3(Z + e_Z) + s_4 = 0 \quad (7)$$

with  $X, Y, Z$  the control point coordinates, and  $e_X, e_Y, e_Z$  are their random error components. It is noted that linear features can also be supported by this model.

Whereas there is little control on the type of control information that will be available to the adjustment procedure, obtaining tie information is practically an implementation concern. As the algorithm is surface based, surface elements that are identified in the overlapping areas of the laser strips are the natural candidates to serve as tie entities. In a similar fashion to photogrammetric strip adjustment surfaces that are introduced into the adjustment with no a priori surface model are considered as tie objects (tie surfaces). Their approximated parameters are computed from the data and refined within the process of the strip adjustment. The identification and selection of tie surfaces is part of the strip adjustment algorithm.

If the aim is to develop an autonomous laser strip adjustment procedure the algorithm should identify suitable tie regions in the data and extract the necessary information. The identification of tie surfaces in the data is conducted under the framework of surface segmentation. Segmentation here concerns with the separation of unsuitable regions and points that are unusable for the purpose of laser strip adjustment, e.g., vegetation points or forested regions where not too many points are reflected from the ground, and with the partition of the smooth regions into segments. The choice of a segmentation (and not for example a filtering method) is based on the realization that identifying suitable points and regions for performing the strip adjustment is insufficient. The dependency

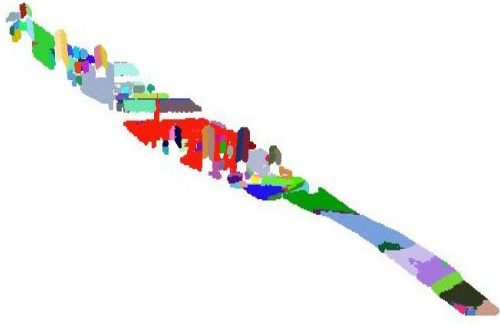


Figure 1: Segmentation of the overlapping part of two parallel strips

on surface parameters implies that noisy data can result in noisy parameters. Therefore, attenuation of the noise component in the laser data, or in other words regularization of the laser surfaces becomes mandatory.

The segmentation algorithm that was developed for this purpose is based on clustering the laser points (see Filin, 2002). The implementation is based on computing a feature vector for each laser point followed by an unsupervised classification of the attributes in a feature space. In the feature space each point is represented by its feature vector, where the values of the feature vector determine the laser point coordinates in this space. Clusters are then identified according to proximity of points in feature space. Validation and refinement phases follow the extraction of clusters from the feature space. The validation phase concerns verification that indeed all of the cluster points are part of one surface, and the refinement phase tests the extension of the cluster to neighboring points or neighboring clusters. The validation and refinement phases are controlled by the fitting accuracy of an analytical surface to the point clusters, where upper and lower bounds for the fitting accuracy are set to avoid under and over segmentation. The choice of fitting accuracy as a control fits well to the strip adjustment application. In general, segmentation algorithms tend to aggregate points that are part of the same physical surface, sometimes on the expense of the overall accuracy of the fitted surface. In the current case where the reconstruction of the actual physical surface is of somewhat less importance the accuracy criterion allows to generate surface with a given level of accuracy and include points within it that indeed belong to it. The results of the point clustering algorithm are presented in Figure 1 and show that the accuracy criterion allows still to reconstruct well the physical surfaces. As one can notice the segmentation results segment both the ground and detached objects, which is different than filtering algorithms that usually extract the bare earth only.

#### 4 Discussion and Results

The surface based model allows the application of the strip adjustment procedure over general surfaces and with the segmentation algorithm natural as well as man-made surface can be incorporated into the adjustment. Estimation of the biases does not require the existence of any distinct landmarks

or flat horizontal surfaces either as control or tie entities in the overflow area. As a result there are only little restrictions on initial condition in which the model can be applied. The algorithm does not require knowledge of the correspondence between the laser points and their actual location on the ground, mostly because of the association of points to surfaces from the outset. As surface elements are defined here explicitly via the point clustering algorithm, problems due to occlusion or height jumps that occur when comparing points to TIN based surfaces are avoided with this representation. Equation 4 allows the derivation of criteria for the estimation of the different parameters. As can be noticed, over horizontal or near horizontal areas the positional biases cannot be estimated well or estimated very weakly. The estimation of these biases require sloped surfaces. The inability to recover some errors over horizontal surfaces indicates the simple fact that the effect of these errors is unnoticeable. Following similar analysis to the one performed in Filin (2003a), one can see that the estimation of the positional offsets requires surfaces with slopes in different directions. Steeper slopes contribute to smaller variances and the variation in normal directions reduces the correlation between the estimates. Experience shows that even modest slopes of about 10 percent are sufficient for obtaining estimates with small variances.

Considering the approximation of the system observations. In general the geolocation of a laser point requires 14 observations – eight system measurements (GPS, INS, and the laser scanner measurements), and six more for the offset vector and the mounting bias. The user is usually provided only with the three coordinates of the laser point. Therefore the error recovery model requires these observations to be recovered. Equation 4 shows however that for the offsets the influence of the observations does not appear on the left-hand-side of the equation but only in the right-hand-side which refers to the differences between the laser point position and the control or tie surface, these differences can be computed by the given laser point coordinates. As a result for the computation of the offsets, there is no need for approximation of the system observations. The data that is usually provided in the form of the  $x, y, z$  coordinates of the laser point can be regarded as sufficient.

The application of the model with the computation of the offsets per strip is demonstrated over the Eelde area in the Netherlands. The dataset consists of twenty strips that are composed of two sub-blocks of ten parallel strips each where one sub-block crosses the other. The flight configuration is illustrated in Figure 2. No control information was available for the adjustment thus forcing an adjustment with tie surfaces only. To avoid rank deficiency an adjustment with a fixed datum constraint was applied by fixing the offset of the first strip to zero.

Tie surfaces were extracted in the overlapping regions of the parallel strips of one of the two sub-blocks (see Figure 2), and in the overlapping region between the two sub-blocks. As Figure 2 shows the same region could have appeared in four individual strips, and thus be segmented four times. To avoid multiple segmentations of the same area, each area was segmented only in one strip and corresponding laser points from other strips were later referred to that segment. The choice of which strip to segment and then what region in the strip to segment was performed by ordering the strip and

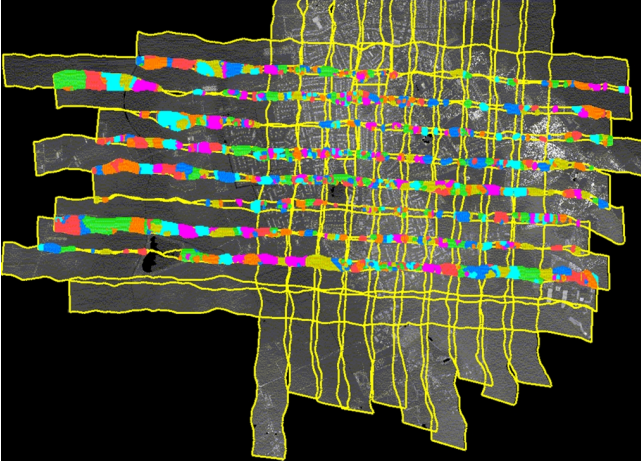


Figure 2: Tie surfaces for adjustment of the parallel strips in the Eelde block

by boolean operations to find the relevant overlapping areas that were not segmented so far. Association of counterpart points (i.e., from other strips) to a segment was performed as follows, for each counterpart strip points that are in or close to a segment were collected and a plane was fitted through them, points that exceeded a preset tolerance were rejected. This procedure introduced a safeguard to exclude erroneous points to enter the adjustment from the outset. Result of the extracted tie surfaces for the an adjustment of only one sub-block are illustrated in Figure 2. These segments serve and the tie surfaces for the adjustment.

The results of the adjustment of the whole block are summarized in Table 1, the variances of the estimated offsets are very small and are therefore not listed. As Table 1 shows the magnitude of the planimetric offsets is bigger than the height offsets by an order of magnitude. The positional offsets reach the order of tens of centimeters whereas the height offsets are on the order of a few centimeters, the biggest among them is four centimeters only. These results indicate that a 1D adjustment of the data is insufficient for the adjustment of airborne laser data. The relatively small height offset can be attributed to the higher level of accuracy in the height determination, or to a 1D adjustment of the data that was performed before the data was delivered. Figure 3 shows the offsets prior to the adjustment and the results after the dataset was corrected for the offsets. As can be seen there are noticeable planimetric offsets before the adjustment whereas the offsets in height can hardly be noticed. The offsets were eliminated after the adjustment parameters were introduced. The results of the adjustment were also evaluated by a comparison of the fitting accuracy of a surface to a segment (that consist of points from more than one strip) before and after the adjustment. The results show an impressive improvement. For tilted surfaces where offsets are noticeable the fitting accuracy for surface that was reduced from about 35cm before adjustment (where the fitting accuracy of a segment from one strip only was 5cm) to a fitting accuracy of about 6cm after adjustment. Post adjustment results show indeed that the positional offsets were eliminated.

Table 1 shows that the offsets within a sub-block are more or less of the same order, however they are not the same. Figure 4 that shows the variation in the magnitude of the offsets

| ID. | $\delta X_0$ [m] | $\delta Y_0$ [m] | $\delta Z_0$ [m] |
|-----|------------------|------------------|------------------|
| P1  | 0.05             | -0.01            | 0.04             |
| P2  | -0.17            | -0.30            | 0.02             |
| P3  | -0.24            | 0.01             | 0.02             |
| P4  | -0.08            | -0.25            | 0.02             |
| P5  | -0.16            | -0.11            | 0.01             |
| P6  | -0.11            | -0.21            | 0.04             |
| P7  | -0.25            | -0.06            | 0.01             |
| P8  | -0.11            | -0.14            | 0.00             |
| X9  | -0.27            | 0.29             | 0.01             |
| X1  | -0.46            | -0.33            | 0.02             |
| X2  | -0.43            | -0.49            | 0.01             |
| X3  | -0.56            | -0.21            | 0.02             |
| X4  | -0.20            | -0.31            | 0.01             |
| X5  | -0.48            | -0.07            | 0.02             |
| X6  | -0.33            | -0.11            | 0.02             |
| X7  | -0.34            | -0.02            | 0.01             |
| X8  | -0.18            | -0.11            | 0.01             |
| X9  | -0.21            | -0.37            | 0.03             |
| X10 | -0.34            | 0.28             | 0.01             |

Table 1: Offsets between for the individual strips. Strips from the parallel sub-block are denoted by P, strips from the crossing sub-block are denoted by X.

(the offsets norm) for the cross strips sub-block and indicates this also graphically. It is therefore not advisable to consider the offsets constant for a whole block. To test for variations of the offsets within a strip the offsets were computed for smaller sections within the strip but did not reveal significant changes, therefore the strip unit seems to suit here. A more detailed inspection of a few horizontal surfaces have indicated that there are some trends that appear to arrive from angular biases (such as mounting or INS biases). The magnitude of these trends is of smaller order but still require treatment. Extension of this work will concern with their elimination.

## 5 Concluding remarks

Reaching the potential accuracy of laser data and eliminating artifacts requires the removal of systematic errors from the data. A strip adjustment formulation enables removing both errors that were not properly eliminated before takeoff and ones that occurred during the mission. By using a system driven solution in the current modeling the actual errors in the system can be removed. The proposed model offers a natural way for eliminating the systematic errors as it constrains the laser points to the surface. The selection of a surface based model enables using general topography and natural and man-made surfaces for the adjustment and do not require distinct object in the overflown region. Results of applying the model on a block consisting 20 strips among which ten strips were taken in a "cross-strip" pattern have demonstrated the existence of significant positional offsets in the data of one order of magnitude bigger than the ones in height.

## REFERENCES

Bretar, F., Pierrot-Deseilligny, M., Roux, M., 2003. Estimating image accuracy of airborne laser data with local 3D-offsets, *International Archives of Photogrammetry and Re-*

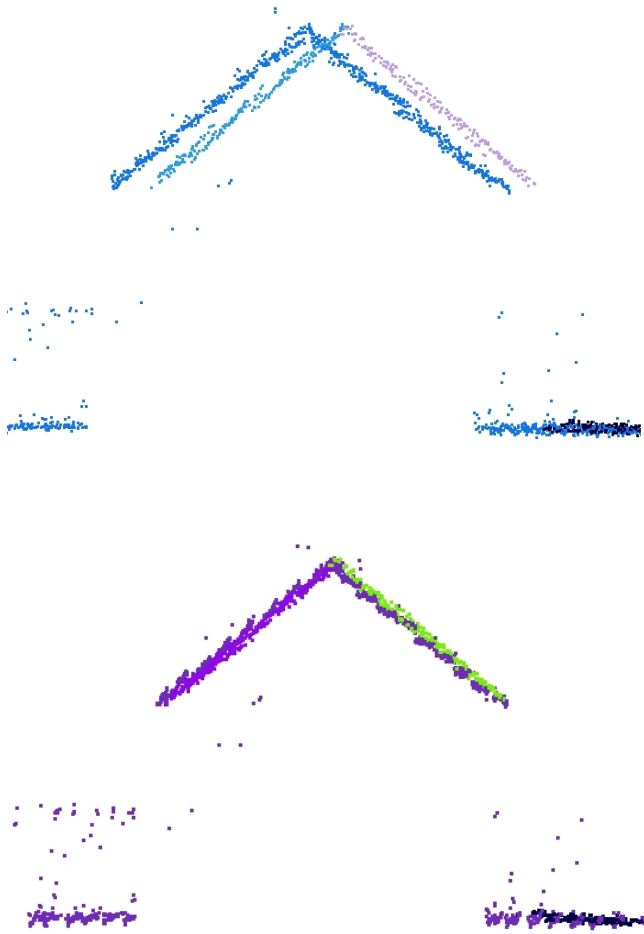


Figure 3: Offsets between two strips before and after the adjustment

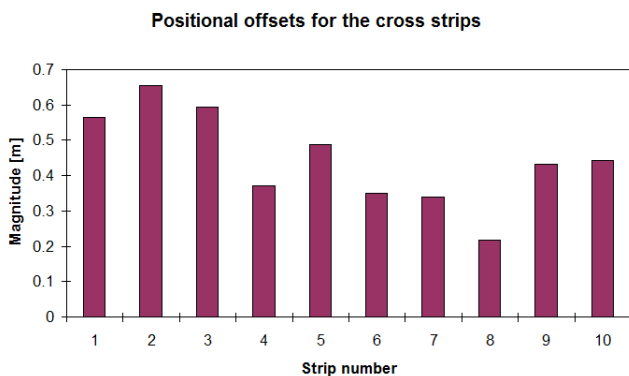


Figure 4: Graph of the positional offset magnitude for the cross strips

*ote Sensing*, **34**(3-W13), 20–26.

Burman, H., 2002. Laserstrip adjustment for data calibration and verification. *International Archives of Photogrammetry and Remote Sensing*, **34**(3A): 67–72.

Crombaghs, M., E. De Min and R. Bruegelmann, 2000. On the Adjustment of Overlapping Strips of Laser Altimeter Height Data. *International Archives of Photogrammetry and Remote Sensing*, **33**(B3/1): 230–237.

Filin, S., 2002, Surface clustering from airborne laser scanning data. *International Archives of Photogrammetry and Remote Sensing*. **34**(3A):117–124.

Filin, S., 2003. Recovery of Systematic Biases in Laser Altimetry Data Using Natural Surfaces. *Photogrammetric Engineering & Remote Sensing*, **69**(11), 1235–1242.

Filin, S., 2003, Analysis and Implementation of a laser strip adjustment model *International Archives of Photogrammetry and Remote Sensing*. **34**(3-W13), 65–70.

Kager, H. and Kraus, K., 2001. Hieght discrepancies between overlapping laser scanner strips. Proceedings of Optical 3D Measurement Techniques V, October, Vienna, Austria: 103–110.

Kornus, W. and Ruiz, A., 2003. Strip adjustment of LiDAR data. *International Archives of Photogrammetry and Remote Sensing*. **34**(3-W13), 47–50.

Ridgway, J. R., J. B. Minster, N. Williams, J. L. Bufton and W. B. Krabill, 1997. Airborne Laser Altimeter Survey of Long Valley California. *Int. J. Geophysics*, **131**: 267–280.

Schenk, T., 2001. Modeling and analyzing systematic errors of airborne laser scanners. *Technical Notes in Photogrammetry* No. 19, Department of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, Columbus, OH., 40 pages.

Vaughn, C. R., J. L. Bufton, W. B. Krabill and D. L. Rabine, 1996. Georeferencing of Airborne Laser Altimeter Measurements. *Int. J. Remote Sensing*, **17**(11): 2185–2200.