ERROR BUDGET OF LIDAR SYSTEMS AND QUALITY CONTROL OF THE DERIVED POINT CLOUD

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ABSTRACT:

The improving capability of the direct geo-referencing technology is having a positive impact on the widespread adoption of LiDAR systems for the acquisition of dense and accurate surface models over extended areas. A typical LiDAR system consists of three main components: a GPS system to provide position information, an IMU unit for attitude determination, and a laser unit to provide the range (distance) between the laser-beam firing point and the ground point (laser footprint). The measured ranges are coupled with the position and attitude information from the GPS/IMU integration process as well as the bore-sighting parameters relating the system components to derive the ground coordinates of the LiDAR footprints. Unlike photogrammetric techniques, the derivation of the point cloud from the LiDAR measurements is not a transparent process. In other words, the raw system measurements are not always provided to the system user. Moreover, the coordinate computation of the LiDAR footprints is not based on redundant measurements, which are manipulated in an adjustment procedure. Consequently, one does not have the associated measures (e.g., variance component of unit weight and variance-covariance matrices of the derived parameters), which can be used to evaluate the quality of the final product. This paper is concerned with providing a tool for the quality control (QC) of the LiDAR point cloud. The objective of the QC procedure is to verify the accuracy of the LiDAR footprints. In other words, the QC procedure would test whether the expected accuracy has been achieved or not. The paper will start with a brief discussion of the LiDAR mathematical model relating the system measurements to the ground coordinates of the point cloud. Then, an analysis of possible systematic and random errors and their impact on the resulting surface will be outlined. Following the discussion of the error sources and their impact on the accuracy of the LiDAR footprints, a QC tool will be proposed. The paper will conclude by experimental results from a real dataset involving overlapping strips from operational LiDAR system.

1. INTRODUCTION

The direct acquisition of a high density and accurate 3D point cloud has made LiDAR systems the preferred technology for the generation of topographic data to satisfy the needs of several applications (e.g., digital surface model (DSM) creation, digital terrain model (DTM) generation, orthophoto production, 3D city modeling, and forest mapping). A typical LiDAR system consists of three main components: a GPS system to provide position information, an IMU unit for attitude determination, and a laser unit to provide the range (distance) between the laser-beam firing point and the ground point (laser footprint). The measured ranges are coupled with the position and attitude information from the GPS/IMU integration process as well as the bore-sighting parameters relating the system components to derive the ground coordinates of the LiDAR footprints. Although the use of LiDAR data for different applications has increased significantly in the past few years, the user community still lacks standard and efficient procedures for evaluating the quality of the provided point cloud. Compared to photogrammetric and other surveying techniques, the computation of the LiDAR footprints is not based on redundant measurements, which are manipulated in an adjustment procedure. Consequently, standard measures for evaluating the quality of the final product, such as the a-posteriori variance factor and variance-covariance matrices of the derived coordinates, are not available. A commonly used procedure to evaluate the data accuracy compares the LiDAR surface with independently collected control points. Besides being expensive, this procedure does not provide accurate verification of the horizontal quality of the LiDAR footprints. Such inability is a major drawback since the horizontal quality of the LiDAR footprints is known to be inferior to the quality of these points in the vertical direction.

In the past few years, several methods have been developed for evaluating and/or improving LiDAR data quality by checking the compatibility of LiDAR footprints in overlapping strips (Kilian et al., 1996; Crombaghs et al., 2000; Maas, 2000; Bretar et al., 2004; Vosselman, 2004; Pfeifer et al., 2005). In Crombaghs et al. (2000), a method for reducing vertical discrepancies between overlapping strips is proposed. Since the horizontal quality of the derived point cloud is considerably lower than the vertical one, this approach is not sufficient to evaluate the overall quality of the LiDAR data. In Kilian et al. (1996), an adjustment procedure similar to the photogrammetric strip adjustment was introduced for detecting discrepancies and improving the compatibility between overlapping strips. The drawback of this approach is relying on distinct points to relate

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overlapping LiDAR strips and control surfaces. Due to the irregular nature of the LiDAR footprints, the identification of distinct points (e.g., building corners) is quite difficult and not reliable. More suitable primitives have been suggested by Maas (2000), where the correspondence is established between discrete points in one LiDAR strip and TIN patches in the other one. The correspondences are derived through a least-squares matching procedure where normal distances between conjugate point-patch pairs are minimized. The drawback of this work is that simple shifts were used as the transformation function relating conjugate point-patch pairs. The validity of such a model was not completely justified. Moreover, the estimated shifts were not used to derive an indication of the point cloud quality. Bretar et al., (2004) proposed an alternative methodology for improving the quality of LiDAR data using derived surfaces from photogrammetric procedures. The main disadvantage, which limits the practicality of this methodology, is relying on having aerial imagery over the same area. In addition, the proposed approach uses an affine transformation to relate LiDAR and photogrammetric surfaces without sufficient justification. In Pfeifer et al. (2005) and Vosselman (2004) other methods were developed for detecting discrepancies between overlapping strips. Detected discrepancies were used for strip adjustment procedures rather than system and data evaluation.

The objective of this paper is to propose a cost-effective and meaningful quality control (QC) method, which is based on analyzing the compatibility of LiDAR data in overlapping strips. More explicitly, the objective of the presented research is to develop a tool for detecting the presence of systematic biases as well as inspecting the noise level in the point cloud with a satisfactory level of automation (i.e., requiring minimum user interaction). The paper will start with a brief discussion of the LiDAR mathematical model relating the system measurements to the ground coordinates of the point cloud. Then, an analysis of possible systematic and random errors and their impact on the resulting surface will be outlined. Following the discussion of the error sources and their impact on the accuracy of the LiDAR footprints, a QC tool will be proposed. The paper will conclude by experimental results from a real dataset involving overlapping strips from operational LiDAR system. The results have proven the feasibility of the introduced methodology to evaluate the quality of the LiDAR data. More specifically, the proposed measure detected the presence of systematic biases and the data noise level. Future research will focus on relating the detected discrepancies between overlapping strips to the system biases.

2. LIDAR MATHEMATICAL MODEL

The coordinates of the LiDAR footprints are the result of combining the derived measurements from each of its system components, as well as the bore-sighting parameters relating such components. The relationship between the system measurements and parameters is embodied in the LiDAR equation (Vaughn et al., 1996; Schenk, 2001; El-Sheimy et al., 2005), Equation 1. As it can be seen in Figure 1, the position of the laser footprint, $\vec{X}_G$, can be derived through the summation of three vectors ($\vec{X}_o$, $\vec{P}_G$ and $\vec{\rho}$) after applying the appropriate rotations: $R_{yaw,\ pitch,\ roll}$, $R_{\Delta\omega,\Delta\phi,\Delta\kappa}$ and $R_{\alpha,\beta}$. In this equation, $\vec{X}_o$ is the vector between the origins of the ground and IMU coordinate systems, $\vec{P}_G$ is the offset between the laser unit and IMU coordinate systems (bore-sighting offset), and $\vec{\rho}$ is the laser range vector whose magnitude is equivalent to the distance from the laser firing point to its footprint. The term $R_{yaw,\ pitch,\ roll}$ stands for the rotation matrix relating the ground and IMU coordinate systems, $R_{\Delta\omega,\Delta\phi,\Delta\kappa}$ represents the rotation matrix relating the IMU and laser unit coordinate systems (angular bore-sighting), and $R_{\alpha,\ \beta}$ refers to the rotation matrix relating the laser unit and laser beam coordinate systems with α *and* β being the mirror scan angles. For a linear scanner, which is the focus of this paper, the mirror is rotated in one direction only (i.e., α is equal to zero). The involved quantities in the LiDAR equation are all measured during the acquisition process except for the bore-sighting angular and offset parameters (mounting parameters), which are usually determined through a calibration procedure.

3. LIDAR ERROR BUDGET

The quality of the derived point cloud from a LiDAR system depends on the random and systematic errors in the system measurements and parameters. A detailed description of LiDAR random and systematic errors can be found in Huising and Pereira (1998), Baltsavias (1999), and Schenk (2001). The magnitude of the random errors depends on the accuracy of the system's measurements, which include position and orientation measurements from the GPS/IMU unit, mirror angles, and ranges. Systematic errors, on the other hand, are mainly caused by biases in the bore-sighting parameters relating the system components as well as biases in the system measurements (e.g., ranges and mirror angles). In the following sub-sections, the impact of random and systematic errors in the system measurements and parameters on the reconstructed object space will be analyzed.

3.1 Random Errors

The purpose of studying the impact of random errors is to provide sufficient understanding of the nature of the noise in the derived point cloud as well as the achievable accuracy from a given flight and system configuration. In this work, the effect of random errors in the system measurements is analyzed through a simulation. The simulation process starts from a given surface and trajectory, which are then used to derive the system measurements (ranges, mirror angles, position and orientation information for each pulse). Then, noise is added to the system measurements, which are later used to reconstruct the surface through the LiDAR equation. The differences between the noise-contaminated and true coordinates of footprints are used to represent the impact of a given noise in the system measurements. The following list summarizes the effect of noise in the system measurements.

- Position noise will lead to similar noise in the derived point cloud. Moreover, the effect is independent of the system flying height and scan angle.
- Orientation noise (attitude or mirror angles) will affect the horizontal coordinates more than the vertical coordinates. In addition, the effect is dependent on the system flying height and scan angle.

- Range noise mainly affects the vertical component of the derived coordinates. The effect is independent of the system flying height. The impact, however, is dependent on the system's scan angle.

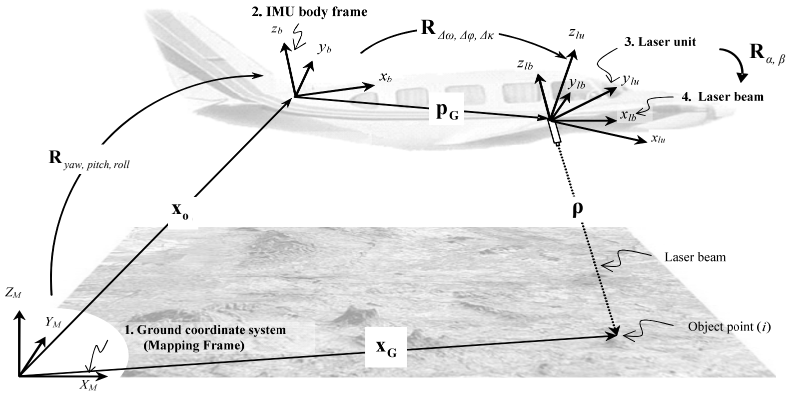


Figure 1. Coordinate systems and involved quantities in the LiDAR equation

$$\vec{X}_G = \vec{X}_o + R_{yaw,\, pitch,\, roll}\ \vec{P}_G + R_{yaw,\, pitch,\, roll}\ R_{\Delta\omega,\, \Delta\phi,\, \Delta\kappa}\ R_{\alpha,\, \beta} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix} \tag{1}$$

Through the proposed simulation, it could be noticed that noise in some of the system measurements affects the relative accuracy of the derived point cloud. For instance, a given attitude noise in the GPS/INS derived orientation affects the nadir region of the flight trajectory less significantly than off nadir regions. Such a phenomenon is contrary to derived surfaces from photogrammetric mapping where the measurements noise does not affect the relative accuracy of the final product. An additional conclusion that could be drawn from the simulation experiments is that the introduction of noise in the system measurements does not lead to systematic discrepancies between conjugate features in overlapping strips.

3.2 Systematic Errors

In this work, the impact of systematic errors/biases in the bore-sighting parameters (spatial and rotational) on the derived point cloud will be analysed. A simulation process was accomplished for that purpose. The process starts from a given simulated surface and trajectory, which are then used to derive the system measurements (ranges, mirror angles, position and orientation information for each pulse). Then, biases are added to the system parameters, which are used to reconstruct the surface through the LiDAR equation. The differences between the bias-contaminated and true coordinates of the footprints within the mapped area are used to represent the impact of a given bias in the system parameters or measurements. Due to the presence of systematic errors in the system parameters, the bias-contaminated coordinates of conjugate points in overlapping strips will show systematic discrepancies. The following conclusions could be drawn from the simulation experiments:

(1) The discrepancies caused by the bore-sighting offset and angular biases can be modelled by shifts and a rotation across the flight direction. Therefore, a six-parameter rigid-body transformation (three shifts and three rotations) can be used to express the relationship between conjugate features in overlapping strips.

(2) The discrepancies can be used for diagnosing the nature of the systematic errors in the system parameters.

4. QUALITY CONTROL METHOD

The proposed quality control tool is based on evaluating the degree of consistency among the LiDAR footprints in overlapping strips to check the internal/relative quality of the LiDAR data. The conceptual basis of the QC methodology is that conjugate surface elements, which can be identified in overlapping strips, should match as well as possible. If consistent discrepancies are detected, then one can infer the presence of biases in the system parameters and/or measurements. Other than the ability to detect systematic errors in the data acquisition system, the proposed methodology will also evaluate the noise level in the data by quantifying the goodness of fit between conjugate surface elements in overlapping strips after removing systematic discrepancies.

To reliably evaluate the consistency between overlapping strips, one must address the following questions:

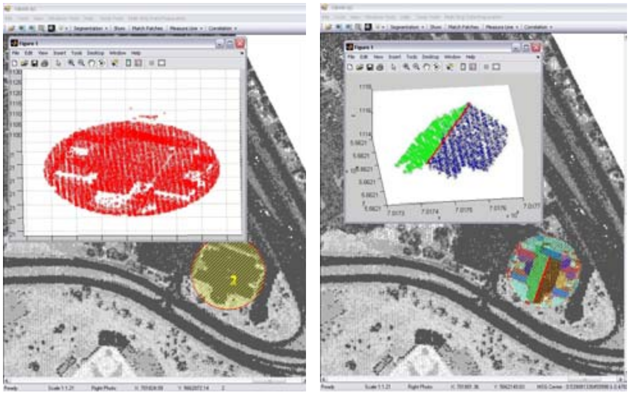
- What is the appropriate transformation function relating overlapping strips in the presence of systematic biases in the data acquisition system?
- What are the appropriate primitives, which can be used to identify conjugate surface elements in overlapping strips comprised of irregular sets of non-conjugate points?
- What is the possibility of automatic derivation of these primitives?
- What is the possibility of automated identification of conjugate primitives in overlapping strips?
- What is the appropriate similarity measure, which utilizes the involved primitives and the defined transformation function to describe the correspondence of conjugate primitives in overlapping strips?

The answer to the first question has been already established in section 3.2, where it has been verified through a simulation procedure that conjugate points in overlapping strips are related to each other through a transformation function involving constant shifts and a rotation angle across the flight direction. Therefore, a six-parameter rigid-body transformation (three shifts and three rotation angles) can be used as the transformation function relating overlapping strips in the presence of the bore-sighting spatial and angular biases. The answers to the remaining questions depend on the nature of the utilized primitives. The following subsections present the answers to the above questions as they pertain to the selected primitives.

4.1 Primitives Extraction and Matching

Since the LiDAR footprints are irregularly distributed, no point-to-point correspondence can be assumed between overlapping strips. In this regard, other primitives must be investigated. In this work, the use of linear features derived from the intersection of neighbouring planar patches is proposed. LiDAR provides high redundancy in planar surfaces. Therefore, the plane parameters can be derived with high accuracy using an adjustment procedure (e.g. plane fitting). The larger the planar surface, the greater will be the point cloud noise reduction. Therefore, high accuracy linear features can be extracted by intersecting neighboring fitted planes. To do so, an environment for the extraction and matching of linear features in overlapping strips was developed. The process starts by displaying the LiDAR intensity images for overlapping strips where the operator selects an area where linear features might exist (e.g. roof ridge line). The user clicks on the centre of the area after defining the radius of a circle, within which the original LiDAR footprints will be extracted. It should be noted that the LiDAR intensity images are only used for visualization purposes. The user needs to establish the area of interest in one of the strips and the corresponding areas in the other strips are automatically defined. Figure 2a shows the specified area in one of the strips as well as the original LiDAR footprints in that area. Then a segmentation technique (Kim et al., 2007) is used to identify planar patches in the point cloud within the selected area. This segmentation procedure is independently run on the point cloud for all the overlapping strips. The outcome from such segmentation is aggregated sets of points representing planar patches in the selected area (bottom right portion in Figure 2b). For the linear features extraction, neighbouring planar patches are identified and the plane parameters determined. Then, the neighboring planes are intersected to produce an infinite straight-line. Using the segmented patches, the infinite line and a given buffer, the end points for the intersected line can be defined (top left portion in Figure 2b). This procedure is repeated for several areas within the overlap portion in the involved strips.

The outcome of the extraction procedure is a set of linear features in overlapping strips. Due to the nature of the LiDAR data acquisition (e.g., scan angle, surface normal, surface reflectivity, occlusions), there is no guarantee that there is one-to-one correspondence between the extracted primitives from overlapping strips. To solve the correspondence problem, one has to utilize the attributes of the extracted primitives. Conjugate linear features can be automatically matched using the normal distance, parallelism, and the percentage of overlap between candidate lines in overlapping strips (Figure 3). A graphic visualization of matched linear features is presented to the user for final confirmation of the validity of the matched primitives.



(a) (b)

Figure 2. Area of interest selection and LiDAR point cloud extraction (a), and extracted linear features by intersection of segmented planar patches in the area of interest (b)

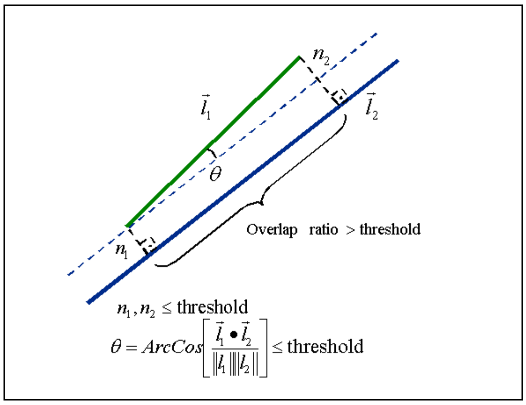


Figure 3. Matching of conjugate linear features in overlapping strips

4.2 Similarity Measure

In this section, the similarity measure, which incorporates the matched primitives together with the established transformation function to mathematically describe their correspondence, is introduced. Conjugate lines will be represented by their end

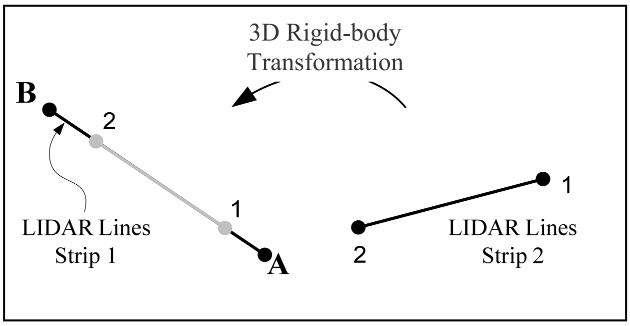


Figure 4. Underlying concept for the incorporation of linear features in a line-based approach for the determination of the transformation parameters.

points, which need not be conjugated. In order to compensate for the non-correspondence between the line end points, we will introduce the necessary constraints to describe the fact that the line segment from the LiDAR strip 2 coincides with the conjugate segment from the overlapping LiDAR strip 1 after applying the 3D rigid-body transformation (Figure 4). The mathematical representation of this constraint for these points is shown in Equation 2.

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} + R_{(\Omega,\Phi,K)} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \lambda_1 \begin{bmatrix} X_B - X_A \\ Y_B - Y_A \\ Z_B - Z_A \end{bmatrix} \tag{2a}$$

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} + R_{(\Omega,\Phi,K)} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \lambda_2 \begin{bmatrix} X_B - X_A \\ Y_B - Y_A \\ Z_B - Z_A \end{bmatrix} \tag{2b}$$

Where:

$(X_T, Y_T, Z_T)^T$ is the translation vector between the strips.

$R_{(\Omega,\Phi,K)}$ is the required rotation matrix for the co-alignment of the strips, and λ_1 and λ_2 are the scale factors.

By subtracting Equation 2a from Equation 2b, and eliminating the scale factors (by dividing the first two rows by the third one in order) result in Equation 3 which relates the rotation elements of the transformation to the coordinates of the points defining the line segments.

$$\frac{(X_B - X_A)}{(Z_B - Z_A)} = \frac{R_{11}(X_2 - X_1) + R_{12}(Y_2 - Y_1) + R_{13}(Z_2 - Z_1)}{R_{31}(X_2 - X_1) + R_{32}(Y_2 - Y_1) + R_{33}(Z_2 - Z_1)}$$

$$\frac{(Y_B - Y_A)}{(Z_B - Z_A)} = \frac{R_{21}(X_2 - X_1) + R_{22}(Y_2 - Y_1) + R_{23}(Z_2 - Z_1)}{R_{31}(X_2 - X_1) + R_{32}(Y_2 - Y_1) + R_{33}(Z_2 - Z_1)} \tag{3}$$

The estimation of the two rotation angles (the azimuth, and the pitch angle along the line) is possible by writing the equations 3a and 3b for a pair of conjugate line segments. On the other hand, the roll angle across the line cannot be estimated. Therefore, a minimum of two non-parallel lines is needed to recover the three elements of the rotation matrix (Ω, Φ, K). To allow for the estimation of the translation parameters, the terms in equations 2a and 2b are re-arranged and the scale factors eliminated (by dividing the first two rows by the third one) to come up with Equation 4. Overall, to recover all six parameters of the transformation function, a minimum of two non-coplanar line segments is required. For a complete description of this approach refer to Habib et al., 2004.

$$\frac{(X_T + x_1 - X_A)}{(Z_T + z_1 - Z_A)} = \frac{(X_T + x_2 - X_A)}{(Z_T + z_2 - Z_A)}$$

$$\frac{(Y_T + y_1 - Y_A)}{(Z_T + z_1 - Z_A)} = \frac{(Y_T + y_2 - Y_A)}{(Z_T + z_2 - Z_A)} \tag{4}$$

Where:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = R_{(\Omega,\Phi,K)} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \text{ and, } \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = R_{(\Omega,\Phi,K)} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}$$

4.3 Noise Level Verification

So far, we have introduced a methodology for detecting systematic errors in the data acquisition system. It is important to emphasise that for a LiDAR system with only random errors, the estimated transformation parameters should be zeros for the translation and rotation parameters. In other words, the expected values will not change with varying noise levels in the LiDAR point cloud. In this section, we are interested in a similarity measure for the evaluation of the noise level in the data.

In this work, the noise level in the LiDAR data will be evaluated by quantifying the goodness of fit between conjugate primitives after removing existing discrepancies between overlapping strips. This can be accomplished by computing the average normal distance between conjugate linear features after applying the estimated transformation parameters.

5. EXPERIMENTAL RESULTS

To evaluate the validity of the proposed QC methodology, experiments were performed using a real dataset. The dataset used in the experiments covers and urban area consisting of three strips as shown in Figure 5. The specifications of this dataset are shown in Table 1.

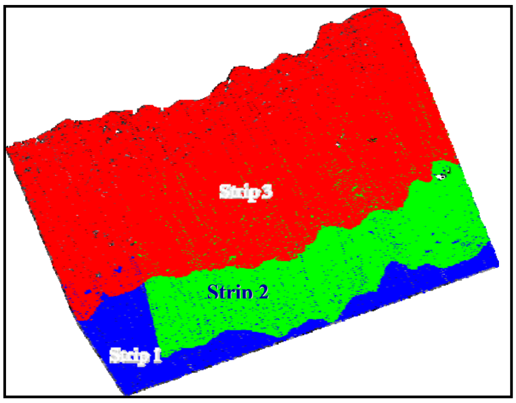


Figure 5. Dataset used in the experiments

Sensor Model	Optech 2050
Flying Height	~1000 m
Ground Point Spacing	~0.75 m
1 survey day	3 strips
Horizontal accuracy	50 cm
Vertical accuracy	15 cm

Table 1. Dataset specifications

Using the semi-automated procedure described in section 4.1, we extracted conjugate lines in the three overlapping strips (Figure 6). Table 2 shows the estimated transformation parameters. As it can be seen in this table, there is a significant discrepancy between conjugate lines in overlapping strips especially in the X direction. Such discrepancies indicate the presence of biases in the system parameters and/or measurements.

	Strips 1&2	Strips 2&3	Strips 1&3
Transf. Parameters/ # of lines	14	11	13
X_T (m)	0.39	0.72	-0.09
Y_T (m)	0.06	-0.17	0.21
Z_T (m)	0.01	0.01	-0.14
Ω (°)	-0.0174	-0.0300	0.0119
Φ (°)	-0.0096	-0.0093	0.0011
K (°)	0.0027	-0.0133	-0.0212
RMSE (m)	0.12	0.15	0.20

Table 2. Estimated transformation parameters using conjugate linear features in overlapping strips together with the RMSE after applying the transformation

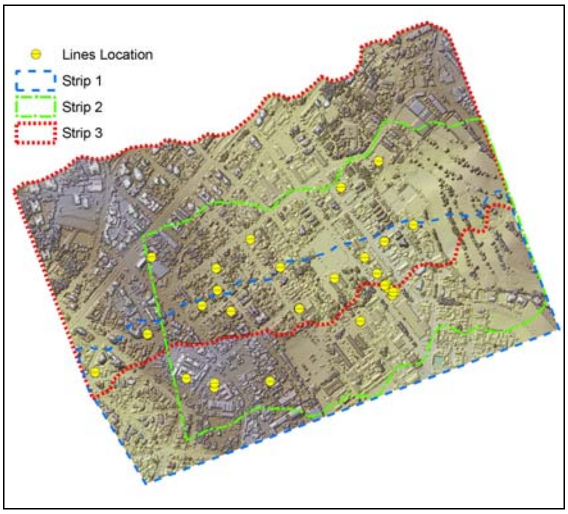


Figure 6. Extracted lines in the three overlapping strips

The noise level was evaluated by computing the average normal distance between conjugate linear features after applying the estimated transformation parameters. The results are reported in Table 2.

6. CONCLUSIONS

This paper has outlined a new tool for the evaluation of LiDAR data quality. The paper started with a brief analysis of random and systematic errors in the system measurements and parameters and their impact on the derived point cloud coordinates. From this analysis some conclusions regarding the mathematical relationship between conjugate surface elements in overlapping strips could be drawn. It was concluded that a rigid body transformation is an appropriate model for relating conjugate points in the presence of the biases in the bore-sighting parameters (spatial and rotational). Following such an analysis, the paper introduced a quality control procedure based on the use of linear features, which can be used to check for the presence of systematic errors in the data acquisition system as well as evaluating the noise level in the delivered point cloud. The proposed procedure has a satisfactory level of automation requiring minimal interaction from the operator (just a few clicks on the intensity image). The results from the real data have shown that collected LiDAR data might exhibit significant incompatibilities due to insufficient calibration procedures.
Future research will focus on relating the detected discrepancies between overlapping strips to the system biases. Moreover, we will be using the estimated transformation parameters to remove the bias effect from the point cloud. In addition, the estimated system biases will be compared to those derived from rigorous calibration procedures. The presented models for linear scanner will be also expanded to include elliptical LiDAR systems. Also, we will be developing some standards and specifications that will allow for the acceptance or rejection of delivered point cloud to the end user. With the wide spread adoption of LiDAR systems for topographic data acquisition, we believe that this research is critical to strengthen the users' confidence in the delivered point cloud especially in the absence of traditional measures, which are provided by other mapping techniques.

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