TRIANGULATION OF AIRBORNE THREE-LINE IMAGES USING UNIT QUATERNION

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ABSTRACT:

In this paper the unit quaternion is used as a substitute for describing the attitude of a camera to overcome the shortcomings of Euler angles on the description and interpolation of orientation, corresponding collinearity equation is then derived and linearized. At the same time the Spherical linear Interpolation (Slerp) of unit quaternion is introduced into the bundle adjustment of three-line imagery with orientation image method. By making use of Slerp formula to accomplish uniform interpolation of camera attitude on the sampling cycles between two orientation images, new mathematic model based on unit quaternion for the reconstruction of the exterior orientation parameters of three-line images is set up. Two ADS40 datasets have been used for validating related algorithms proposed in this paper. Experimental results show that bundle adjustment based on unit quaternion achieves the accuracy of about one pixel both in horizontal and height directions when locating a full control point at each corner of the block. It provides us an ideal and robust approach for the calibration and triangulation of three-line array CCD sensor.

1. INTRODUCTION

Since the year of 2000, two kinds of airborne digital cameras have been taken into world market, one of them is characteristic of frame CCD sensor, the other one is characteristic of three-line CCD sensor. Under current technological conditions, three-line sensors such as ADS40 of Leica Geosystems, HRSC-AX of DLR, have some advantages over frame cameras because of its larger side coverage and higher aerial photography efficiency (Tempelmann, 2000; Scholten, 1999).To recover the exterior orientation (EO) parameters of each scan line of airborne pushbroom sensors, GPS/IMU system must be equipped to acquire auxiliary orientation data, which is the indispensable information for the geometrical rectification of twisted raw images and its georeferencing.

For the triangulation of three-line imagery, the approach which was first proposed by Hofmann is most widely used (Hofmann, 1984; Ebner, 1988). Here the exterior orientation parameters are estimated only at certain time intervals named orientation images (OI). The Lagrange polynomials then model the assumed smooth variations of sensor position and attitude between adjacent OIs. There are some shortcomings for this traditional OI approach. Firstly the attitude of sensor is described by three Euler angles omega, phi, kappa, so the orientation interpolation between adjacent OIs is difficult because of the periodicity of Euler angles. The numeric range of input angles must be restricted manually to ensure interpolation results reasonable. Furthermore the approach can not accomplish smooth interpolation. Secondly the IMU misalignment model described by Euler angles is fairly complicated and not robust enough when image strip with 90-degree kappa angle exists in the block. These limitations can be overcome by the introduction of quaternion.

In this paper a new georeferencing technique of three-line images based on quaternion is proposed, in which the EO angular elements omega, phi, kappa are replaced by unit quaternion. Corresponding collinearity equation is reconstructed, and then the IMU misalignment model is rebuilt by quaternions to realize the simplicity and linearity because that only multiplication and addition operations are included. Additionally, the Spherical Linear Interpolation (Slerp) is introduced into orientation image approach to accomplish smooth and uniform interpolation of camera attitude. Two ADS40 datasets are used to validate related algorithms and experimental results are discussed in detail.

2. BASICS OF QUATERNION

2.1 Definition

Quaternion is the extension of complex numbers in 4D space, which was first invented by Irish mathematician William Hamilton in 1843. It is normally expressed as follows (Horn, 1987; Dunn, 2002):

$$\dot{q} = q_0 + q_1 \mathrm{i} + q_2 \mathrm{j} + q_3 \mathrm{k}, \quad q_i (i = 0,1,2,3) \in \Re$$

where q_0 is the real part and $q_1 \mathrm{i} + q_2 \mathrm{j} + q_3 \mathrm{k}$ is the imaginary part. $\mathrm{i}, \mathrm{j}, \mathrm{k}$ are the bases of quaternion, for which following relations hold：

$$\mathrm{ij} = -\mathrm{ji} = \mathrm{k}, \ \mathrm{jk} = -\mathrm{kj} = \mathrm{i}, \ \mathrm{ki} = -\mathrm{ik} = \mathrm{j}, \ \mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = -1 \quad (1)$$

The conjugate of a quaternion, denoted $\dot{q}^*$, is obtained by negating the vector portion of the quaternion:

$$\dot{q}^* = q_0 - q_1 \mathrm{i} - q_2 \mathrm{j} - q_3 \mathrm{k}$$

According to the product rules given in equation (1), the multiplication of quaternion can be derived (Horn, 1987) :

$$\dot{q}\dot{r} = Q(\dot{q})\dot{r} \quad \dot{r}\dot{q} = \overline{Q}(\dot{q})\dot{r} \tag{2}$$

where $Q(\dot{q})$ and $\overline{Q}(\dot{q})$ are both orthogonal matrices:

$$Q(\dot{q}) = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix}, \overline{Q}(\dot{q}) = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & q_3 & -q_2 \\ q_2 & -q_3 & q_0 & q_1 \\ q_3 & q_2 & -q_1 & q_0 \end{bmatrix}$$

It is not difficult to see that quaternion multiplication is associative, but not commutative.

2.2 Representing Rotations

Supposing that $\vec{z}$ and $\vec{z}'$ are two vectors or two points in 3D space, $\dot{q}$ is a unit quaternion. According to the characteristics of quaternion multiplication, following equation can be derived (Diebel, 2006):

$$\begin{bmatrix} 0 \\ \vec{z}' \end{bmatrix} = \dot{q} \begin{bmatrix} 0 \\ \vec{z} \end{bmatrix} \dot{q}^{-1} = \overline{Q}(\dot{q})^T Q(\dot{q}) \begin{bmatrix} 0 \\ \vec{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & M(\dot{q}) \end{bmatrix} \begin{bmatrix} 0 \\ \vec{z} \end{bmatrix} \tag{3}$$

Where

$$M(\dot{q}) = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_3q_1 - q_0q_2) & 2(q_3q_2 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

Therefore

$$\vec{z}' = M(\dot{q})\vec{z}, \quad \vec{z} = M(\dot{q})^T \vec{z}' \tag{4}$$

Considering that both $Q(\dot{q})$ and $\overline{Q}(\dot{q})$ are orthogonal matrices, matrix $M(\dot{q})$ constructed by the production of $Q(\dot{q})$ and $\overline{Q}(\dot{q})$ is also orthogonal, so equation (4) expresses a transformation of rotation. It means that formulation $\dot{r}' = \dot{q}\dot{r}\dot{q}^{-1}$ performs a coordinate rotation from $\vec{z}$ to $\vec{z}'$, where $\dot{r}$ and $\dot{r}'$ are the pure imaginary unit quaternion corresponding to $\vec{z}$ and $\vec{z}'$ respectively, that is, $\dot{r} = \begin{bmatrix} 0 & \vec{z} \end{bmatrix}^T$ and $\dot{r}' = \begin{bmatrix} 0 & \vec{z}' \end{bmatrix}^T$.

2.3 Spherical Linear Interpolation (Slerp)

Quaternion is very suitable to orientation interpolation, and the standard way is spherical linear interpolation. The Slerp operation is useful because it allows us to smoothly interpolate between two orientations (Zhan, 2004). If we look upon a unit quaternion as a point on the surface of a 4D sphere, Slerp will interpolate around the shortest arc that connects the two quaternions along the surface of the sphere.

Given two unit quaternions $\dot{q}_1$ and $\dot{q}_2$ with an inclination angle of θ, the Slerp function is given by:

$$\dot{q}(t) = C_1(t)\dot{q}_1 + C_2(t)\dot{q}_2 \tag{5}$$

Where $C_1(t)$, $C_2(t)$ are the projections of $\dot{q}(t)$ on $\dot{q}_1$ and $\dot{q}_2$ respectively.

$$C_1(t) = \frac{\sin(1-t)\theta}{\sin\theta} \qquad C_2(t) = \frac{\sin t\theta}{\sin\theta}$$

$$\theta = \cos^{-1}(\dot{q}_1 \cdot \dot{q}_2) = \cos^{-1}(q_{10}q_{20} + q_{11}q_{21} + q_{12}q_{22} + q_{13}q_{23})$$

The greatest advantage of Slerp is that the interpolation is smooth. It can avoid all the problems that plagued with the interpolation of Euler angles (Dunn, 2002).

3. COLLINEARITY EQUATIONS DESCRIBED BY UNIT QUATERNION

Collinearity equations are the rigorous image forming equations for central projection, which are usually considered as the most important equations in photogrammetry. The collinearity equations between object space coordinates (X,Y,Z) and image space coordinates (x,y) are (Wang, 2007):

$$\begin{cases} x = -f\dfrac{\overline{X}}{\overline{Z}} = -f\dfrac{a_1(X - X_S) + b_1(Y - Y_S) + c_1(Z - Z_S)}{a_3(X - X_S) + b_3(Y - Y_S) + c_3(Z - Z_S)} \\ y = -f\dfrac{\overline{Y}}{\overline{Z}} = -f\dfrac{a_2(X - X_S) + b_2(Y - Y_S) + c_2(Z - Z_S)}{a_3(X - X_S) + b_3(Y - Y_S) + c_3(Z - Z_S)} \end{cases} \tag{6}$$

Where (X_S, Y_S, Z_S) are the spatial coordinates of the sensor perspective centre at the moment when the image is taken; f is the focal length of the sensor; $a_i, b_i, c_i (i = 1, 2, 3)$ are the elements of a rotation matrix R which is determined by three Euler angles $(\omega, \varphi, \kappa)$.

If the three EO angular parameters $(\omega, \varphi, \kappa)$ in equations (6) which are used to model the attitude of a sensor are replaced by unit quaternion, then the collinearity equations based on unit quaternion can be obtained. Here the above two formulas don't change themselves, only that the rotation matrix R is not constructed by Euler angles $(\omega, \varphi, \kappa)$ any more but the quaternion components (q_0, q_1, q_2, q_3):

$$R = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

And soon, the EO parameters change to $(X_S, Y_S, Z_S, q_0, q_1, q_2, q_3)$ correspondingly. To solve these parameters in aerial triangulation or single image resection, the collinearity equations must be linearized.

Applying Taylor expansion of x, y with respective to EO parameters $(X_S, Y_S, Z_S, q_0, q_1, q_2, q_3)$, we obtain the following error equations:

$$\begin{cases} v_x = a_{11}\Delta X_S + a_{12}\Delta Y_S + a_{13}\Delta Z_S + a_{14}\Delta q_0 + a_{15}\Delta q_1 \\ \quad + a_{16}\Delta q_2 + a_{17}\Delta q_3 - a_{11}\Delta X - a_{12}\Delta Y - a_{13}\Delta Z - l_x \\ v_y = a_{21}\Delta X_S + a_{22}\Delta Y_S + a_{23}\Delta Z_S + a_{24}\Delta q_0 + a_{25}\Delta q_1 \\ \quad + a_{26}\Delta q_2 + a_{27}\Delta q_3 - a_{21}\Delta X - a_{22}\Delta Y - a_{23}\Delta Z - l_y \end{cases} \quad (7)$$

Where

$$a_{11} = \frac{1}{\bar{Z}}(a_1 f + a_3 x), \ a_{12} = \frac{1}{\bar{Z}}(b_1 f + b_3 x), \ a_{13} = \frac{1}{\bar{Z}}(c_1 f + c_3 x)$$

$$a_{21} = \frac{1}{\bar{Z}}(a_2 f + a_3 y), \ a_{22} = \frac{1}{\bar{Z}}(b_2 f + b_3 y), \ a_{23} = \frac{1}{\bar{Z}}(c_2 f + c_3 y)$$

$$a_{14} = 2\left(-\frac{xy}{f}q_1 + (f + \frac{x^2}{f})q_2 + yq_3\right)$$

$$a_{15} = 2\left(\frac{xy}{f}q_0 + yq_2 - (f + \frac{x^2}{f})q_3\right)$$

$$a_{16} = 2\left(-(f + \frac{x^2}{f})q_0 - yq_1 - \frac{xy}{f}q_3\right)$$

$$a_{17} = 2\left(-yq_0 + (f + \frac{x^2}{f})q_1 + \frac{xy}{f}q_2\right)$$

$$a_{24} = 2\left(-(f + \frac{y^2}{f})q_1 + \frac{xy}{f}q_2 - xq_3\right)$$

$$a_{25} = 2\left((f + \frac{y^2}{f})q_0 - xq_2 - \frac{xy}{f}q_3\right)$$

$$a_{26} = 2\left(-\frac{xy}{f}q_0 + xq_1 - (f + \frac{y^2}{f})q_3\right)$$

$$a_{27} = 2\left(xq_0 + \frac{xy}{f}q_1 + (f + \frac{y^2}{f})q_2\right)$$

$$l_x = x + f\frac{\bar{X}}{\bar{Z}}, \ l_y = y + f\frac{\bar{Y}}{\bar{Z}}$$

This approach has been tested by single image resection (Jiang, 2007; Wang, 2007). Experimental results show that it can get correct solutions under a larger range of initial values than traditional way. It is a useful and valuable supplement to traditional photogrammetry principle. In this paper this method is extended to the bundle adjustment of three-line images while taking the advantages of unit quaternion on the description and interpolation of orientation.

4. BUNDLE ADJUSTMENT OF THREE-LINE IMAGES WITH ORIENTATION IMAGE APPROACH

The principle of orientation image interpolation is illustrated in figure 1, in which k-1 to k+2 these four sampling cycles with equal time intervals are named as OIs. During the triangulation process, only the EO parameters of the OIs are updated, while those of other sampling cycles are interpolated from adjacent OIs by Lagrange polynomials.

In this paper, the classical OI model is modified. The Lagrange polynomials are still employed for interpolating the coordinates of prospective centres, but the Slerp formula is used for camera attitude interpolation. This time the attitude is modelled by unit quaternion (q_0, q_1, q_2, q_3).
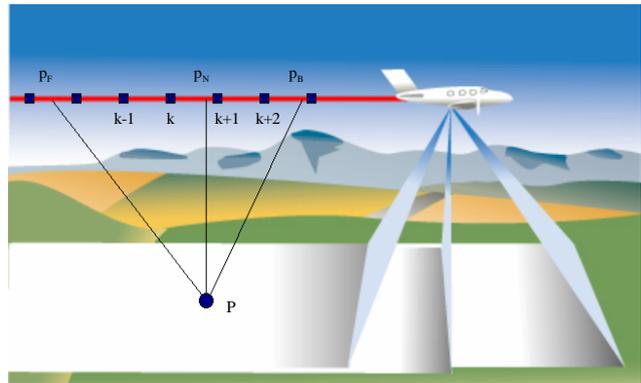
Figure 1. Orientation image interpolation

As illustrated in figure 1, the image point p_N of ground object P on nadir image strip is captured on scan line j, which is located between orientation images k and $k+1$. If the EO parameters of these two OIs are expressed as $(X_S^k, Y_S^k, Z_S^k, \dot{q}^k)$ and $(X_S^{k+1}, Y_S^{k+1}, Z_S^{k+1}, \dot{q}^{k+1})$ respectively, then the EO linear elements (X_S^j, Y_S^j, Z_S^j) of scan line j can be calculated by equation (8), which is implemented in ORIMA (Hinsken, 2002):

$$\begin{cases} X_S^j = t \cdot X_S^k + (1-t) \cdot X_S^{k+1} - \delta X_j \\ Y_S^j = t \cdot Y_S^k + (1-t) \cdot Y_S^{k+1} - \delta Y_j \\ Z_S^j = t \cdot Z_S^k + (1-t) \cdot Z_S^{k+1} - \delta Z_j \\ \delta X_j = t \cdot X_{GPS}^k + (1-t) \cdot X_{GPS}^{k+1} - X_{GPS}^j \\ \delta Y_j = t \cdot Y_{GPS}^k + (1-t) \cdot Y_{GPS}^{k+1} - Y_{GPS}^j \\ \delta Z_j = t \cdot Z_{GPS}^k + (1-t) \cdot Z_{GPS}^{k+1} - Z_{GPS}^j \end{cases} \quad (8)$$

Where $t = \frac{t_{k+1} - t_j}{t_{k+1} - t_k}$, and $(\delta X_j, \delta Y_j, \delta Z_j)$ are the constant items for interpolating error compensation computed from GPS/IMU observations.

The interpolation of sensor attitude of scan line j is carried out by Slerp equation (5), which can be represented in components form:

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} C_1 q_{10} + C_2 q_{20} \\ C_1 q_{11} + C_2 q_{21} \\ C_1 q_{12} + C_2 q_{22} \\ C_1 q_{13} + C_2 q_{23} \end{bmatrix} \qquad (9)$$

Where

$$\dot{q}^k = \begin{bmatrix} q_{10} \\ q_{11} \\ q_{12} \\ q_{13} \end{bmatrix}, \quad \dot{q}^{k+1} = \begin{bmatrix} q_{20} \\ q_{21} \\ q_{22} \\ q_{23} \end{bmatrix}, \quad \dot{q}^j = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

For the collinearity equations corresponding to image point p_N in figure 1, the unknowns include not only the object coordinates of tie points, but also the EO parameters $(X_S^k, Y_S^k, Z_S^k, \dot{q}^k)$ and $(X_S^{k+1}, Y_S^{k+1}, Z_S^{k+1}, \dot{q}^{k+1})$ of adjacent OIs when Slerp is adopted in OI interpolation.

Applying Taylor expansions of equations (8) and (9) with respect to the EO parameters of adjacent OIs, the first-order differential expressions of the EO elements of scan line j can be derived. Substituting them into equation (7) the linearized collinearity equations described by unit quaternion, the error equations of image point observations based on unit quaternion can be obtained.

Considering that the norm of unit quaternion which is used to model camera attitude is equal to one, the correction items $(\Delta q_0, \Delta q_1, \Delta q_2, \Delta q_3)$ are not independent. Parameter adjustment with constraints, which is discussed in detail in (Huang, 1992), is an ideal approach for the aerial triangulation of three-line images based on unit quaternion.

For GPS/IMU supported aerial triangulation, the attitude and positioning information of the sensor acquired by GPS/IMU system should be treated as uncorrelated observations. Usually the coordinates of perspective centre are introduced with time-dependent parameters to compensate the drift effects of GPS system. In view of that the axes of IMU can not aligned with that of the camera perfectly, the relationship between IMU angular observations and exterior orientation angular elements can be modelled by boresight misalignment model, in which the rotation matrix constructed by EO angles $(\omega, \varphi, \kappa)$ are described by concatenated multiplication of the rotation matrices built by IMU observations $(\omega_{IMU}, \varphi_{IMU}, \kappa_{IMU})$ and misalignment angles (e_x, e_y, e_z). This conventional IMU misalignment model is complicated and non-linearized, in which lots of trigonometric functions are involved. Using quaternion the multiplication of two rotation matrices in above model can be simply represented as the production of corresponding unit quaternions, thus only multiplication and addition of quaternion components are involved. Moreover, the calculation of anti-tangent function in traditional model is also avoided. So the model is more robust and has distinct technological advantages over traditional ones.

5. TESTS AND ANALYSES

5.1 Input Data

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Two ADS40 datasets of Waldkirch test field provided by Leica Geosystems® are used for the verification of related algorithms proposed in this paper. The flight was carried out twice in May 2002 (WK0205) and September 2004 (WK0409), and their flying height above ground was 2000m and 3400m with the GSD of 21cm and 36cm respectively. WK0205 block consisted of four west-east (WE) strips and two cross strips; WK0409 consisted of two WE strips and two cross strips. Only the forward, nadir and backward panchromatic images were used in the following experiments. Thirty ground points had been surveyed, and most of them located at the end of zebra crossing. The object coordinates of these 30 points had been calculated at first by multi-ray forward intersection using post-processed POS/AV data. The direct georeferencing errors on horizontal and height were 0.31m and 1.03m respectively for WK0205 block, and 0.88m and 0.29m respectively for WK0409 block.

5.2 Experimental Results of the Block of West-East Strips

According to the OEEPE test on integrated sensor orientation, GPS/IMU aided combined adjustment using very a few control points achieves equivalent accuracy to traditional aerial triangulation (Heipke, 2002). But for airborne three-line images, the question that how many ground control points (GCPs) are needed for triangulation when GPS/IMU data is introduced is of great interest. So different GCP configurations consisting of zero to five GCPs have been designed for the test, other points are treated as check points (CKPs). Table 1 and 2 are the results of WK0205 and WK0409 datasets, for both the cross strips are not considered this moment.

No. of GCPs	No. of CKPs	σ_0 [μm]	RMS[m] XY	RMS[m] Z	RMS/GSD XY	RMS/GSD Z
0	30	3.7	0.323	1.023	1.5	4.9
1A	29	3.7	0.235	0.244	1.2	1.2
1B	29	3.7	0.299	0.277	1.4	1.3
2A	28	3.7	0.253	0.249	1.2	1.2
3A	27	3.7	0.211	0.222	1.0	1.1
4A	26	3.7	0.212	0.229	1.0	1.1
4A + 1B	25	3.7	0.212	0.228	1.0	1.1

Note: A = point at block centre, B = point at block corner.

Table 1. Triangulation results of WK0205 W-E strips

No. of GCPs	No. of CKPs	σ_0 [μm]	RMS[m] XY	RMS[m] Z	RMS/GSD XY	RMS/GSD Z
0	29	3.3	0.596	0.332	1.7	0.9
1A	28	3.3	0.382	0.326	1.1	0.9
1B	28	3.3	0.483	0.345	1.4	1.0
2A	27	3.3	0.359	0.273	1.0	0.8
3A	26	3.3	0.357	0.260	1.0	0.7
4A	25	3.3	0.374	0.211	1.1	0.6
4A + 1B	24	3.3	0.368	0.194	1.0	0.6

Table 2. Triangulation results of WK0409 W-E strips

In above two tests, about 1 pixel accuracy in both horizontal and height directions can be obtained, which proves that the triangulation approach based on unit quaternion proposed in this paper is correct and effective. Checking into the experimental results we can see that block adjustment without ground control points can not improve the positioning accuracy of EO parameters in object space, if GPS/IMU data are introduced as weighted observations. Figure 2 and 3 illustrate the positioning residuals on check points, in which obvious systematic effects are existed. It shows that the error of the orientation data provided by GPS/IMU equipment is systematic. Therefore, a few ground control points are needed to make the best of GPS/IMU data.

On the condition that only one ground control point is used, the positioning accuracy on checking points will be significantly improved. As can be seen, the horizontal and height accuracy of the two experimental data sets is better than 1.4 GSD, which is much closed to the best result. Meanwhile, when only one control point is used to control the orientation of the block, the point locating at the centre is more effective than that on the corners. Furthermore, when three ground control points distributed on different corners of the block are taken into triangulation, the systematic error of GPS/IMU can be thoroughly compensated. The horizontal positioning error is about 1 ground pixel, while the height positioning accuracy is better that 1.1 ground pixels. However, more control points are not responsible for more favourable accuracy.

From the above points, it can be seen that the auxiliary position and attitude data provided by GPS/IMU system is of great inner precision; as a result the amount of control points required by GPS/IMU supported aerial triangulation can be significantly reduced. Generally speaking, if there is a ground control point located in each corner of the survey area, ideal adjustment result can be obtained and both the horizontal and vertical positioning error on checking points is about 1GSD.
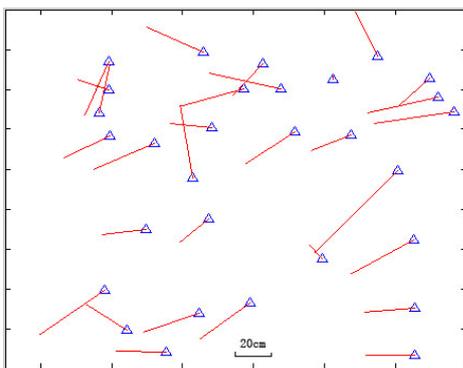
Figure 2. Horizontal errors of triangulation without GCP

5.3 Experimental Results of the Block with Cross Strips

In the GPS supported aerial triangulation, in order to determine the drift parameters of GPS system with high degree of accuracy, two rows of height control points or two cross strips at each end of the block are needed (Yuan, 2001). However, thanks to the introduction of inertial measurement unit (IMU), the performance and reliability of GPS/IMU systems is enormously improved. Therefore, in GPS/IMU supported triangulation we are more concerned with the necessity or the positioning accuracy improved by cross strips. So, in this paper,

all image strips of these two ADS40 datasets are selected for adjustment, corresponding experimental results are listed in table 3 and table 4.
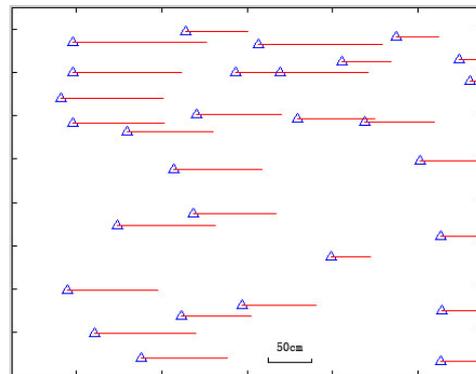
Figure 3. Height errors of triangulation without GCP

No. of GCPs	No. of CKPs	σ_0	RMS[m]		RMS/GSD	
		[μm]	XY	Z	XY	Z
0	30	3.7	0.367	1.034	1.7	4.9
1A	29	3.7	0.240	0.212	1.2	1.0
1B	29	3.7	0.250	0.238	1.2	1.1
2A	28	3.7	0.206	0.243	1.0	1.2
3A	27	3.7	0.196	0.228	0.9	1.1
4A	26	3.7	0.195	0.219	0.9	1.0
4A + 1B	25	3.7	0.194	0.221	0.9	1.0

Table 3. Triangulation results of all WK0205 strips

No. of GCPs	No. of CKPs	σ_0	RMS[m]		RMS/GSD	
		[μm]	XY	Z	XY	Z
0	29	3.3	0.593	0.196	1.7	0.6
1A	28	3.3	0.372	0.235	1.1	0.7
1B	28	3.3	0.415	0.310	1.2	0.9
2A	27	3.3	0.361	0.370	1.0	1.0
3A	26	3.3	0.338	0.276	1.0	0.8
4A	25	3.3	0.349	0.222	1.0	0.6
4A + 1B	24	3.3	0.343	0.222	1.0	0.6

Table 4. Triangulation results of all WK0409 strips

Comparing table 3, 4 with table 1, 2 respectively, we find that when no ground control points are used, because of the significant systematic error of the GPS/IMU observations of cross strips, the effect of block adjustment is limited and the positioning accuracy on check points is equivalent to that of direct georeferencing, If there are few control points (less than three) participate in triangulation, block adjustment with additional cross strips can improve the positioning accuracy of GPS/IMU data to a small extent, which is less than 0.2 GSD both in horizontal and height directions. Meanwhile, if the strength of ground control exceeds that locating three control points at different corners of the block, only about 0.1 GSD improvement of accuracy can be obtained when cross strips are involved into triangulation, which is not of essential difference comparing to above results.

Therefore, considering such factors as flight costs and the workload of data storage and post-processing, cross strips are not cost-effective and are not necessary to be adopted in practical applications.

6. CONCLUSIONS

In this paper, quaternion is introduced into the block adjustment with orientation image method of three-line images. The collinearity equations expressed by unit quaternion are derived, and then corresponding error equations of image point coordinates observations are deduced in detail. Experimental results show that the Slerp based block adjustment with orientation image method is an ideal technique for the aerial triangulation of three-line images. It not only enables desired positioning precision, which is about 1 GSD in both horizontal and height directions, but also effectively avoids the limitations existing in attitude interpolation by Euler Angles. Therefore it is of great potential in such practical projects as the block adjustment of three-line images and the calibration of GPS/IMU systems.

REFERENCES

Diebel J., 2006. Representing Attitude: Euler Angles, Unit Quaternion, and Rotation Vectors. Stanford University. CA, USA. http://ai.stanford.edu/~diebel/attitude.html (accessed 6 Oct. 2006).

Dunn, F., Parberry, I., 2002. *3D math primer for graphics and game development*. Wordware Publishing Inc, Texas, pp.147-193.

Ebner, H., Muller, F., Zhang, S.L., 1988. Studies of Object Reconstruction from Space Using Three Line Scanner Imagery. *International Archives of Photogrammetry and Remote Sensing*, Vol. 27 (11), pp.242-249.

Heipke, C., Jacobsen, K., Wegmann, H., 2002. Analysis of the Results of the OEEPE Test "Integrated Sensor Orientation". *OEEPE Official Publication No. 43*. Frankfurt, Germany.

Hinsken, L., Miller, S., Tempelmann, U., Uebbing, R., Walker, S., 2002. Triangulation of LH Systems' ADS40 Imagery Using ORIMA GPS/IMU. *Proceedings of ISPRS Commission III "Photogrammetric Computer Vision"*. Graz, Austria.

Hofmann, O., Nave, P., 1984. DPS-A Digital Photogrammetric System for Production Elevation Models (DEM) and Orthophotos by Means of Linear Array Scanner Imagery. *PE&RS*, 50(8), pp.1135-1143.

Horn, B. K.P., 1987. Closed-from Solution of Absolute Orientation Using Unit Quaternion. *Journal* of the *Optical Society of America*, Vol. 4(4), pp. 629-643.

Huang, W.B, 1992. *Modern Adjustment Theory and Its Application (in Chinese)*. PLA Press, Beijing, pp. 48-56.

Jiang, G.W., Jiang T., Wang, Y., Gong H., 2007. Space Resection Independent of Initial Value Based on Unit Quaternions (in Chinese). *Acta Geodaetica et Cartographica Sinica,* 36(2), pp. 169-175.

Scholten F., Sujew S., Wewel F., et al, 1999. The High Resolution Stereo Camera (HRSC)-Digital 3D-Image Acquisition, Photogrammetric Processing and Data Evalution. *Proceedings ISPRS Joint Workshop "Sensors and Mapping from Space 1999"*. Hannover, Germany.

Tempelmann, U., Börner, A., Chaplin, B., Hinsken, L., Mykhalevych, B., Miller, S., Recke, U., Reulke, R., Uebbing, R., 2000. Photogrammetric Software for the LH Systems ADS40 Airborne Digital Sensor. *International Archives of Photogrammetry and Remote Sensing*, Amsterdam, the Netherlands, Vol. XXXIII, part B2, pp. 552-559.

Tempelmann, U., Hinsken, L., Recke, U., 2003. ADS40 Calibration and Verification Process. *Proceedings of "Theory, Technology and Realities of Inertial/GPS Sensor Orientation"*, Castelldefels, Spain.

Wang, Y., Jiang, T., Jiang, G.W., 2007. Space Resection of Single Image Described by Unit Quaternion(in Chinese). *Journal of Zhengzhou Institute of Surveying and Mapping*. 24(2), pp. 133-135.

Wang Z.Z., 2007. *Principles of Photogrammetry (in Chinese)*. Wuhan University Press, Wuhan, pp. 1-18.

Yuan, X.X., 2001. *The Theory and Application of GPS Supported Aerial Triangulation (in Chinese)*. Surveying and Mapping Press, Beijing.

Zhan, H.S., Li, G.X., Wang, H.L., 2004. *Math Method in 3D Games and Computer Graphics (in Chinese)*. Tsinghua Press, Beijing, pp. 46-50.