

# ACCURACY OF LACUNARITY ALGORITHMS IN TEXTURE CLASSIFICATION OF HIGH SPATIAL RESOLUTION IMAGES FROM URBAN AREAS

M. N. Barros Filho<sup>a,\*</sup>, F. J. A. Sobreira<sup>b</sup>

<sup>a</sup> Faculty ESUDA, R. Barão de Itamaracá, 460/1802, 52020-070, Recife, Brazil  
- mbarrosfilho@gmail.com

<sup>b</sup> UNICEUB - Centro Universitário de Brasília, Brasília - DF, Brazil  
- fabiano.sobreira@gmail.com

Commission III, WG III/4

**KEY WORDS:** Accuracy, Algorithms, Texture, Classification, High Resolution, Urban

## ABSTRACT:

Lacunarity based measures can be described as texture recognition approaches that provide a flexible yet theoretically consistent mean of characterizing the morphology of urban spatial patterns across different scales. This paper proposes a comparison between the Gliding-Box and Differential Box-Counting algorithms based on the concept of lacunarity to recognize and classify textures of urban areas with different inhabitability conditions through the analysis of binary and grayscale images from 30 @ Quickbird sensor image samples from Recife (Brazil), captured in October 2001, with 250 x 250 meters in size. Results show that the Differential Box Counting algorithm applied in grayscale images improves the discrimination between textures from urban areas with different inhabitability conditions, and it reveals a strong correlation between urban morphology and socioeconomic patterns.

## 1. INTRODUCTION

Satellite images are rich sources of information about earth surface, and provide wide coverage, frequent updates and relatively low costs. Over the last years, with the significant improvements in image resolutions and digital image processing techniques, remote sensing activities have been increasingly focused on urban environments. The development of techniques to extract and classify high resolution satellite images is an important task in urban planning, especially those from developing countries which lack reliable and accurate maps to represent their rapid and informal growth.

Despite current advances in remote sensing technologies, accurately classifying high spatial resolution satellite images into urban land-use classes remains a challenge. Urban areas are composed of objects with different forms and materials, and high spatial resolution images from these areas lead to a more complex combination of surfaces with different spectral reflectance (Myint, 2007).

Besides that, high spatial resolution images have, in general, low spectral resolution (a small number of bands), and this may make difficult the distinction between different urban features with similar reflectance in the visible wavelength (Donnay et al, 2001). Traditional per pixel spectral classification algorithms like maximum likelihood do not take into account the spatial arrangement of neighborhood pixels, and spectral attributes alone cannot provide good classification results.

Texture is a description of the spatial variability of pixel tones in a digital image, and it may improve image classification of urban areas. Texture analysis of digital images aims to recognize and to distinguish spatial arrangements of gray levels values, based on methods which measure the spatial variability

of pixel tones in an image. The higher the variability, the less homogeneous or uniform will be the image texture (Barros Filho and Sobreira, 2005).

A texture pattern is scale dependent. It may varies significantly according to the size and spatial resolution of a digital image. A very small image may contains parts of a pattern, and it may not be able to characterize the whole pattern, whereas a large image may be composed of more than one single pattern and could not be able to properly describe it as well. In the same way, a pixel in a low spatial resolution image may represent an integrated sign of many patterns smaller than the pixel size. As the spatial resolution increases the image pixels could become smaller than the analyzed pattern, generating spectral noises that degraded image classification (Mesev, 2003).

Lacunarity based measures provide a flexible yet theoretically consistent mean of characterizing texture patterns across different scales. Experiments with binary @ Quickbird images from the city of Recife (Brazil) showed that it is possible to distinguish texture patterns of urban areas with different inhabitability conditions. Urban areas with better inhabitability conditions had high lacunarity values than those with worse conditions. These differences, however, tend to decrease as the scales become finer (Barros Filho, 2006; Barros Filho and Sobreira, 2007). These results are coherent with another experiment done in the city of Campinas (Brazil), when textures of binary @ Ikonos images were analyzed (Barros Filho and Sobreira, 2005).

Experiments with binary @ CCD/CBERS-2 image data of 20 m spatial resolution from the same city (Recife, Brazil), showed an opposite relation between lacunarity and inhabitability: in general, image from urban areas with better inhabitability conditions have lower lacunarity values than those from poor

---

\* Corresponding author.

areas. This is because small gaps, very common in slums, cannot be detected in low spatial resolution images (Barros Filho, 2007).

Despite these relevant findings, binary image samples do not represent the real textures of the original images. Lacunarity results strongly depend on the image threshold (Alves Junior and Barros Filho, 2005). Moreover, some valuable information about the spatial arrangement of the image tones may be lost when a gray-level image with 8 bits, for instance, is converted to a binary one with only 1 bit. In order to avoid this problem, new lacunarity algorithms have been proposed to work with grayscale images (Dong, 2000; Myint et al, 2006).

In this context, the present paper focus on the effects of spatial scales and radiometry of high spatial resolution satellites images in texture classification of urban areas with different inhabitability conditions. For this purpose, we calculated lacunarity measures of binary and grayscale images through different algorithms. Before introducing lacunarity, some characteristics of the fractal theory should be described.

## 2. FRACTALITY

As said above, any texture pattern is scale dependent. Actually, there is not a single or preferred scale to characterize a texture pattern. According to Mandelbrot (1982), the length of a coastline in a map depends on the ruler size used to measure it. In other words, if the coastline is measured with a large ruler, many details will not be calculated. Whereas if this same coastline is measured with a smaller ruler, a large number of measurement would be required, and bigger would be its length. The more accurately we try to measure the length of a fractal curve, the longer we find it. As the size of the ruler tends to zero, the length of the curve tends to infinite.

Considering this relation, it is possible to generalize that any spatial pattern is governed by a scaling law: the bigger your size  $r$  is, the smaller will be its quantity  $N(r)$ . Mathematically, this relationship can be represented by a logarithm graph which plots the pattern size (X axis) by its frequency (Y axis). The slope of the regression line in this graph corresponds to the fractal dimension  $D$ , and express the level of irregularity of the pattern or its space filling efficiency (Mandelbrot, 1982).

Fractal dimension is based on the hypothesis that spatial patterns are self-similar, that is, they repeat themselves among many scales and exhibit a certain hierarchical dependency when they are simultaneously analyzed in different scales. This hierarchical dependency may provide valuable information to characterize such patterns. Many image texture analyses have been used to differentiate urban land-use classes, applying and comparing diverse fractal dimensions.

There are many algorithms to estimate the fractal dimension of an image. One of the most applied is the Box-Counting algorithm. According to this algorithm, an image is systematically covered with grids. Each one is composed by adjacent boxes of size  $1/2^n$ , where  $n$  takes increasing integer values from 0 to infinite. For each successive grid, the number of boxes  $N(n)$  of this size which are needed to cover the image are counted. The fractal dimension is defined as:

$$D = \lim_{n \rightarrow \infty} \frac{\ln N(n)}{\ln 2^n} \quad (1)$$

Where  $D$  = fractal dimension  
 $n$  = any integer value  
 $N(r)$  = number of boxes of size  $r$

Fractal dimension ( $D$ ) reveals the degree of gray tones concentration in an image (De Keersmaecker et al, 2003). The more uniform the spatial distribution of the mass (a set of selected pixels) in an image, the more close to 2 will be its fractal dimension. Otherwise, if the mass is concentrated in a single point of the image,  $D$  will be zero. Thus, in general, dispersed spatial patterns have values of  $D$  closer to 2 than aggregated ones.

Despite its importance in understand spatial patterns, fractal dimension cannot provide a complete description of urban patterns because such patterns are not exactly self-similar. In other words: images are not really fractals. They do not exhibit the same structure at all scales. Images with similar fractal dimension can have different textures (Mandelbrot, 1982; Lin and Yang, 1986). Individual elements when aggregated gain new properties that cannot be explained by their original properties. This means that inference of results from different spatial aggregated levels may lead to ecological fallacy.

## 3. LACUNARITY

The concept of lacunarity was established and developed from the scientific need to analyze multi-scaling texture patterns in nature (mainly in medical and biological research), as a possibility to associate spatial patterns to several related diagnosis. Regarding texture analysis of urban spaces registered by satellite images, lacunarity is a powerful analytical tool as it is a multi-scalar measure, that is to say, it permits an analysis of density, packing or dispersion through scales. In the end, it is a measure of spatial heterogeneity, directly related to scale, density, emptiness and variance. It can also indicate the level of permeability in a geometrical structure.

Lacunarity can be defined as a complementary measure of fractal dimension or the deviation of a geometric structure from its translational invariance (Gefen et al, 1984). It permits to distinguish spatial patterns through the analysis of their gap distribution in different scales (Plotnick et al, 1996). Gaps in an image can be understood as pixels with a specific value (e.g. foreground pixels in binary images) or a certain interval of values (in grayscale images). The higher the lacunarity of a spatial pattern, the higher will be the variability of its gaps in an image, and the more heterogeneous will be its texture.

There are many algorithms to calculate lacunarity of an image. Among them, two algorithms have been commonly used: Gliding-Box and Differential Box-Counting. In the next section it will describe the basic characteristics of each one.

### 3.1 Gliding-Box

The Gliding-Box algorithm was proposed by Allain and Cloitre (1991). According to this algorithm a box of size  $r$  slides over an image. The number of gliding-box with radius  $r$  and mass  $M$  is defined as  $n(M,r)$ . The probability distribution  $Q(M,r)$  is obtained by dividing  $n(M,r)$  by the total number of boxes.

Lacunarity at scale  $r$  is defined as the mean-square deviation of the variation of mass distribution probability  $Q(M,r)$  divided by its square mean.

$$L(r) = \frac{\sum_M M^2 Q(M,r)}{\left[ \sum_M M Q(M,r) \right]^2} \quad (2)$$

where  $L(r)$  = lacunarity at box size  $r$   
 $M$  = mass or pixels of interest  
 $Q(M,r)$  = probability of  $M$  in box size  $r$

The Gliding-Box algorithm when applied to binary images (images with only 1 bit) counts only the foreground pixels. This is because each pixel in a binary image can only have one of two possible values (either background or foreground). Whereas in greyscale images, one pixel can have many values. In an 8 bits image, for instance, each pixel can have  $2^8$  values. In this case it measures the average intensity of pixels per box which is the difference between the maximum and minimum intensity value at each box of size  $r$  (Karperien, 2007).

### 3.2 Differential Box Counting

The Differential Box-Counting (DBC) algorithm was proposed by Dong (2000) based on the Gliding-Box algorithm described before, and the Differential Box-Counting algorithm proposed by Sarkar and Chaudhuri (1992) to fractal dimension estimation. According to this algorithm, a gliding-box of size  $r$  is placed at the upper corner of an image window of size  $W \times W$ . The window size  $W$  should be an odd number to allow the computed value to be assigned to a central pixel, and  $r < W$ . Depending on the pixel values within the  $r \times r$  gliding-box, a column with more than one cube may be necessary to cover the maximum pixel value by stacking cube boxes on the top of each other. If the minimum and maximum pixel values within a given column fall in cubic box  $u$  and  $v$ , respectively. Then, the relative height of the column will be (Myint et al, 2006):

$$n_r(i, j) = v - u + 1 \quad (3)$$

where  $n_r(i, j)$  = relative height of column at  $i$  and  $j$   
 $V$  = cubic box with maximum pixel value  
 $U$  = cubic box with minimum pixel value

When the gliding-box slides over the  $W \times W$  image window, the mass will be:

$$M_r = \sum_{i,j} n_r(i, j) \quad (4)$$

where  $M_r$  = mass of the grayscale image  
 $n_r(i, j)$  = relative height of column at  $i$  and  $j$

Then, the mass  $M$  in equation 2 is replaced by  $M_r$  in equation 3 to obtain the lacunarity in the  $W \times W$  window. The lacunarity value is assigned to the central pixel of the window and the  $W \times W$  window slides throughout the whole image.

Figure 1 illustrates an example of a column with three cubic boxes  $r \times r \times r$  (boxes 1, 2, and 3) on a image window  $W \times W$ .

The vertical bars within the column represent the pixel gray level values. The red square represent the central pixel of the window which is assigned the lacunarity value. The minimum pixel value is inside box 1 (which correspond to box  $u$  of equation 3). The maximum pixel value is inside box 3 (box  $v$  in equation 3). So, the relative height of the column at this particular location will be:  $3 - 1 + 1 = 3$ .

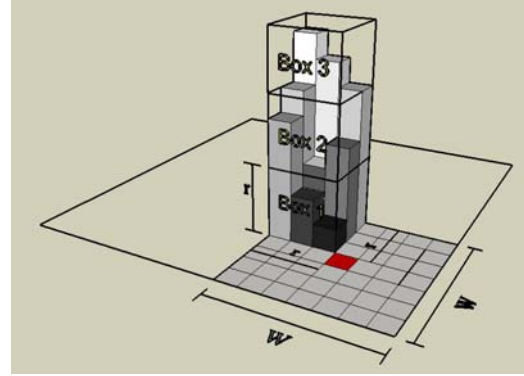


Figure 1. Example of the DBC algorithm

## 4. EXPERIMENTS

The image samples selection firstly requires the construction of a methodology to identify urban areas with high and low inhabitability conditions. A socioeconomic index was created from Census 2000 data of the city of Recife (Brazil). The index values range between 0 and 1. The closer to 1 an urban area is, the higher its inhabitability conditions. Then, these values were geo-referenced to each census sector's centroid of the city. Finally, these centroid's points were interpolated by Ordinary Kriging, generating a raster surface which was used as reference to the image samples selection (Barros Filho, 2006).

Then a RGB composition of a © Quickbird image, with spatial resolution of 0.70 meters, was laid over the raster surface described above, and 30 image samples with 250 x 250 meters were selected from this image. Fifteen of them are from urban areas with high inhabitability condition, while the other fifteen are from areas with low inhabitability conditions. These image samples were then enhanced through a histogram equalization and converted to grayscale and binary images.

Appendixes A and B show, respectively, all the grayscale and binary image samples generated from the 30 RGB images of urban areas with high (images with prefix A) and low (images with prefix B) inhabitability condition. As we can see in those images, the grayscale image offer more information than binary ones. For instance, we can clearly distinguish the shadows of the buildings from the trees and the roads in the grayscale images, while this is not possible to do in the binary images of the same areas. Moreover, buildings are more clearly identified in grayscale images than in the binary ones. Some black areas of the binary images which seem to be non-built areas are actually rooftops with different materials.

Finally, Gliding-Box and Differential Box-Counting lacunarity were estimated for 11 box sizes (in pixels): 2 x 2, 4 x 4, 5 x 5, 8 x 8, 10 x 10, 16 x 16, 20 x 20, 32 x 32, 40 x 40, 80 x 80, and 160 x 160. The maximum box size corresponds to 45% of the image size. The boxes were set up to slides 2 by 2 pixels (on the

horizontal and vertical axes) over the images. The image background was defined as black.

### 5. RESULTS AND CONCLUSIONS

In this section we make a brief discussion about the lacunarity results obtained with the two algorithms presented before, considering their possibilities in the discrimination of textures from the image samples selected in the last section. In order to permit a better comprehension of the difference between these algorithms, the mean lacunarity, *Lm*, of the selected box sizes was calculated and compared through a statistical discriminant analysis.

Table 1 shows the discriminant analysis results from the mean lacunarity values of binary images As and Bs under the Gliding-Box algorithm. As we can see, only 50% of these images were correctly classified (images not shaded in Table 1), that is, the previous group defined by the inhabitability conditions of the urban area where the image was extracted, is the same as the posterior group generated through the discriminant analysis of the mean lacunarity value of this particular image.

Image	Lm	Score	Group	
			Previous	Posterior
A1	1.29	1.21	1	1
A2	1.25	0.20	1	1
A3	1.22	-0.27	1	2
A4	1.23	-0.09	1	2
A5	1.28	0.85	1	1
A6	1.29	11.9	1	1
A7	1.22	-0.25	1	2
A8	1.23	-0.16	1	2
A9	1.22	-0.35	1	2
A10	1.23	-0.24	1	2
A11	1.24	0.18	1	1
A12	1.23	-0.13	1	2
A13	1.21	-0.50	1	2
A14	1.23	-0.23	1	2
A15	1.29	11.06	1	1
B1	1.22	-0.40	2	2
B2	1.22	-0.41	2	2
B3	1.24	0.01	2	1
B4	1.23	-0.13	2	2
B5	1.27	0.77	2	1
B6	1.19	-10.38	2	2
B7	1.20	-0.88	2	2
B8	1.23	-0.20	2	2
B9	1.19	-0.97	2	2
B10	1.26	0.53	2	1
B11	1.25	0.22	2	1
B12	1.21	-0.57	2	2
B13	1.26	0.62	2	1
B14	1.24	0.01	2	1
B15	1.23	-0.13	2	2

Table 1. Discriminant analysis results for binary images under the Gliding-Box algorithm

A better distinction between images As and Bs is generated when the same Gliding-Box algorithm is applied to grayscale images (table 2). In this case, 80% of the image samples were correctly classified. So, we can conclude that the higher the radiometric resolution of an image, the better will be its texture discrimination and therefore a better socioeconomic pattern distinction of image samples.

Image	Lm	Score	Group	
			Previous	Posterior
A1	1.15	0.03	1	1
A2	1.18	0.67	1	1
A3	1.14	-0.03	1	2
A4	1.14	-0.03	1	2
A5	1.18	0.63	1	1
A6	1.18	0.68	1	1
A7	1.14	-0.10	1	2
A8	1.17	0.49	1	1
A9	1.18	0.66	1	1
A10	1.16	0.20	1	1
A11	1.15	0.10	1	1
A12	1.16	0.26	1	1
A13	1.16	0.22	1	1
A14	1.15	0.00	1	2
A15	1.18	0.64	1	1
B1	1.10	-0.96	2	2
B2	1.10	-0.83	2	2
B3	1.11	-0.74	2	2
B4	1.11	-0.77	2	2
B5	1.16	0.30	2	1
B6	1.12	-0.54	2	2
B7	1.14	-0.02	2	2
B8	1.14	-0.03	2	2
B9	1.11	-0.63	2	2
B10	1.12	-0.55	2	2
B11	1.12	-0.49	2	2
B12	1.31	32.47	2	1
B13	1.11	-0.76	2	2
B14	1.10	-0.86	2	2
B15	1.11	-0.77	2	2

Table 2. Discriminant analysis results for grayscale images under the Gliding-Box algorithm

Table 3 shows the discriminant analysis results from grayscale images under the Differential Box-Counting algorithm. In this case, 90% of the selected images were correctly classified. Only 2 image samples (B5 and B7) were assigned to different groups. Considering these results, we can conclude that the DBC applied in grayscale images is the most appropriate algorithm to discriminate texture from urban areas with different inhabitability conditions.

Image	Lm	Score	Group	
			Previous	Posterior
A1	1.14	1.14	1	1
A2	1.17	5.95	1	1
A3	1.14	0.71	1	1
A4	1.14	0.68	1	1
A5	1.17	5.57	1	1
A6	1.17	5.93	1	1
A7	1.14	0.16	1	1
A8	1.16	4.51	1	1
A9	1.17	5.87	1	1
A10	1.15	2.38	1	1
A11	1.15	1.64	1	1
A12	1.15	2.84	1	1
A13	1.15	2.60	1	1
A14	1.14	0.87	1	1
A15	1.17	5.61	1	1
B1	1.09	-6.31	2	2
B2	1.10	-5.28	2	2
B3	1.11	-4.67	2	2
B4	1.10	-4.84	2	2
B5	1.15	3.12	2	1
B6	1.11	-3.17	2	2
B7	1.14	0.69	2	1
B8	1.14	0.62	2	1
B9	1.11	-3.91	2	2
B10	1.11	-3.28	2	2
B11	1.12	-2.81	2	2
B12	1.13	-1.39	2	2
B13	1.10	-4.83	2	2
B14	1.10	-5.54	2	2
B15	1.10	-4.87	2	2

Table 3. Discriminant analysis results for grayscale images under the Differential Box Count algorithm

**REFERENCES**

Allain, C. and Cloitre, M., 1991. Characterizing lacunarity of random and deterministic fractal sets. *Physics Review A*, 44, pp. 3552-3558.

Alves Júnior, S. and Barros Filho, M., 2005. Multiscale measurements of fragmented cities: enhancing urban analysis through lacunarity based measures. In: *VIII GeoComputation International Conference*, University of Michigan, USA.

Barros Filho, M. and Sobreira, F., 2007. Urban textures: a multi-scale analysis of sociospatial patterns. In: *Proceedings of X Computers in Urban Planning and Urban Management*, CUPUM, Foz de Iguacu, Brazil.

Barros Filho, M., 2006. *As múltiplas escalas da diversidade intra-urbana*. Phd Thesis in Urban Development. Federal University of Pernambuco, Brazil.

Barros Filho, M. and Sobreira, F., 2005. Analysing spatial patterns in slums: a multiscale approach. In: *CASA Working Papers*, 87, Centre for Advanced Spatial Analysis. University College London, London, UK. [http://www.casa.ucl.ac.uk/working\\_papers/paper87.pdf](http://www.casa.ucl.ac.uk/working_papers/paper87.pdf) (accessed 16 Aug. 2006)

Der Keersmaecker, M. et al., 2003. Using fractal dimensions to characterize intra-urban diversity. The example of Brussels. *Geographical Analysis*, 35(4), pp. 310-328.

Donnay, J. et al., 2001. Remote sensing and urban analysis. In: *Remote sensing and urban analysis*. Taylor & Francis, London, pp.3-18.

Dong, P., 2000. Test of a new lacunarity estimation method for image texture analysis. *International Journal of Remote Sensing*, 21(17), pp. 3369-3373.

Gefen et al., 1984. Phase transitions on fractals: III infinitely ramified lattices. *Journal of Physics A*, 17, pp.1277-1289.

Karperien, A., 2007. *User's guide for FracLac v.2.5.*, School of Community Health, Faculty of Science, Charles Sturt University, Albury-Wodonga, Australia. <http://rsb.info.nih.gov/ij/plugins/fractalac/FLHelp/Introduction.htm> (accessed 20 Sep. 2007).

Lillesand, T. et al., 2004. *Remote sensing and image interpretation*. John Wiley & Sons. New Jersey, pp70-72.

Lin, B.; Yang, Z., 1986. *A suggested lacunarity expression for Sierpinski carpets*. *Journal of Physics A*, 19, L49-L52.

Mandelbrot, B., 1982. *The fractal geometry of nature*, Freeman, New York.

Mesev, V., 2003. Remotely sensed cities: an introduction. In: *Remotely sensed cities*, Taylor & Francis, London, pp.1-13.

Myint, S., 2007. Urban mapping with geospatial algorithms. In: *Urban remote sensing*, Taylor and Francis, pp.109-135.

Myint, S. et al., 2006. Urban textural analysis from remote sensor data: lacunarity measurements based on the differential box counting method. *Geographical Analysis*, 38, pp. 371-390.

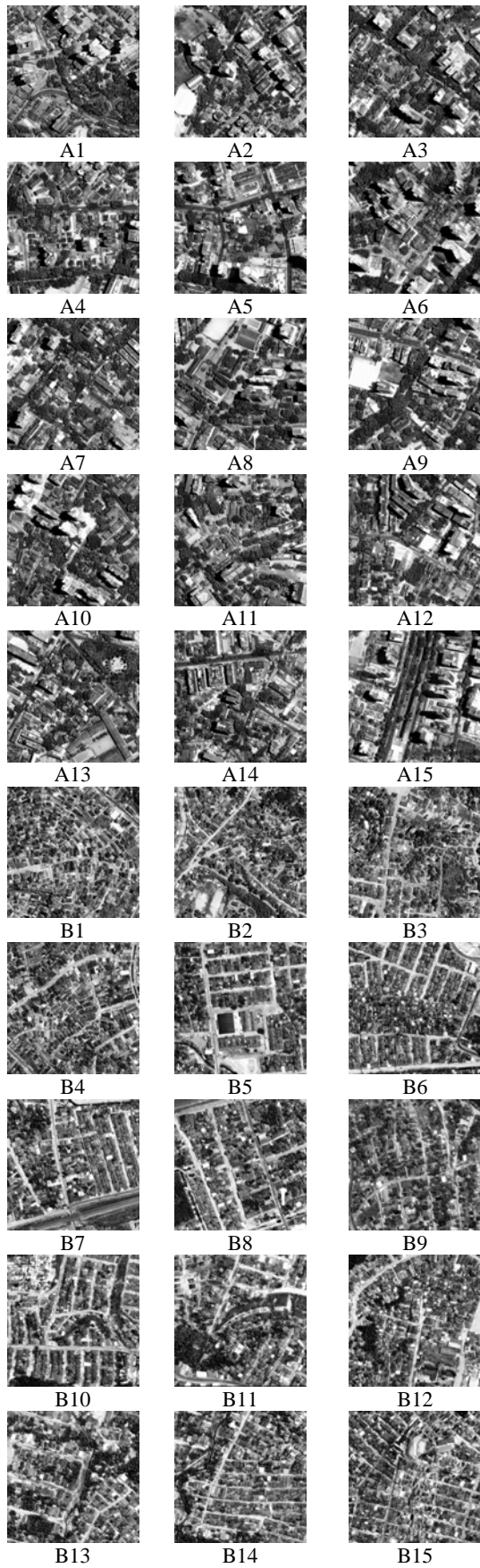
Plotnick, R. et al., 1996. Lacunarity analysis: a general technique for the analysis of spatial patterns. *Physical Review*, 55(5), pp.5461-5468.

Sarkar, N. and Chaudhur, B., 1992. An efficient approach to estimate fractal dimension of texture images. *Pattern Recognition*, 25, pp. 1035-1041.

**ACKNOWLEDGEMENTS**

The authors are very grateful to Soe Myint from Arizona State University for explanations about equation 3, and to Audrey Karperien who develops and maintains the software FracLac at Charles Sturt University, Australia, for explanations about the algorithms applied in this paper. Mauro Barros Filho would like to thank FACEPE for the financial support to this research.

APPENDIX A. GRAYSCALE IMAGES SELECTED



APPENDIX B. BINARY IMAGES SELECTED

