CALIBRATION OF A LOW COST MEMS INS SENSOR FOR AN INTEGRATED NAVIGATION SYSTEM

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ABSTRACT:

The method, the procedure and the results of the calibration of a low cost MEMS INS sensor are described and discussed. The reduced cost of the MEMSs sensors is very advantageous, but these sensors are characterized by much large errors. The accurate calibration of the sensors is very important for the determination of the systematic errors, like bias, scale factor and misalignment of the axes. For this aim, it has been used the multi-position calibration method, which doesn't require the precise aligned mounting of the sensors with either local level frame or to the vertical direction. The equations in the static case, for the accelerometers sensors are shortly described; the results obtained with the Kalman filter in the experimentation of the tri-axial sensor ADIS 16350 are then illustrated.

1. INTRODUCTION

The integration of GPS and inertial systems is very interesting for several technological applications (Mohinder et al. 2001), (Mohinder et al. 2007), (Sansò, 2006) . The MEMS technology allows the realization of new miniaturized low cost sensors. Over the last few years many MEMSs systems have been developed with growing performances, very small size and light weight. These sensors, matched with miniaturized GPS receivers, are the basis of complete low cost and low weight navigation systems (Godha, 2006), (Niu and El-Sheimy, 2005), (Shin, 2001).

The results obtained by such sensors are generally less accurate than the ones typically associated with tactical-grade inertial sensors. However, by means of high quality integration algorithms, it is possible to obtain significant positioning accuracy improvements with respect to GPS systems, when there is limited satellite availability.

The IMU measurements are independent of the GPS signal outages and the frequencies of acquisition are very high. The IMU measures the angular velocities and linear accelerations. The matrix rotation is obtained using the angular rates by gyroscopes and the measured forces are rotated in the required frame using the rotation matrix. The velocities and positions of the body are obtained by integration of the accelerations and angular rates.

Unfortunately the integration process is very sensible to the systematic errors of the IMU. The acceleration bias introduces an error in the velocity proportional to the time t, and an error in the position proportional to t^2. Still worse the gyro bias introduces a quadratic error in the velocity and a cubic error in the position (El Sheimy, 2006).

The performance of integrated GPS/INS system depends of the signal quality of GPS and the correct prediction measurements of the INS, when the GPS signals are blocked.

The position and velocity drift depends of bias, scale factor, misalignment and sensor noise. Then it is very important an accurate procedure of calibration for the initialization of the IMU.

In the case of the gyroscopes generally the following equation is used:

$$l_\omega = \omega + b_\omega + S\omega + N\omega + \varepsilon(\omega) \tag{1}$$

where l_ω is the measured angular rate, ω is the true angular rate, b_ω is the gyroscope instrument bias, S is the linear scale factor matrix, N is the non-orthogonality matrix and $\varepsilon(\omega)$ is the sensor noise.

In the case of the accelerometers generally the following equation is used:

$$l_f = f + b_f + S_1 f + S_2 f^2 + Nf + \delta g + \varepsilon(f) \tag{2}$$

where l_f is the measured acceleration, f is the true specific force, b_f is the accelerometer instrument bias, S_1 and S_2 are the linear and non linear scale factor matrices, N is the non-orthogonality matrix, δg is the deviation from theoretical gravity and $\varepsilon(f)$ is the accelerometer noise.

Generally the position drift is estimated dependent of gyro and accelerometer bias instability, using the following approximate equation:

$$\begin{aligned}\delta p(t) \approx \delta p_0 + \delta v_0 \Delta t + \delta b_{0a}\frac{\Delta t^2}{2} + \delta b_{0g}\, g\frac{\Delta t^3}{6} + \\ + \delta\alpha_0\, g\frac{\Delta t^2}{2} + \delta H_{0z}(V\Delta t) + SF_{0a}\frac{\Delta t^2}{2} + \\ + SF_{0g}(\Delta H_z)(V\Delta t)\end{aligned} \tag{3}$$

where:
δp_0 = position error at time t_0
δv_0 = velocity error at time t_0
Δt = $t - t_0$ = total elapsed time
δb_{0a} = residual accelerometer bias at time t_0 (uncompensated)
δb_{0g} = residual gyro bias at time t_0 (uncompensated)
$\delta\alpha_0$ = Horizontal misalignment at time t_0
δH_{0z} = Azimuth misalignment multiplied by the approximate distance $V\Delta t$
SF_{0a} and SF_{0g} = accelerometer and gyroscope residual scale factor errors
F = sensed acceleration
g = gravity acceleration ($\approx$ 9,81 m/sec^2)

The equations above show the importance of bias, scale-factor and non-orthogonality errors.

The appropriate use of the Kalman filter and the introduction of these parameters like additional states of a filtering algorithm, produce their valuation and the correction of velocity and position drift. But the filter may converge very slow, and may even diverge if inappropriate starting values are given.

For MEMSs sensors, the bias, scale factor and misalignment instabilities are much large and it is important an appropriate procedure for the correct evaluation of the initial parameters of these factors.

2. INS CALIBRATION METHODS AND THE MULTI-POSITION CALIBRATION METHOD

The calibration of the IMU is the process of comparing the instruments outputs with known reference information and the determination of the coefficients in the output equation, that agree the reference information (Chatfield, 1997).

For the tactical grade IMUs the process of calibration requires the inertial system installed on a very levelled platform and the IMU oriented with one of the three axes x, y and z perpendicular to the levelled plane, first in up and then in down position (Titterton and Weston, 2004).

These six positions allow the bias and scale factors determination by solving the equations:

$$b = \frac{l_f^{up} + l_f^{down}}{2} \tag{4}$$

$$S = \frac{l_f^{up} + l_f^{down} - 2\times k}{2\times k} \tag{5}$$

where l_f^{up}, l_f^{down} are the sensor measures in the *up* and *down* positions, while K is the known reference value, equal, in the accelerometers case, to earth gravity, and in the gyroscopes case, to earth rotation vector.

For the determination of the misalignment of the sensor axes, we need to evaluate, at the same time, all error factors by using the misalignment matrix. This matrix contains in the diagonal terms the bias and scale factors too (Niu, 2002).

The system, in the case of the accelerometers, has the form:

$$\begin{bmatrix} l_{ax} \\ l_{ay} \\ l_{az} \end{bmatrix} = \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_{ax} \\ b_{ay} \\ b_{az} \end{bmatrix} \tag{6}$$

where the diagonal elements m_{xx}, m_{yy}, m_{zz}, are the scale factors, the non-diagonal elements are the non-orthogonality factors and the b elements are the bias factors.

The six positions of the IMU on the levelled plate produce 18 equations (6x3) and allow, by the mean square procedure, the bias, scale and axis misalignment factors determination.

This calibration procedure is very delicate and expensive and also very difficult to realise with an IMU made with MEMS sensors.

Then they have been studied and proposed calibration procedures, that don't require the precise alignment of IMU with given directions, and a perfectly levelled positioning plate.

An effective system is the multi-position calibration method (Shin and El Sheimy, 2002), based on 18 different and independent positions of the sensors. The sensors are collocated on a locally levelled plane. The axes of the reference sensors and the reference levelled plan form three angles characterized by directors cosines α, β e γ.

The acceleration components on the group of three axis of the sensors, in the static case, are:

$$\begin{aligned} gx &= g\cos\alpha \\ gy &= g\cos\beta \\ gz &= g\cos\gamma \end{aligned} \tag{7}$$

The equations of the calibration model are:

$$gx^2 + gy^2 + gz^2 = |g|^2(\cos\alpha^2 + \cos\beta^2 + \cos\gamma^2) = |g|^2 \tag{8}$$

This model is valid for the calibration of the accelerometers; we can use the known terrestrial rotation velocity for the gyroscopes calibration equations.

For the MEMS sensors the bias, scale and misalignment factors are very bigger than the same factors of the tactical grade sensors. Thus it is more effective the use of the modified multi-position calibration method (Syed, et al. 2007).

This method needs only the approssimate sensor positions, and we don't need to know exactly the sensor attitude, respect to the three axes of the reference plane. Besides, the MEMS sensors aren't very accurate and the angular velocity error is bigger than the terrestrial velocity rotation. Then for the gyroscopes calibration we use a rotation plate.

The described method (equations 7, 8) takes into account the bias, axes misalignment and scale factors. The non-orthogonality of the sensor axes is given by three misalignment angles: given a fixed x axis, the misalignment of y axis is obtained by a rotation angle θ_{yz} of y axis respect to z axis, and by the rotation angles of the z axis θ_{zx} and θ_{zy}, respect to x and y axis.

By using the rotation matrices, and assuming small values for the above defined rotation angles, we obtain the formulation of the problem:

$$\begin{bmatrix} gx_1 \\ gx_2 \\ gx_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\vartheta_{gyz} & 1 & 0 \\ \vartheta_{gzx} & -\vartheta_{gzy} & 1 \end{bmatrix} \begin{bmatrix} gx \\ gy \\ gz \end{bmatrix} \tag{9}$$

where gx_1, gx_2, and gx_3 are the components of the acceleration along the misaligned axes.

The equation (9) can be rewritten, taking into account both bias and scale factors; we obtain the equation:

$$\begin{bmatrix} l_{gx} \\ l_{gy} \\ l_{gz} \end{bmatrix} = \begin{bmatrix} 1+s_{gx} & 0 & 0 \\ -\vartheta_{gyz} & 1+s_{gy} & 0 \\ \vartheta_{gzx} & -\vartheta_{gzy} & 1+s_{gz} \end{bmatrix} \begin{bmatrix} gx \\ gy \\ gz \end{bmatrix} + \begin{bmatrix} b_{gx} \\ b_{gy} \\ b_{gz} \end{bmatrix} \tag{10}$$

where l_{gx}, l_{gy}, l_{gz}, are the sensor measures, s_{gx}, s_{gy} and s_{gz} are the scale factors and b_{gx} b_{gy} b_{gz} the bias factors along the axes x, y e z of the reference frame.

By using the abovementioned model, the state equations have the form:

$$gx^2 + gy^2 + gz^2 - |g|^2 = 0 \tag{11}$$
$$\omega x^2 + \omega y^2 + \omega z^2 - |\omega|^2 = 0 \tag{12}$$

for accelerometers and gyroscopes respectively.

The unknowns of these equations are the misalignment parameters, the scale factors and the bias factors, and can be solved by using a least squares procedure.

The combined system of (11) and (12) uses parameters properly weighed (Krakiwsky, 1990) and must be linearized. For writing the state equations, the precise determination of the sensor axes is not required. To have a number of equation exceeding the unknowns number, it is sufficient to position the IMU in different orientations, as possible distinct and independent each other.

3. THE STATIC CASE - THE ANALISYS OF THE ACCELEROMETER RESULTS WITH KALMAN FILTER

In this section we describe the test performed on an IMU, for the determination of bias, scale and misalignment factors of the accelerometers.

First we have analyzed the accelerometers measures of the tri-axis sensors, obtained in the static case, by using a Kalman filter.

The utilized instrumentation is a MEMS IMU made by Analog Devices: the ADIS 16350.

The ADIS 16350 is a complete triple axis gyroscope and triple axis accelerometer inertial sensing system. For the tri-axis accelerometer the measurement range is ± 10 g, with 14 bit resolution, the axis non-orthogonality is ± 0,25 degrees at 25°C and the bias is 0,7 mg at 25 °C, with a 4 mg/°C temperature coefficient.

In the model of static case, with constant acceleration, we can write the following state equations:

$$\begin{cases} x_{t+1} = x_t + v_t \Delta t \\ v_{t+1} = v_t + a_t \Delta t \\ a_{t+1} = a_t + \varepsilon_{t+1} \end{cases} \tag{13}$$

where:
x is the position, v is the velocity and a is the acceleration; t and t+1 indicate the time, Δt is the time rate of acquisition and ε_t is the model stochastic error.

For the tri-dimensional case, the matrix formulation is:

$$\mathbf{x}_{t+1} = \mathbf{\Phi}_t \mathbf{x}_t + \boldsymbol{\varepsilon}_t \tag{14}$$

where **x** is the unknowns vector with nine components (three of position, three of velocity and three of acceleration), $\mathbf{\Phi}$ is the state matrix having 9x9 dimension, and $\boldsymbol{\varepsilon}$ is the model stochastic error vector with nine components.

The observation equation is:

$$\mathbf{z}_t = \mathbf{H}\mathbf{x}_t + \mathbf{v}_t \tag{15}$$

where **z** is the vector of the observation of the tri-axial sensor with three components (the acceleration measures of the sensor), **H** is the measurements matrix (3x9) and **v** is the noise sensor vector.

The rate acquisition was 1/200 sec.

The Kalman filter has been applied, and the results of position, velocity and acceleration obtained without and with Kalman filter, have been compared.

Figures 1 and 2 show the acceleration without Kalman filter and with Kalman filter, for 9000 and for 500 acquisitions. Figures 3 and 4 show velocity and position obtained with Kalman filter. Figure 5 shows the position without Kalman filter.

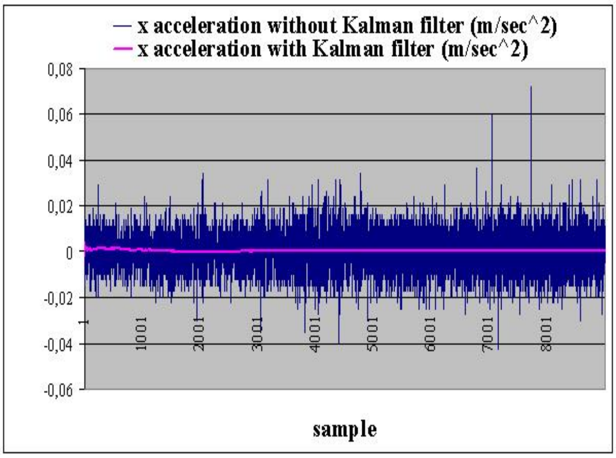

Figure 1. acceleration along x axis without and with Kalman filter

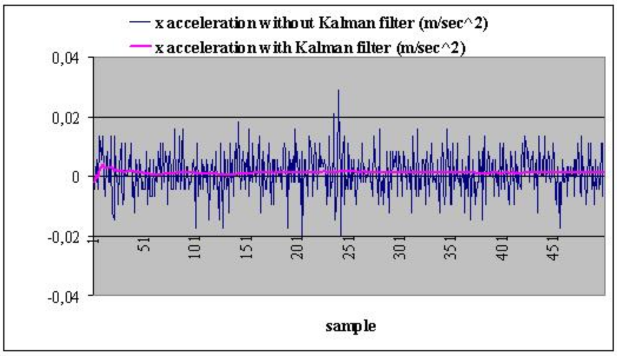

Figure 2. x acceleration for 500 acquisitions

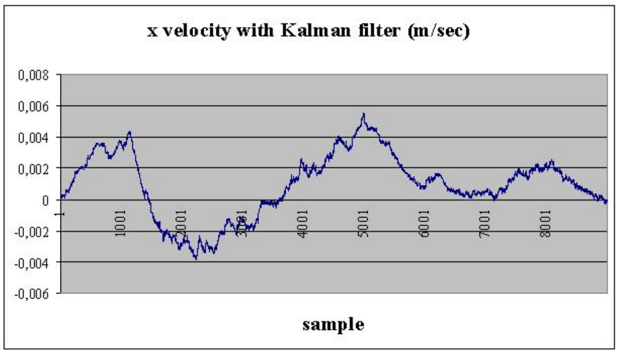

Figure 3. Velocity with Kalman filter

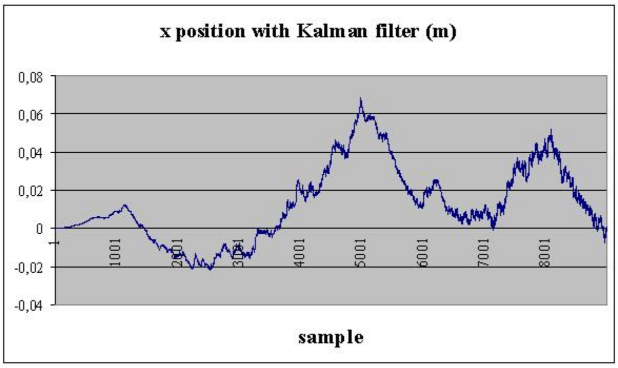

Figure 4. Position with Kalman filter

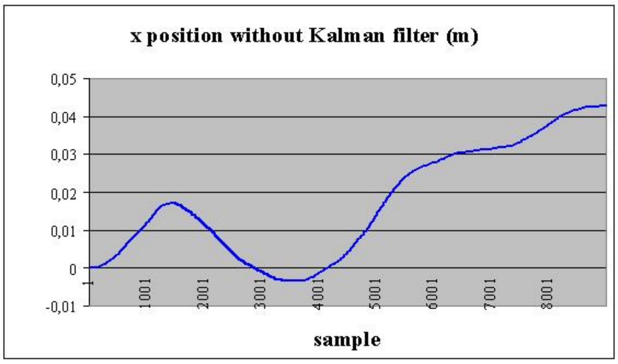

Figure 5. Position without Kalman filter

The above drawings show the Kalman filter effect on the accelerations and the decreasing drift of the position.

4. THE ACCELEROMETER CALIBRATION

The described multi position calibration method has been used for determination of bias, scale and misalignment factors of the ADIS 16350 IMU.

The procedure has been applied only for the tri-axial accelerometers.

The IMU has been placed in 39 different positions: the angle between two successive positions is about 45°. The orientations have been obtained by positioning the sensor on a rotating instrument (Figure 6). The precise rotation is unknown, however, in multi position calibration method, the precise attitude of sensors is not required; only the independence of equations for the rapid convergence of results is needed.

Figure 6. The instrument used for positioning the MEMS INS

The complete convergence of results required about 10 iterations of calculus. Figure 7 shows the convergence of the bx parameter; in figure 8, an enlargement of figure 7 is shown.

The calibration results are shown in Table 1, where:

- bx, by, bz are the bias factors divided by the gravity acceleration g;
- sx, sy and sz are the scale factors;
- θ_{yz}, θ_{zx}, θ_{zy} are the misalignment angles in radians.

In the first column are reported the parameters, while in the second column the relevant variances are shown.

The parameter values are in some cases higher than the ones declared by the datasheet, also by taking into account the influence of temperature.

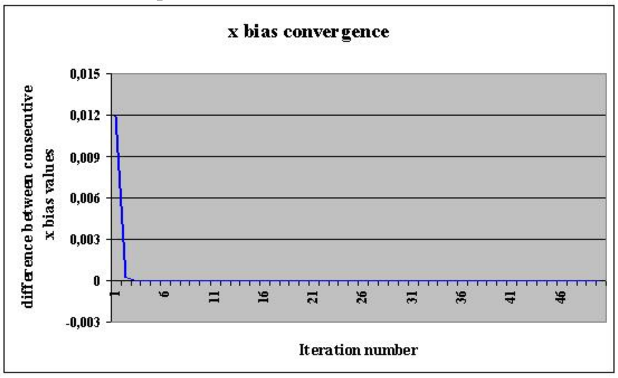

Figure 7. bx convergence

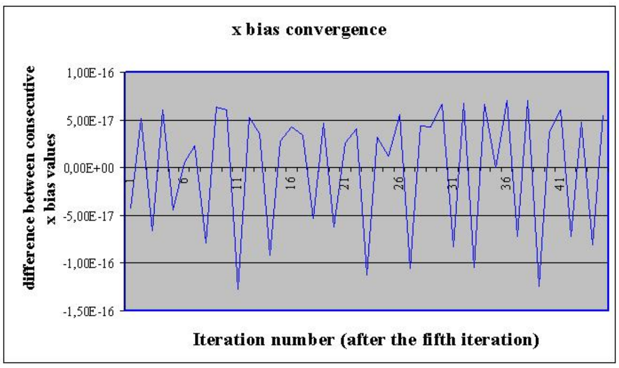

Figure 8.Oscillation of bx parameter after the fith iteration

bx (g)	0,012133	7,807 e-07
by (g)	0,023295	3,876 e-07
bz (g)	-0,03593	4,380 e-07
sx	0,00775	1,491 e-06
sy	0,008838	6,123 e-07
sz	0,008041	5,746 e-07
θ_{yz} (rad)	-0,00086	2,056 e-06
θ_{zx} (rad)	0,01142	1,597 e-06
θ_{zy} (rad)	-0,00049	2,554 e-06

Table 1. Accelerometers bias, scale and disalignment factors (first column) and variances (second column)

The sensors are differently influenced by temperature. The temperature difference among the three axes, is about 1-2 °C. Besides, about two minutes after the turn on are needed, to stabilize the temperature. This is shown in Figure 8.

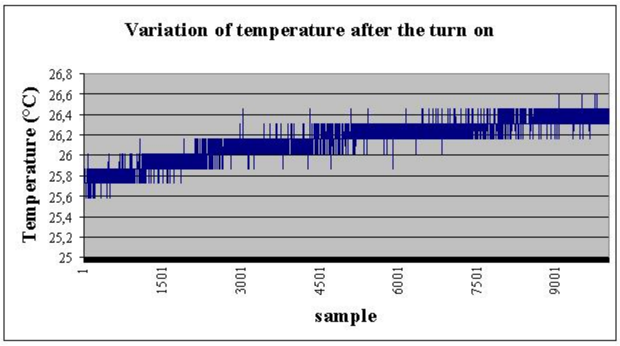

Figure 8. Variation of temperature after the turn-on.

5. CONCLUSION AND FUTURE WORKS

The multi-position method has been used for the calibration of a MEMS INS. The method is very fast and efficient. The convergence of the results, in the accelerometers calibration case, is relatively rapid and the procedure is not expensive.

With reference to the calibrated sensor (ADIS 16350), the results showed the strong influence of temperature.

In the future, the implementation of a new modified multi calibration method, which theoretical formulation we set up, is foreseen.

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