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GEOMETRIC MODEL OF PROJECTION PLANE FOR BILINEAR BUNDLE OF RAYS

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ABSTRACT: The development of aerial photography methods and those of photogrammetric plotting has set forward some problems which go beyond the theory connected with isomorphous representation of orthoscopic bundle of rays and planes on projection plane. In this paper a new approach to the projection plane is proposed, the said plane being regarded as a result of reflection of bilinear bundle consisting of two crossing lines and the totality of rays and second-order surfaces passing through those lines.

There are theoretical and applied tasks which overstep the limits of classic photogrammetry and orthoscopic optics. They were devepoped with airsurvey methods as well as with image photogrammetric processing methods aimed at map compilation and updating. Among those there is a developement of theory and methods for photogrammetric processing of images, obtained by non-instant exposure cameras. They include processing of pictures, taken by blind-slot, slot, scaner, panoramic cameras, as well as by photo-television and conventional images, and parts of images (fragments, copies etc.) with unknown or aproximately known parameters of photo exposure or copy-making.

Studies of the emerged tasks could not evade the projecting reflection theory of one of the most important branches of mathematics. It is discriptive geometry which is the theoretical base of photogrammetry. The orthoscopic pencil of projecting straight lines (rays) and planes, being one of the main geometrical patterns of discriptive geometry was proved to be the only type of bundles which can be isomorphically reflected on a projecting plane. There are other types of bundles which can be simply reflected at a projecting plane. One of such bundles is a bilinear pencil of projective rays and planes. The name is given because the two crossing straight lines a_s and a_s' are the guiding lines of the bundle. They are geometrical places of rays' intersection points (Fig.1).

The before mentioned name of "anamorphotic bundle" (1, 2,3) is applied for particylar case of bilinear bundle with orthodox crossing guiding lines.

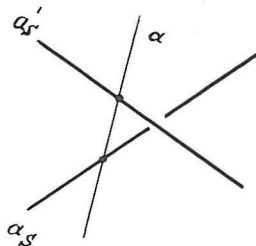Bilinear bundle of rays differs from the orthoscopic one. There are two intersecting straight lines, which are guiding lines of the bundle and of the multiplicity of all rays, planes and second order linear surfaces, but not just single point and multiplicity of lines and planes appertained to that point.

The difference of bilinear bundle from orthoscopic one is in fact, that it has to be put in simple conformity to the projecting plane. The bundle rays conform to the points of projecting plane as in case of orthoscopic bundle; while not only planes, but second order linear surfaces conform to the straight lines.

a_s' α

α_s

Fig.1

The argument that bilinear bundle can be put into simple conformity to the projecting plane is as follows.

One can draw a plane through each guiding line a_s and a_s' of the bundle (Fig.2) so as it would pass through intersecting point P of projecting plane and the other guiding line of the bundle. But in this case all the three planes H, H' and P would intersect along single straight line VV'. They would make up a single bundle of planes with axis of VV'. This axis is a special ray of the bilinear bundle as it fully belongs to the projecting plane while being the

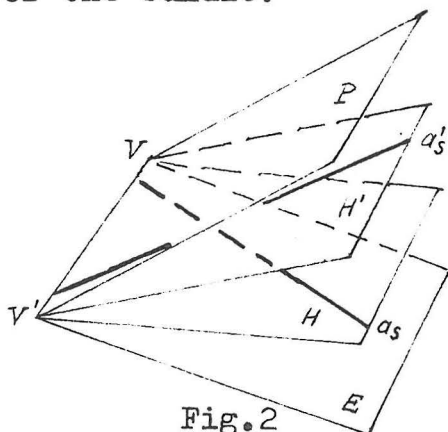common guiding line of all the second order linear surfaces of the bundle.

Fig.2

The following inference can be drawn from the above. If any second order linear surface passes through both guiding lines of bilinear bundle and two its planes make single bundle of planes with a third one, which does not belong to the bilinear bundle, then this linear surface would intersect the third plane along a straight line only, as the given third plane would be tangential to any linear surface of the bundle, drawn through its guiding lines. However a multiplicity of different second order linear surfaces can be drawn through the guiding lines of the bundle, with each of them intersecting the above mentioned plane along a single straight line, which does not coincide with the other ones. The points of this straight line would appertain both to the rays and to the straight line. Hence on the one hand this plane is isomorphically projected on a bilinear bundle while as it was mentioned above, rays of the bundle correspond to the points of plane, whereas linear surfaces and planes of the bundle correspond to its straight lines; on the other hand this plane is a projecting one, as it consists only of points and straight lines, interconnected by the rule of identity, i.e. by the reciprocal affiliation.

If one would draw any planes E through axis VV' of a bundle of three planes H,H' and P, then all these planes would be projecting ones, as related both to the bilinear bundle and between themselfs. Therefore a bilinear bundle provides projecting transformation of the planes of its guiding lines have common points with intersecting line of these planes. It is expedient to name the intersection line of projecting bilinear bundle planes as "biperspective"axis, which is simular to the perspective axis in an orthoscopic bundle. Thus bilinear pencil of projecting rays is naturally considered as a bundle of linear surfaces of the second order. They have one common straight line of one series (the axis of biperspective) and two common straight lines of another series (the guiding lines of bundle); the latter are featured by intercompliance of the straight lines of linear surfaces' series, as well as by a build up of interprojecting planes' bundles with axis, intersecting the guiding lines of the bundle.

Interlocation of Bundle and Projecting Plane

A bilinear bundle can be in three different positions as regards to the projecting plane.

1. Both guiding lines of the bundle (Fig.2) have the only common points of their own with the projecting plane.

In this case the bundle has one special surface. Its guiding lines have common non-proper points with projecting plane, which make up one common non-proper straight line.
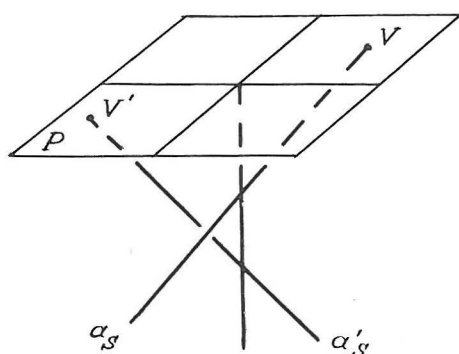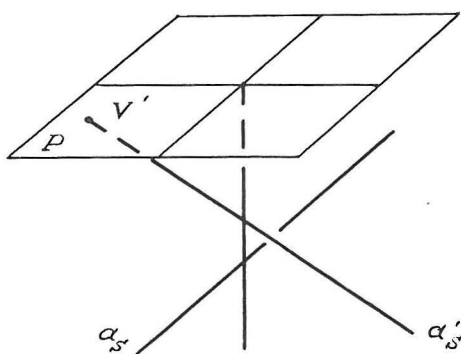
Fig.3

Fig.4

This surface is a hiperbolic paraboloid.

2. One of the bundle's guiding lines (Fig.4) has a proper point with a projecting plane, whereas the second, the special one, has a non-proper point.

In this case the bundle has one special plane, passing through the special guiding line and having one proper common point with the second guiding line. The special plane has one common non-proper point with the projecting plane.

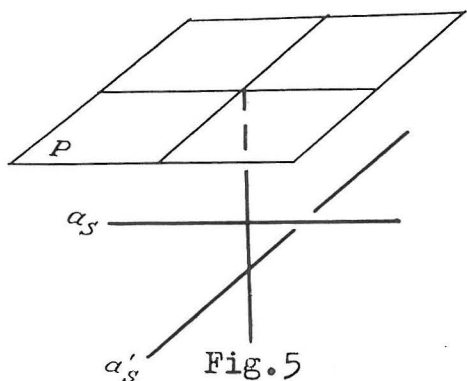3. Both guiding lines of the bundle are special straight lines.

Fig.5

In this case two special planes belong to it. They have common non-proper straight line with the projecting plane, but not with the belonging to them guiding lines - common non-proper points. The bilinear bundle with two special guiding lines has a common with the plane non-proper ray* and therefore it is brought into a simple compliance with the affine-projecting plane.

Geometrical model of Projecting Plane with Bilinear Bundle of Projecting Rays

As it was mentioned the bilinear bundle provides for projective transformation of planes (Fig.2) provided the transformed planes (biperspectively conjugate ones) P and E and two other planes H and H' make up a single bundle of planes. Any other plane, which does not belong to the bundle will be tangential only to the linear surface of the bilinear bundle, which passes through intersection line of conjugated planes. Therefore it will cleave the bilinear bundle along second order curves only, with the exeption of the above mentioned straight line and the straight line passing through common with this plane points of guiding lines of

* This ray passes through non-proper intersection point of the guiding lines of bundle with the projecting plane.

014.

the bundle.

Thus in common case the bilinear bundle transforms a straight line of the plane into a conformable to it second order curve of another plane conjugated by the bundle with the first one. It is evident, that one should introduce a notion of <u>first and second order projecting planes.</u>

The geometrical meaning of the first order projecting plane in a bilinear bundle unlike the geometrical meaning of projecting plane in orthoscopic bundle will be contained in regularity of points and straight lines 1-2-3 formation. Namely: the projecting plane in bilinear bundle is obtained in itself by two superbundles of straight lines with centres S_1 and S_2 (Fig.6). They are in multiperspective compliance with each other so that any two simple rays, taken from the superbundles, make up straight forward line of points 1-2-3 or straight line crossing corresponding rays. In this case intersection points of the projecting plane with bilinear bundle guiding lines, are cetres S_1, S_2 of straight lines' superbundles, while straight lines (rays) are intersecting lines of this plane with planes of plane bundles, with guiding lines of bundles as their axis. In the described case a non-proper straight line of the projecting plane is made up by pairs of parallel rays passing accordingly through both centres of the superbundles, thus it is a closed line from all sides - a non-proper ellipsis of the first order.

The two centres of superbundles V and V' (S and S') and the crossing point O of bilinear bundle with the projecting plane make up its natural coordinate triangle.
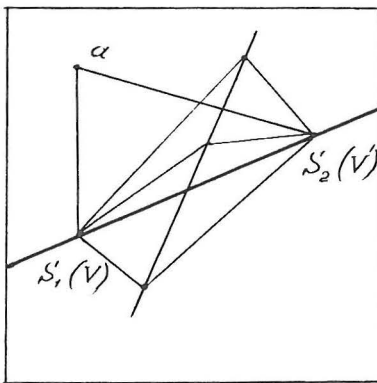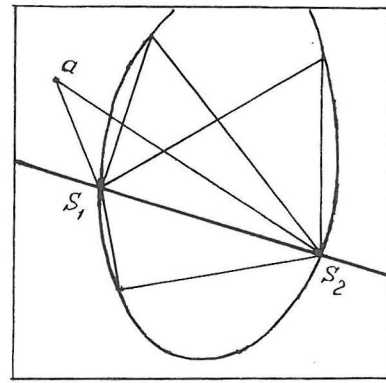

Fig.6   Fig.7

The first order affine-projecting plane in bilinear bundle is made up of two sets of parallel straight lines' intersection, e.i. superbundles with non-proper centres.

Thus bilinear bundle, unlike the orthoscopic one, provides for full affine transformation of planes if they are parallel to the guiding lines of bundle.

Geometrical meaning of second order projecting plane in a bilinear bundle is in availability of the two superbundles

of straight lines, which are not in a poliperspective compliance, but a policorrelative one. As a result, any pair of simple bundles, taken from the superbundle, will produce a curved line of points of the second order at the intersection of compliant rays (Fig.7). Thus a second order projecting plane consists only from points of the second order, intersecting in centres of the superbundles. It is expedient to name such a plane a "biperspective" one.

The non-proper second order curve, as in the foregoing case, is obtained by parallel rays, passing through both centres of the superbundles. It is a non-proper ellipsis of the second order cut by a double straight line passing through centres of the superbundles.

The second order affine-projecting plane will be obtained if a bilinear bundle, isomorphically plotted on a first order projecting plane, will be cut by another plane which does not belong to the plane bundle with the common guiding line of bundle as the axis and with the plane passing so, that the bundle relative to it will turn out a special one. The selected plane will pass parallel to the guiding lin s of the bundle and it will cleave the bundle so, that cutting lines will be two sets of cutting each other and parallel to each other second order curves.

Particular cases of Bilinear Bundle of Projecting Rays

There are particular cases with bilinear bundle. The first one is when the guiding lines of bundle do not cross but intersect, and the second one is when one of the guiding lines renders non-proper - that is it continues to the infinity.

In the first case the bilinear bundle turns out into well known orthoscopic bundle of projecting rays and planes.

The second case is a new (Fig.8) insufficiently known bundle with single guiding line. It can be named monolinear or cylindrical one, as it is obtained by cylindrical lenses, as well as with some assumption with the images obtained by airsurvey cameras with slot locks.
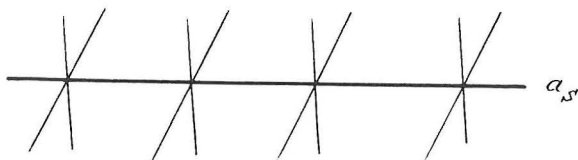

Fig.8

All the projecting rays of cylindrical (monolinear) bundle intersect the guiding line so, as to remain parallel to some plane. It means that such a bundle consists of rays, planes and second order linear surfaces, which are hyperbolic paraboloids.

This bundle can be in two positions as regards to the projecting plane.

The first position (Fig.9) is when the guiding line has one proper common point with the projecting plane, while the bundle has one special linear surface with all the straight lines of it parallel to that plane.

The second position (Fig.10) is when the guiding line has one common non-proper point with the projecting plane,
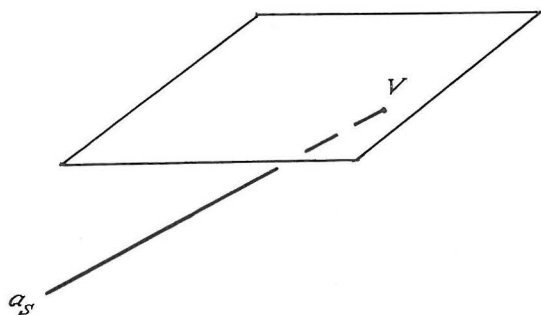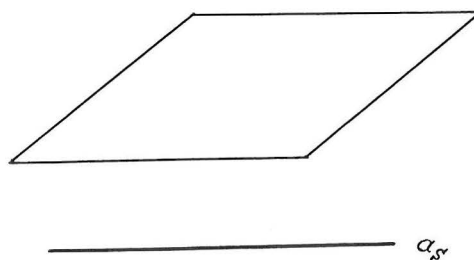
Fig.9   Fig.10

and one special plane. In this case the cylindrical bundle
itself is a special one.

This bundle, like a bilinear one, can be isomorphically
reflected on the projecting plane. In this case points of
the plane will be identical to the rays of the bundle, and
straight lines will be identical to the linear surfaces of
bundle.

Conclusions

1. The study of essence of non-instant exposure came-
ras' image build up, as well as geometrical properties of
colinear deformed photo images, for example as a result of
image transformation with rectifier, unprecise blow up of
images, affine deformations, etc., render revelation of a
new, more complicated pencil of projecting rays. It is bili-
near and it consists of two crossing straight lines (guiding
lines) and multiplicity of lines (rays) cleaving at the sa-
me time both guiding lines. This bundle, unlike the ortho-
scopic one, provides for most complete projective transfor-
mation of planes including all the particular cases.

2. In photogrammetric image processing there are pho-
togrammetric instruments partially taking into account mo-
dern requirements of photogrammetry in more common image
transformation, as well as instruments which provide for
subcorrection of cameras' focal lenghtes along X and Y axis.
Their optical systems provide for panchratic affine trans-
formation of images along any direction. At the same time
till now there was no efficient theoretical base for nece-
ssity of the above innovations. Evidently comparetively slow
inculcation of the new image transformation methods in pho-
togrammetry is explained by this lack of theoretical base.

3. The discovery of geometrical essence of projective
reflection in non-orthoscopic bilinear pencil of projecting
rays, validity of a new outlook at the geometrical model of
projecting plane in this bundle as at a regular set of points
and lines made up by the two superbundles of rays belonging
to the plane itself, provides more valid theoretical solution
of photogrammetric tasks of air-space image processing. The

theory includes not only non-central projection images' processing, but central projection ones as well, because on the one hand, as it was mentioned, the latter could be aggravated by systematic errors, including those of colinear origin; on the other hand they often require a considerable blow up of images, thus their conventional processing in image size becomes either impossible or unexpidient.

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