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AN ONLINE BUNDLE BLOCK ADJUSTMENT FOR ANALYTICAL PLOTTER

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ABSTRACT

The paper describes an approach to the solution of the adjustment problem for normally distributed data utilizing the Bayesian estimator applied to photogrammetric bundle block adjustment.

Under the assumption of uncorrelated observations (eg. image coordinates x', y') this approach does not require the inversion of matrices it also allows a sequential introduction of single consecutive observations (omitability for online adjustment) as well as the implementation of an error-snooping algorithm.

Finally the results of an attempt at extrapolation of timing requirements is included.

INTRODUCTION

In the data acquisition phase of photogrammetric block adjustment, the image coordinates are measured in pairs of x' and y' for each point in sequence one after the other.

The classical data processing phase can be started only after all data has been acquired and is thus generally unsuitable for on-line procedures; a more readily adopted estimator has to be found. Eg. E.M. Mikhail and R.F. Helmering proposed in {2} a recursive approach utilizing the implementation of additional data (observations) as well as delection of rejected data (eg. blunders) based on the least squares estimator.

A similar formulation for the estimator procedure is given in this paper although explained in terms of using a more general justification for this kind of approach.

The mathematical model for this adjustment is taken to be the complete set of information about the functional description of the physical relation between image and ground surface locations as well as their stochastic model.

While the functional model is adopted unchanged from the conventional Bundle Block Adjustment, the stochastic model will be extended.

THE STOCHASTIC MODEL

The stochastic model is the description of the probabilistic properties of the variables involved in the functional model regarding distribution functions, variances and covariances.

In surveying and photogrammetry, observations and variables derived from observations are generally regarded as normally distributed. This indeed is the distribution form required for the application of the Least Squares Estimator; it also permits the application of other estimators such as the Bayesian or Maximum Likelihood.

The linear Bayesian estimator will be used within this papers.

Quoting K. Kubik {3}:

> "In these Bayesian methods it is assumed that an 'a priori' probability distribution is already known of the unknown parameters before the sampling commences. The sample is then used to refine this a priori knowledge."

The least squares estimator also requires some knowledge about unknowns before commencing the estimation process in form of approximations to the unknowns; eg. coordinate approximations for ground points, exterior, and often also interior, orientation data, as well as for the "additional parameters" when adopted. But it does not require any other knowledge about their stochastical model than being free variable normally-distributed random data. In practice some of this data is introduced as "observed unknowns" (as proposed here in general).

For instance, the flight elevation can be observed by statoscope; the camera location at the instant of exposure can be determined by comparison of image and maps available, and so on. All these determinations of approximations to the unknowns are actually direct estimation of the unknowns themselves, hence, by their nature observations to which estimates of accuracy can easily by assigned.

This also applies for the coordinates of unknown ground points which might be retrieved from maps or even more conveniently within the program itself from image coordinates and an assumed medium ground elevation including a variance estimation by error propagation law.

Those direct estimations of the unknowns constitute the best knowledge about the unknowns themselves before commencing an adjustment (of additional observations).

From this it can be seen easily that the stochastic model has to be extended in general in order to make use of valuable information by incorportation the unknown approximation as random data together with their probability distribution parameters as one part of the input to the estimation process itself. The second and third parts of the input information are usually the functional model and the observations with their distribution parameters.

THE RECURSIVE ESTIMATOR

The term recursive estimator is used in this paper in order to describe a dynamic estimation process mathematically; such an estimation process allows the refinement of the present knowledge about the unknowns by adding observation information in the same sequence as the data acquisition process provides them.

Before commencing any data acquisition (image coordinate measurement) knowledge about the unknowns is assumed in the form of their approximation in the sense of direct estimations:

$$L_{X_o} = X_o \text{ with variance } \sigma^2_{X_o} \tag{1}$$

The corresponding observation equation formulated for least squares adjustment is

$$L_{X_o} + V_{X_o} = X_o + dX_o \quad ; \quad \sigma^2_{X_o} = Q_{XX} \quad ; \quad \text{with } \sigma_o = 1 \tag{2}$$

or

$$V_{X_o} = 1.dX_o - (L_{X_o} - X_o) \quad ; \quad Q_{XX} \tag{3}$$

Since L_{X_o} and X_o are identical the least squares adjustement problem is solved by minimizing V^TPV simply by setting dX_o = o which indeed confirms that the L_{X_o} are the best known data of the X_o at that stage.

Notice that the coefficient matrix A_X contains zeros and a single 1 per row and that it has in general as many rows as it has columns (a square matrix). Assuming the order of the unknowns in dX_o to be identical to the order of their direct estimations in L_{X_o} the coefficient matrix A_X becomes a unit matrix:

$$A_X = I \quad \therefore \quad V_X = IdX - 0 \quad ; \quad Q_{XX} \tag{4),(5}$$

Adding observations to the present knowledge yields then the observation equation system:

$$V = Adx - \ell \tag{6}$$

$$\text{where } A^T = \| I, A_L^T \| \; ; \quad V^T = \| V_X^T \; ; V_L^T \| \quad ; \quad \ell = \| 0, \ell_L^T \| \tag{7}$$

$$\text{and } \ell_L = \| (L - A_L X_o) \| \tag{8}$$

The resulting normal equation formulated so that dX can be computed then is:

$$d_X = (P_{XX} + A_L^T P_{LL} A_L)^{-1} \quad . \quad A_L^T P_{LL} \ell \tag{9}$$

which is identical to the solution based on the Bayesian estimator (see {1}, page 33, 34) and also to the generalized least squares solution given by Schmid in {5}.

The expansion of the stochastic model as described before actually has upgraded the least squares estimator to take the form of the Bayesian estimator under the assumption of normally distributed random data.

The above solution (9) can be rewritten in a form suitable for a recursive process, as described below.

Equation (6) may be re-arranged:

$$V = AdX - \ell$$

as $$AdX - V - \ell = 0 \qquad (10)$$

This yields two matrix equations

$$dX - V_X = 0 \text{ as defined earlier and} \qquad (11)$$

$$A_L dX - V_L - \ell_L = 0 \qquad (12)$$

or

$$\| A_L, I \| \, \| dX^T, -V_L^T \|^T - \ell_L = 0 \qquad (13)$$

which is the basic formulation for least squares adjustment of condition equations in the form

$$\dot{A}\dot{V} - W = 0 \qquad (14)$$

where $$\dot{A} = \| A_L, I \| ; \qquad \dot{V}^T = \| V_X^T, - V_L^T \| \qquad (15)$$

$$W = \ell_L = \| (L - A_L X_c) \| \qquad (16)$$

with

$$Q = \begin{pmatrix} Q_{XX} & o \\ o & Q_{LL} \end{pmatrix} = \begin{pmatrix} P_{XX}{}^{-1} & o \\ o & P_{LL}{}^{-1} \end{pmatrix} \qquad (17)$$

and

$$E(V) = E(V_X) = E(V_L) = o \qquad (18)$$

we obtain the Normal equation

$$(\dot{A}Q\dot{A}^T)k = \ell_L \quad ; \quad Q^{-1}\dot{V} = \dot{A}^T k \qquad (19)$$

with

$$k = (\dot{A}Q\dot{A}^T)^{-1}\ell_L \qquad (20)$$

and

$$Q^{-1}\dot{V} = \dot{A}^T(\dot{A}Q\dot{A}^T)^{-1}\ell_L \qquad (21)$$

follows

$$\dot{V} = Q\dot{A}^T(\dot{A}Q\dot{A}^T)^{-1}\ell_L \qquad (22)$$

or rearranged and split into two equations

$$\therefore \quad dX = Q_{XX} A_L^T (A_L Q_{XX} A_L^T + Q_{LL})^{-1} \ell_L \qquad (23)$$

$$V_L = -Q_{LL} (A_L Q_{XX} A_L^T + Q_{LL})^{-1} \ell_L \qquad (24)$$

and for the refinement of the cofactor matrix

$$Q_{\bar{X}\bar{X}} = Q_{XX} - Q_{XX} A_L^T (A_L Q_{XX} A_L^T + Q_{LL})^{-1} A_L Q_{XX} \tag{25}$$

or

$$Q_{\bar{X}\bar{X}} = (A_L^T Q_{LL}^{-1} A_L + Q_{XX}^{-1})^{-1} \tag{26}$$

as from above (9).

A comparison of the above formulations with those given in {1}, {2} and {5} immediately shows the identity of the Bayesian estimator and the Least Squares solution under the same assumptions.

Under the assumption of σ_o = unity the cofactor matrix P^{-1} is identical with the variance-covariance matrix Q of the variables concerned:

$P_{XX}^{-1} = Q_{XX}$ = variance-covariance matrix of the unknowns before adding further observations

$P_{\bar{X}\bar{X}}^{-1} = Q_{\bar{X}\bar{X}}$ = variance-covariance matrix of the unknowns after adding observations

$P_{LL}^{-1} = Q_{LL}$ = variance-covariance matrix of the additional observations.

In the case of uncorrelated observations (as here for the image coordinate measurement) Q_{LL} reduces to a square diagonal matrix for all observations. The diagonal elements are the variances of these observations and can simply be set equal to the square of the coordinate measurement accuracy $\sigma x'$ or $\sigma y'$.

Since Q_{LL} is assumed to be a diagonal matrix the same solution will be found for sequential introduction of single observations as for the introduction of all observations at once into equation (23).

In the case of sequential introduction of single observations into (23) the matrix A_L becomes a row vector and the matrix $P_{LL}^{-1} = Q_{LL}$ is a scalar equal to σ_L^2 for any of this single observation.

Since the term $(A_L Q_{XX} A_L^T)$ then results in a scalar, easily obtained by vector multiplications, the term $(A_L Q_{XX} A_L^T + Q_{LL})^{-1}$ requires only the inversion of a scalar, thus providing simple computation tasks.

This applies for equations (23) and (24) as well as for equation (25).

Since one image point is observed usually within two images similtaniously it is more practical to introduce not one but four observations at a time (x', y', x", y"), thus a 4*4 square matrix $(A_L Q_{XX} A_L^T + Q_{LL})$ will have to be inverted. Still a simple computation task.

While equation (23) yields corrections dX to the unknowns, equation (24) allows the computation of the observation residual(s) at the present stage for those observation(s) which are implemented at that stage (not for those implemented at an earlier stage).

The residuals V_ℓ are in general subject to alteration throughout the recursive adjustment; hence (24) can not be used directly in order to compute σ_o - a posteriori.

Although this is so, equation (24) allows a check for blunders previous to the refinement of the unknowns and their stochastic model, Q_{XX}, thus comprising an important tool for the practical application of this approach:

$$Q_{VV} = Q_{LL} - Q_{LL}(A_L Q_{XX} A_L^T + Q_{LL})^{-1} Q_{LL} \tag{27}$$

It is important to stress, that equations (24) and (27) can be computed without applying any corrections to the unknowns or the stochastic model, hence equation (24) and (27) provide the means for "error snooping" as described by W. Baarda {8}, F. Amer {9} and others.

As proposed by Baarda

$$w_i = \frac{|Vi|}{\sigma_o \sqrt{qii}} = \frac{|Vi|}{\sqrt{qii}} \quad ; \quad \sigma_o = 1$$

are checked against F $\{\alpha_o, \beta_o, 1, \infty\}$(with eg. $\alpha_o = 0.001$, $\beta_o = 0.80$) in a two sided test.

A (set of) observation(s) may then be rejected when $w_i > F_i$ with 80% probability for not making an error type II and with 1 ‰ probability of making an error type I.

NUMERICAL LIMITATION OF THE SUGGESTED APPROACH

Usually the control point coordinates are assumed to be error free, thus having zero variance which implies an infinite weight. Hence, P_{XX} would carry weight parameters equal to infinity at those places corresponding to the control point coordinates and Q_{XX} would also carry zero diagonal elements accordingly. While equation (9) would not be solvable in practice with a digital computer unless the infinity large diagonal elements in P_{XX} are approximated with the largest number manageable by that computer, equations (23), (24), (25) and (27) are no problem at all.

Hence equations (23), (24), (25) and (27) provide a generally flexible formulation for digital computers in particular also as observations assumed to be error free will cause no numerical problems.

At the other hand the numerical solution of equations (23), (24) and (25) will become problematic in a case when either the unknowns approximations or the observations or both are regarded as having an infinite variance implying a weight equal to zero. But this indeed has no practical value since data with zero weight are useless in contributing information.

The approach outlined before also extends into the field of FREE NETWORKS. The solution for singular normal equation systems as for instance given by Mittermayer {6} implements the condition E(dXdX) = min, hence, implying equal weighted unknown approximations, although some may require stronger corrections than others. The solution described here in effect implies at all stages because of $dX = V_X$ the condition $E(dXP_{XX}dX) = \min$. Singularity should not occur other than in those cases where the functional model is too weak, that is to say when the unknwons are indeterminate or nearly so, or when the functional model is incorrect.

PRACTICAL LIMITATION OF REAL TIME BUNDLE BLOCK ADJUSTMENT

The suggested approach requires the availability of coordinates of points on the ground, exterior and interior orientation data of the cameras involved at all exposure stations in question and, as well, their variance-covariance matrices at all times during the estimation process. All this data required should be organized within the storage facilities of the computer system which is interfaced with the stereo-comparator module of the analytical plotter.

In practice neither of the above mentioned data types can be or should be stored within the computer's main memory since at present the only practical computer is a mini computer of up to about 256k byte of main storage. The storage requirements for the Q_{XX} matrix alone would by far exceed this capacity.

In order to meet the requirements for on-line processing, peripherial storage devices with high data transfer and access rates are required such as high-speed-fixed-head-disk systems for small blocks or, even better, disk emulation systems such as bulk core memories probably based on bubble memory chips. Currently the latter allow an access rate close to that of the main memory of the computer, thus probably allowing on-line computation of large blocks.

A further reduction of CPU time can be achieved by utilizing the vector processing capacity of a readily available peripheral array processor and also freeing the mini-computer from comparator control procedures which can easily be transferred to microprocessor controllers as anticipated by Helava {7} and referred to by him as "digital projectors". On the other hand, the implementation of peripheral bulk-core-memory and an array processor will increase the cost for an analytical plotter by about 50% at present price levels.

EXPERIMENTATION RESULTS

Timing Considerations:

Although no analytical plotter was abailable for the preparation of this paper some experiments could be performed on the PDP11/34 of the Department of Surveying at the University of Queensland.

The computer system used is a

- DEC - PDP 11/34 with 32k-words memory

with
- one RL01 - disk - drive - (5M-byte storage)
- one DECWRITER,

in no way comparable with the suggested requirements for on-line block adjustments.

In order to perform experiments giving results and indications which might be extrapolated towards an analytical plotter application, a self-calibrating bundle block adjustment on the basis of the previously outlined Bayesian approach was programmed by the author, having the capacity for adjustment small blocks of up to 30 images taken by up to three different cameras and containing not more than 80 ground points (control plus unknown points).

Since the correctness of the final results was checked against those obtained by other authors or programmes (eg. space resections as in {4}, bundle block adjustment of a KOENIGSHUEGEL sub-block with Bauer/Muller programme) the main interest was directed towards timing and its extrapolation for more adquate hardware components.

The total run time for the Koenigshuegel sub-block with six images and 160 image coordinate pair observations (80 comparator measurements with x', y', x", y", = 320 observations) was about 5½ hours.

It was estimated that about 90% of this time was used for mechanical disk operation of I/O. As recommended and technically possible, utilizing a fast peripheral storage device instead of the slow RL01 disk-drive can reduce the I/O data transfer time by a factor of 100 or even more when disk emulation systems are used.

The runtime of 5½ hourse for this block then will be reduced to about 1 hour, hence, from an average 4 min per comparator measurement to 45 sec which is already about the average measurement time per point for a skilled operator (about 1 min).

Furthermore about 9/10 of the CPU - processing time is used for vector manipulation. Utilizing the capacities of an array processor for this task will reduce this by a factor of about 50 to 100. This finally brings the average time interval per point down to about ¼ of a minute.

Taking into account the time required for real time comparator movement control - probably over estimating - to be the same magnitude, the total required time interval between two consecutive point measurements will not exceed 1 minute for small blocks. Even for medium sized blocks the computing time between pointsettings may not considerably exceed 2 minutes and definitely not when the comparator movements are controlled by microprocessors.

CONCLUSIONS

Although this paper does not deal with new estimation procedures, two conclusions can be drawn from it, which the author regards as very important:

1. The Bayesian estimator provides a more natural justification for the use of unknown approximations as stochastic data in such a way that their valuable information contributes towards the refined result together with observations dynamically, including simple error snooping algorithm.

   Appropriate use of the Bayesian approach for adjustment can also solve many of the problems in free networks.

2. Although the time extrapolation done here is rather speculative towards on-line application in "realtime" rather than in "real" time it indicates this possibility for the near future depending on the instrument manufacturer's intention to implement already available hardware developments.

The author strongly believes that the 1980's will bring a new generation of analytical photogrammetric instruments with real time or near real time capabilities for the data evaluation processes utilizing dynamically refined data in data bank systems.

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