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A MATHEMATICAL MODEL FOR THE RECONSTRUCTION OF OBJECTS USING A MULTILENS CAMERA WITH A FOCAL PLANE SHUTTER AND FMC

ABSTRACT

The use of a 180° multi lens photographic equipment in a low altitude quick flying carrier gives reasonable results only if there is an adequate system for Forward Motion Compensation (FMC) in combination with a focal plane shutter. The resulting "dynamic" images then show certain deformations from the central perspective.

A mathematical model for the generation of images by such a system is presented and used for rigorous reconstruction of objects from these dynamic images. The accuracy potential of the whole system is estimated by computer simulations using a synthetic data set.

1. INTRODUCTION

For the special purpose of aerial reconnaissance a high photographic standard of the recorded images is required. Usually cameras of limited metric accuracy are applied in this field. From the point of view of photogrammetry it is of interest to search for the accuracy level which can be attempted, using the photos for object point coordinate determination.

The considerations of this paper are based on a camera with focal plane shutter and with a Forward Motion Compensation (FMC) device. As an example for such a camera may serve the KRb 6/24 from Zeiss, Oberkochen /1/. This camera avoids the projective cylindrical distortions of a panoramic camera on one hand /2/. On the other hand the combination of a focal plane shutter and FMC, allowing low flying altitudes, results in an image geometry which is different from the unique central perspective, usually realized in photogrammetry.

In the following chapter 2 the main characteristics of the KRb 6/24 system are stated and the common effect of the focal plane shutter and FMC on the image coordinates is described.

Based on these facts in the chapters 3 and 4 a mathematical model for the dynamic image generation with the described system is presented and used for rigorous reconstruction of object point coordinates from image coordinate measurements, inner orientation data and recordings of the exterior orientation parameters.

Finally in chapter 5 the accuracy potential of the whole system is estimated by computer simulations.

The use of a focal plane shutter in a photogrammetric equipment without Image Motion Compensation devices and the processing of images obtained by such a system was studied in /4/, /5/.

An introduction to the geometry of a similar camera system, the KRb 8/24 and general recommendations for the reconstruction of objects from "dynamic" images are given by /3/. Some comments on the reconstruction model used in /3/ are outlined in chapter 2.

2. DESCRIPTION OF THE SYSTEM KRb 6/24 AND OF THE COMMON EFFECT OF FOCAL PLANE SHUTTER AND FMC ON THE IMAGE COORDINATES

The camera system KRb 6/24 consists of 5 lenses arranged across the flying direction in such a way that the resulting angular field of the camera is more than 180°. See fig. 1. The optical axes of the lenses are diverted by prisms. The effect of the diversion of the optical axes of ± 36° resp. ± 72° is the same as if the cameras were tilted at ± 36° resp. ± 72° in respect to the vertical line.

All the images from the 5 lenses are recorded on 9.5"(240mm) film. Fig. 2 shows the arrangement of an "image array" realized by a KRb 6/24 camera system. In addition the deforma-

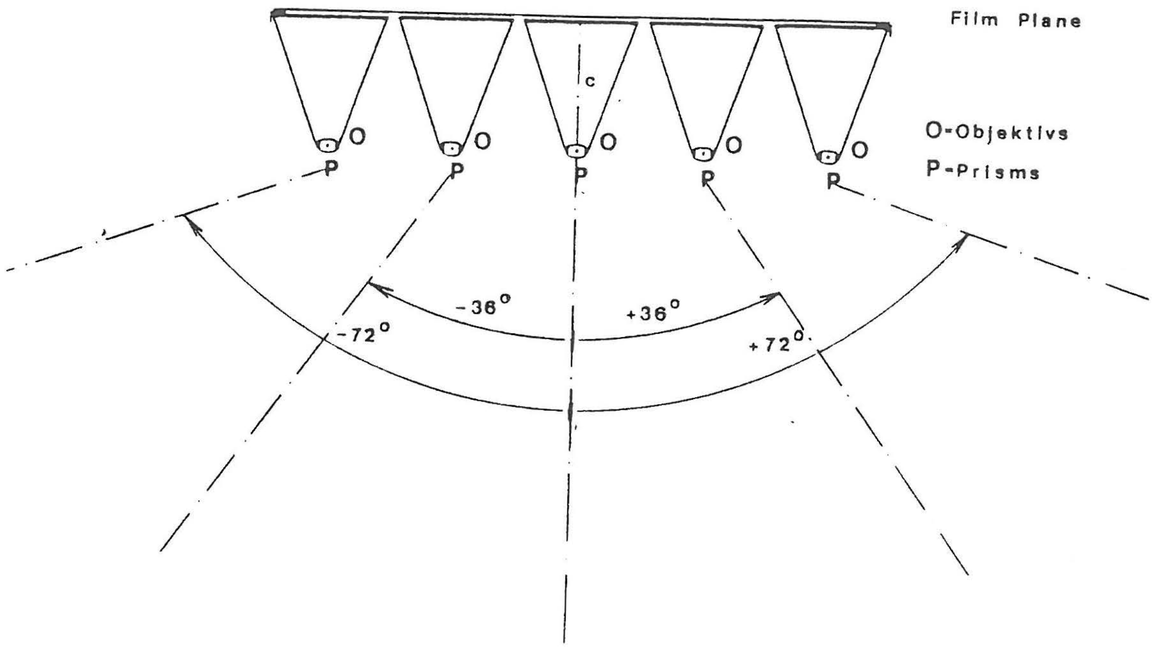


fig. 1 Schematic cross sectional view of the lens / film arrangement in the KRb 6/24

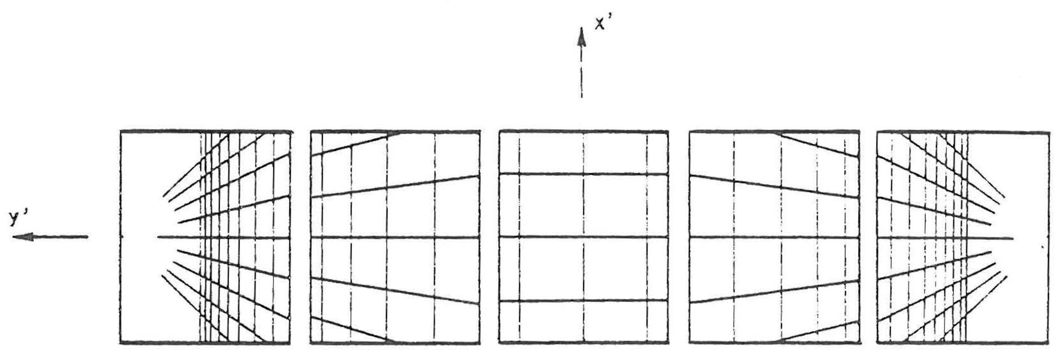


fig. 2 Image array of a KRb 6/24 system showing the deformations of a quadratic flat grid

tion of a flat quadratic object grid in the 5 images of an array is represented. From this arrangement of the photos stereoscopic coverage is obtained for every image pair of two adjacent arrays.

The focal plane shutter moves across the flying direction along the whole array. The images are exposured by a slit of the shutter. The width of the slit is selected in accordance with the prevailing illumination conditions.

Table 1 gives some of the system characteristics for the KRb 6/24 camera, which are necessary for the geometric reconstruction of the images.

Number of lenses	5
Principal distance c	57 mm
Single frame format	50 x 40 mm^2
Total angular field:	
in flight direction	47.4°
across flight direction	182.7°
Angular velocity v/h	max 5.0 rad/sec
Exposure time	1/150 to 1/2000 sec
Shutter slit velocity v_s	max 4.33 m/sec
FMC	rigorous, max 285 mm/sec

Table 1: Characteristics of the camera KRb 6/24

The angular velocity is the ratio between the forward velocity v of the flying platform and the flying height h. The exposure time is a function of the width of the slit and the velocity v_s of the shutter in y'-direction.

The incorporated FMC-device compensates the effect of image motion rigorously by moving the film in x'direction. The velocity v_F of this film movement is a function of the angular velocity v/h, the principal distance c_j, the deflection angle ν_j of the optical axe of lens j of the camera and of the angle α. See figure 3.

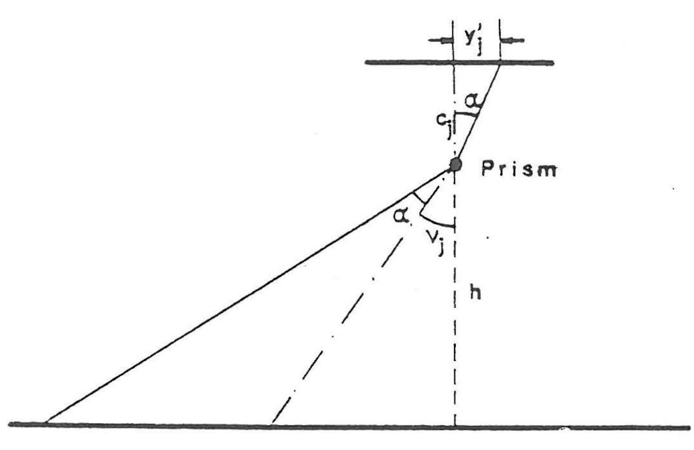


fig. 3 Cross sectional view for lens j

The velocity v_F of the film can be expressed as:

$$v_F = \frac{vc_j}{h} \frac{\cos(\nu_j+\alpha)}{\cos\alpha} \qquad (1)$$

Equation (1) may be transformed into

$$v_F = \frac{vc_j}{h} \frac{(\cos\nu_j \cos\alpha - \sin\nu_j \sin\alpha)}{\cos\alpha} \qquad (2)$$

and with

$$\tan\alpha = y_j' / c_j ,$$

(where y_j' is the image coordinate related to the principal point of the image j of the array) we find

$$v_F = \frac{v}{h} (c_j \cos\nu_j - y_j' \sin\nu_j). \qquad (3)$$

In order to compensate image motion the FMC - device has to move the film with the speed v_F during the exposure time. Evidently equation (3) is valid only for flat terrain.

Using equation (3) the displacement of the film can be calculated by integration of:

$$dx_j' = v_F \cdot dt \qquad (4)$$

where:

dx_j' = motion increment of the film, in the image coordinate system x_j', y_j'

dt = time increment

The time increment can be expressed as a function of the image coordinate increment dy_j' and the velocity v_S of the focal plane shutter

$$dt = \frac{dy_j'}{v_S} . \qquad (5)$$

Using (3) and (5) the equation (4) becomes:

$$dx_j' = \frac{v}{hv_S} (c_j \cos\nu_j - y_j' \sin\nu_j)\, dv_j' . \qquad (6)$$

By integration of the equation (6) one gets

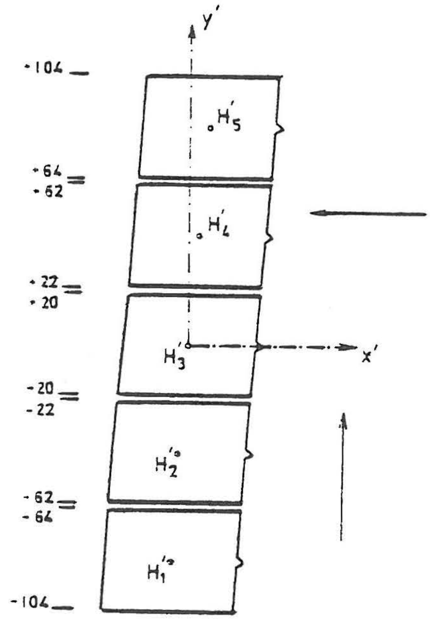
$$\Delta x_j' = \frac{v}{hv_S}(y_j' c_j \cos\nu_j - y_j'^2 \sin\nu_j/2) \qquad (7)$$

where

$\Delta x_j'$ denotes the correction of the x_j'-coordinate that has to be applied for a given y_j'-coordinate as a function of c_j, ν_j, v, h and v_S.

From equation (7) one can learn that every slit with y'_j = constant has the same image displacement. Therefore a focal plane shutter slit extending in x'-direction is very adequate for the generation of images.

In the following the Forward Motion Compensation is considered as fullfilled according to (7). That means that an approximate compensation as proposed in /3/ is not further investigated. Even in the case when it is not possible to calculate the value $\Delta x'_j$ the correction can be applied, because the effect of Forward Motion Compensation is also noticeable by the deformation of the image of the frames. Without FMC the frames are well defined rectangles of 50 mm x 40 mm. With FMC however the images of the frames show a type of deformation illustrated by fig. 4.



Direction of the film displacement

x', y' = undivided coordinate system for the whole image array

Direction of the motion of the slit

fig. 4: Deformation of the images of the frames (typical for KRb 6/24 frames)

Whilst the left and right lateral border lines of the centre frame are straight lines, in the side oblique frames the border lines are parabolic curves.

The image coordinate measurements are firstly reduced to the undivided array coordinate system. In a second step the image coordinates x'_{ij} y'_{ij} of the frame j are obtained from the undivided array coordinates x'_i y'_i (fig.4) by the following reduction procedure:

$$x'_{ij} = x'_i + \Delta x'_j \qquad \text{(left image)} \qquad (8)$$
$$y'_{ij} = y'_i - y'_{H_j}$$

Analogously the corresponding image coordinates x''_{ij} y''_{ij} in the right frame j result as:

$$x''_{ij} = x''_i + \Delta x''_j$$
$$y''_{ij} = y''_i - y''_{H_j} \qquad \text{(right image)} \qquad (9)$$

where

$\Delta x'_j$ ($\Delta x''_j$) denotes the correction in x' (x'')- direction depending on y'_j (y''_j) to remove the effect of film movement

and

y'_{H_j} (y''_{H_j}) denotes the y-coordinate of the principal point of the considered left (right) frame of the stereo pair in the undivided array coordinate system.

3. A MATHEMATICAL MODEL FOR IMAGE GENERATION

As shown in fig. 5 for a focal plane shutter slit, one can establish relationships between the ground coordinates of a point P (x_i y_i z_i) and its image coordinates in the image j P'_{ij} (x'_{ij} y'_{ij} $-c_j$) making use of a principal distance c_j and the actual projection centre P'_0 (x'_0 y'_0 z'_0) at time t'. These relations are based on the well known central perspective equations and referenced to the individual principal point H'_j of frame j.

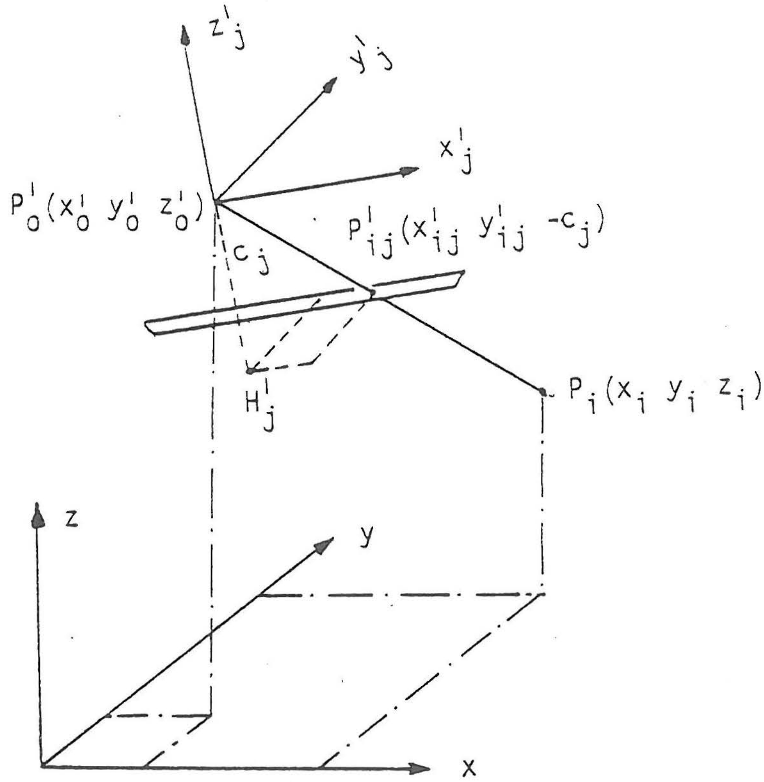


fig. 5 Central perspective imaging in a slit (left image)

With the notations of figure 5:

$x_i \; y_i \; z_i$ = ground coordinates of the point P_i

$x'_o \; y'_o \; z'_o$ = ground coordinates of the projection center P'_o at the time t'

$x'_{ij} \; y'_{ij}$ = image coordinates of the point P_i in the (left) image

c_j = principal distance of the image j.

and a general rotation matrix R'_j resp. R''_j

$$R'_j = R_{\nu_j} \cdot R'_{\omega\phi\kappa} = \begin{bmatrix} r'_{11,j} & r'_{12,j} & r'_{13,j} \\ r'_{21,j} & r'_{22,j} & r'_{23,j} \\ r'_{31,j} & r'_{32,j} & r'_{33,j} \end{bmatrix} \tag{10}$$

for each image of an array, the image coordinates may be expressed as:

$$x'_{ij} = -c_j \frac{r'_{11,j}(x_i - x'_o) + r'_{12,j}(y_i - y'_o) + r'_{13,j}(z_i - z'_o)}{r'_{31,j}(x_i - x'_o) + r'_{32,j}(y_i - y'_o) + r'_{33,j}(z_i - z'_o)}$$

$$y'_{ij} = -c_j \frac{r'_{21,j}(x_i - x'_o) + r'_{22,j}(y_i - y'_o) + r'_{23,j}(z_i - z'_o)}{r'_{31,j}(x_i - x_o) + r'_{32,j}(y_i - y_o) + r'_{33,j}(z_i - z_o)} \tag{11}$$

$$x''_{ij} = -c_j \frac{r''_{11,j}(x_i - x''_o) + r''_{12,j}(y_i - y''_o) + r''_{13,j}(z_i - z''_o)}{r''_{31,j}(x_i - x''_o) + r''_{32,j}(y_i - y''_o) + r''_{33,j}(z_i - z''_o)}$$

$$y''_{ij} = -c_j \frac{r''_{21,j}(x_i - x''_o) + r''_{22,j}(y_i - y''_o) + r''_{23,j}(z_i - z''_o)}{r''_{31,j}(x_i - x''_o) + r''_{32,j}(y_i - y''_o) + r''_{33,j}(z_i - z''_o)} \tag{12}$$

Note that the coordinates of the projection centres and also the elements of the rotation matrix R_j are time-dependent unknowns. Thus considering different points of an image and even more points of different images of an array this time dependency has to be taken into account.

4. A RIGOROUS MATHEMATICAL MODEL FOR THE GEOMETRIC RECONSTRUCTION OF OBJECTS

The orientation of two adjacent image arrays at the moments t' resp. t" may be described by the following parameters:

$x_o \; y_o \; z_o \; \omega \; \phi \; \kappa$ — Orientation unknowns of the left frame array at the time t', which can be related to a certain y'-coordinate of the undivided array system (s.fig.4)

and

$\Delta x_o \; \Delta y_o \; \Delta z_o \; \Delta\omega \; \Delta\phi \; \Delta\kappa$ — Change of the orientation unknowns from time t' to the time t" corresponding to y" in the right image array

Assuming that the camera is equiped with a recording device for the recording of the coordinates of the projection centre x'_{or} y'_{or} z'_{or}, x''_{or} y''_{or} z''_{or} and of the angles ω'_{or} ϕ'_{or} κ'_{or}, ω''_{or} ϕ''_{or} κ''_{or} at the moments t' and t" one can introduce as "observed values":

$$x_{or} \;\; \Delta x_{or} \;\; y_{or} \;\; \Delta y_{or} \;\; \cdots\cdots \;\; \kappa_{or} \;\; \Delta\kappa_{or} \quad .$$

These "observed unknowns" are calculated from the recorded values according to:

$$\begin{aligned} x_{or} &= x'_{or} \\ \Delta x_{or} &= x''_{or} - x'_{or} \\ &\vdots \\ \kappa'_r &= \kappa'_r \\ \Delta\kappa''_r &= \kappa''_r - \kappa'_r \quad . \end{aligned} \tag{13}$$

Usually the recorded values of the unknowns will be correlated (correlation coefficient ρ), but of equal accuracy. Thus the corresponding covariance matrix K of every two equations of the system (13) (e. g. x_{or} Δx_{or}) results as:

$$K_{x_{or}} = \sigma^2_{x_{or}} \cdot \begin{bmatrix} 1 & (1-\rho_{x_{or}}) \\ (1-\rho_{x_{or}}) & 2(1-\rho_{x_{or}}) \end{bmatrix} \tag{14}$$

where $\sigma^2_{x_{or}}$ is the variance of the recorded values x'_{or} x''_{or}.

By analogy the covariance matrices $K_{y_{or}}$ $K_{z_{or}}$ K_{ω_r} K_{ϕ_r} K_{κ_r} are obtained.

Consequently the 2 x 2 weight matrices will be obtained as:

$$P_{x_{or}} = \sigma_o^2 \cdot K_{x_{or}}^{-1}$$
$$\vdots \qquad (15)$$
$$P_{\kappa_r} = \sigma_o^2 \cdot K_{\kappa_r}^{-1}$$

with σ_o^2 = variance factor.

Denoting with s the distance which is passed by the focal plane shutter between two adjacent recordings, one can calculate for each mission

$$s = \frac{v_s}{\text{number of arrays per second}} \ .$$

From s the reciprocal value q is obtained as

$$q = 1/s \text{ (dimension of } q{:}m^{-1}). \qquad (16)$$

If the velocity v_s of the shutter and the frequency of arrays per second are known, q can be treated as an observation. If not, q will be an additional unknown of the adjustment.

Using the value q the individual orientation parameters related to an image slit of array coordinates y' or y" can be expressed as functions of the orientation unknowns x_o y_o z_o ω ϕ κ .... $\Delta\kappa$:

$$\begin{aligned}
x_o' &= x_o + ((y'-1)\cdot q)\cdot\Delta x_o \\
x_o'' &= x_o + (1+ (y''-1)\cdot q)\cdot\Delta x_o \\
\\
y_o' &= y_o + ((y'-1)\cdot q)\cdot\Delta y_o \\
y_o'' &= y_o + (1+ (y''-1)\cdot q)\cdot\Delta y_o \\
\\
z_o' &= z_o + ((y'-1)\cdot q)\cdot\Delta z_o \\
z_o'' &= z_o + (1+ (y''-1)\cdot q)\cdot\Delta z_o \\
\\
\omega' &= \omega + ((y'-1)\cdot q)\cdot\Delta\omega \\
\omega'' &= \omega + (1+ (y''-1)\cdot q)\cdot\Delta\omega \\
\\
\phi' &= \phi + ((y'-1)\cdot q)\cdot\Delta\phi \\
\phi'' &= \phi + (1+ (y''-1)\cdot q)\cdot\Delta\phi \\
\\
\kappa' &= \kappa + ((y'-1)\cdot q)\cdot\Delta\kappa \\
\kappa'' & \kappa + (1+ (y''-1)\cdot q)\cdot\Delta\kappa
\end{aligned} \qquad (17)$$

In these equations for instance

$x'_o \;\; x''_o$ are the actual values of the x-coordinates of the projection centres at the time given by y' resp. y" for the left resp. right image slit.

The value 1 is used to fix the origin of time in relation to the y' and y" coordinate.

With the above 12 unknowns $x_o \; \Delta x_o \; \ldots\ldots \; \kappa \; \Delta\kappa$ and the additional one introduced as q one gets for every point P_i with the unknown ground coordinates $x_i \; y_i \; z_i$ and the respective image points $P'_{ij} \; (x'_{ij} \; y'_{ij} \; -c_j)$ and $P''_{ij} \; (x''_{ij} \; y''_{ij} \; -c_j)$ 4 nonlinear observation equations.

$$\begin{aligned}
vx'_{ij} &= f_{x'} \; (x_i \; y_i \; z_i \quad x_o \; \Delta x_o \; \ldots\ldots \; \kappa \; \Delta\kappa \; q) - x'_{ij} \\
vy'_{ij} &= f_{y'} \; (x_i \; y_i \; z_i \quad x_o \; \Delta x_o \; \ldots\ldots \; \kappa \; \Delta\kappa \; q) - y'_{ij} \\
vx''_{ij} &= f_{x''} \; (x_i \; y_i \; z_i \quad x_o \; \Delta x_o \; \ldots\ldots \; \kappa \; \Delta\kappa \; q) - x''_{ij} \\
vy''_{ij} &= f_{y''} \; (x_i \; y_i \; z_i \quad x_o \; \Delta x_o \; \ldots\ldots \; \kappa \; \Delta\kappa \; q) - y''_{ij}
\end{aligned} \qquad (18)$$

To the above system some observation equations for the "observed unknowns" may be added. In the case of the observed values κ_r and $\Delta\kappa_r$ the observation equations to be considered are:

$$\begin{array}{lll}
 & & \text{weight matrix} \\
v_\kappa = \kappa - \kappa_r & & P_{\kappa_r} \\
v_{\Delta\kappa} = \Delta\kappa - \Delta\kappa_r & & (2\mathrm{x}2)
\end{array} \qquad (19)$$

Further on it is possible to add an observation equation for the unknown q:

$$v_q = q - q_r \qquad P_{q_r} \qquad (20)$$

where q_r denotes the "observed value" for q.

The whole system of observation equations can be completed by equations for given control points. Every control point will be introduced into the system by forming a respective observation equation for the x y or z coordinate:

$$\begin{array}{ll}
 & \text{weight} \\
v_{x_{i_{pp}}} = x_i - x_{i_{pp}} & P_{x_{i_{pp}}} \\
v_{y_{i_{pp}}} = y_i - y_{i_{pp}} & P_{y_{i_{pp}}} \\
v_{z_{i_{pp}}} = z_i - z_{i_{pp}} & P_{z_{i_{pp}}}
\end{array} \qquad (21)$$

All available observation equations are used to formulate a least squares adjustment. Thus the unknown object coordinates and the orientation parameters for each image array are computed.

To keep the solving system of normal equations small, the coordinates of the objects points are eliminated before the solving procedure starts.

The inversion of the whole system of normal equations is used to describe the accuracy of object coordinates as well as of the camera parameters.

5. RESULTS FROM A SYNTHETIC DATA SET

In order to verify the presented mathematical model some assumptions are necessary.

The realization of the interior orientation is assumed as errorfree. The same applies to the value q, which can be considered as known and errorfree.

For a given symmetric point distribution at the ground and having assumed flat terrain, the dynamic image coordinates were computed for a flying height of 100 m. The distribution of the points in the ground and in the single frames of the array are presented in fig. 6.

As focal length of all 5 lenses c = 56.5 mm was used. A certain value q was introduced and assumed as errorfree.

To demonstrate the effectivness of the rigorous model the following standard deviations where adopted:

$\sigma_o = \pm 30\ \mu m$ (Accuracy of the image coordinates)

$\sigma_{pc} = \pm\ 1.0\ m$ (Accuracy of the $x_0\ y_0\ z_0$ coordinates of the projections centres)

$\sigma_{pa} = \pm\ 0.1\ ^o$ (Accuracy of $\omega\ \phi\ \kappa$, the angles of the camera)

Inverting the normal equations one can calculate the accuracy for the object coordinates under the above assumptions.

In figure 7 to each object point the $\sigma_x\ \sigma_y\ \sigma_z$ values are noted. The dimension of the given values is m.

On the left hand side of figure 7 only the effect of σ_o (accuracy of the image coordinates) is shown. The listed standard deviations present the accuracy level which may be attempted in the ideal case when the accuracy of ground coordinates is affected only by the accuracy of image coordinates measurements. In this special case the elements of exterior orientation are considered as known and errorfree.

On the right hand side the combined effect of $\sigma_o\ \sigma_{pc}$ and σ_{pa} is shown. Due to the standard deviations taken into account for the projection centres and the camera angles the accuracy decreases.

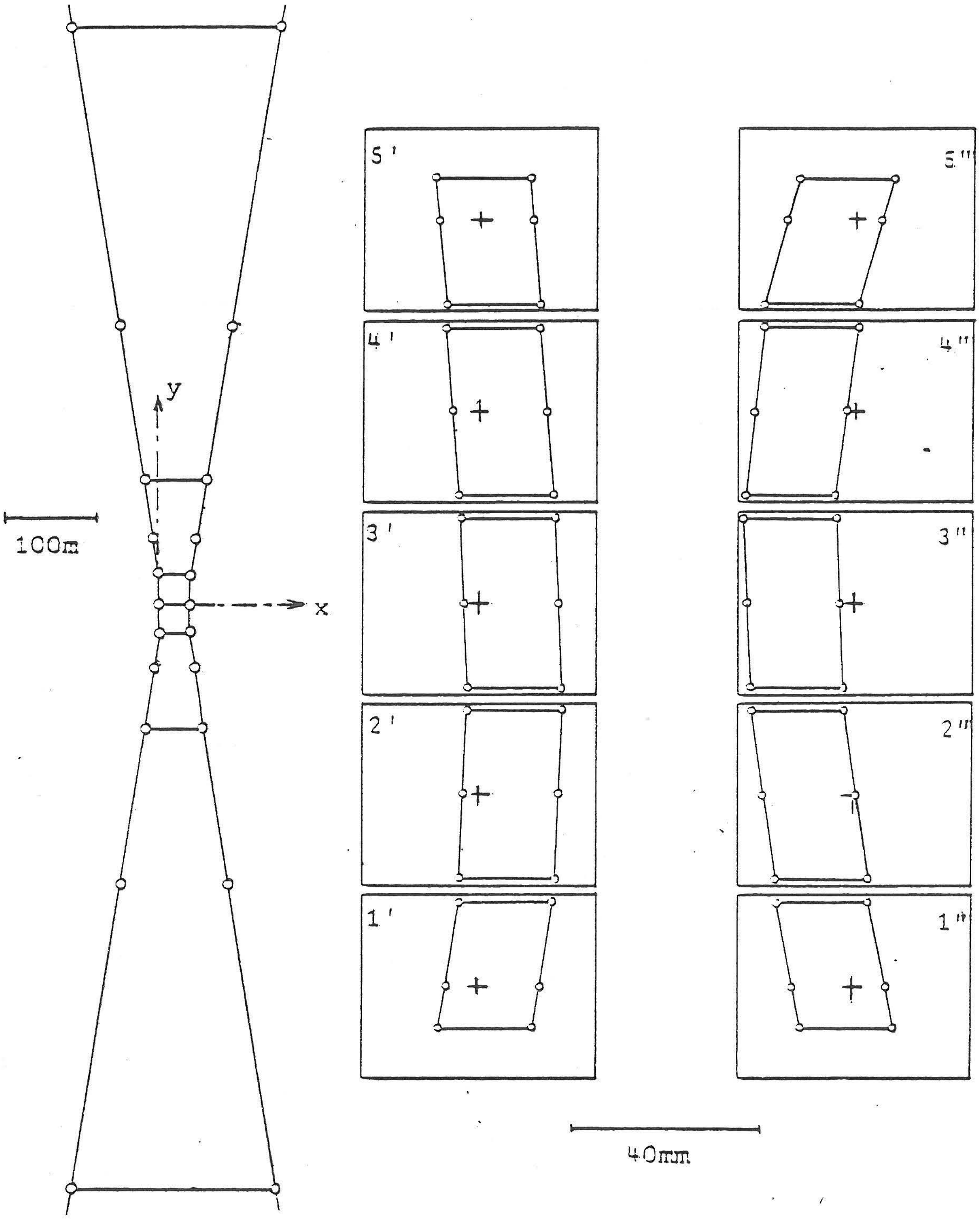


fig. 6 Ground coordinates and related image points for a flying height of h = 100 m

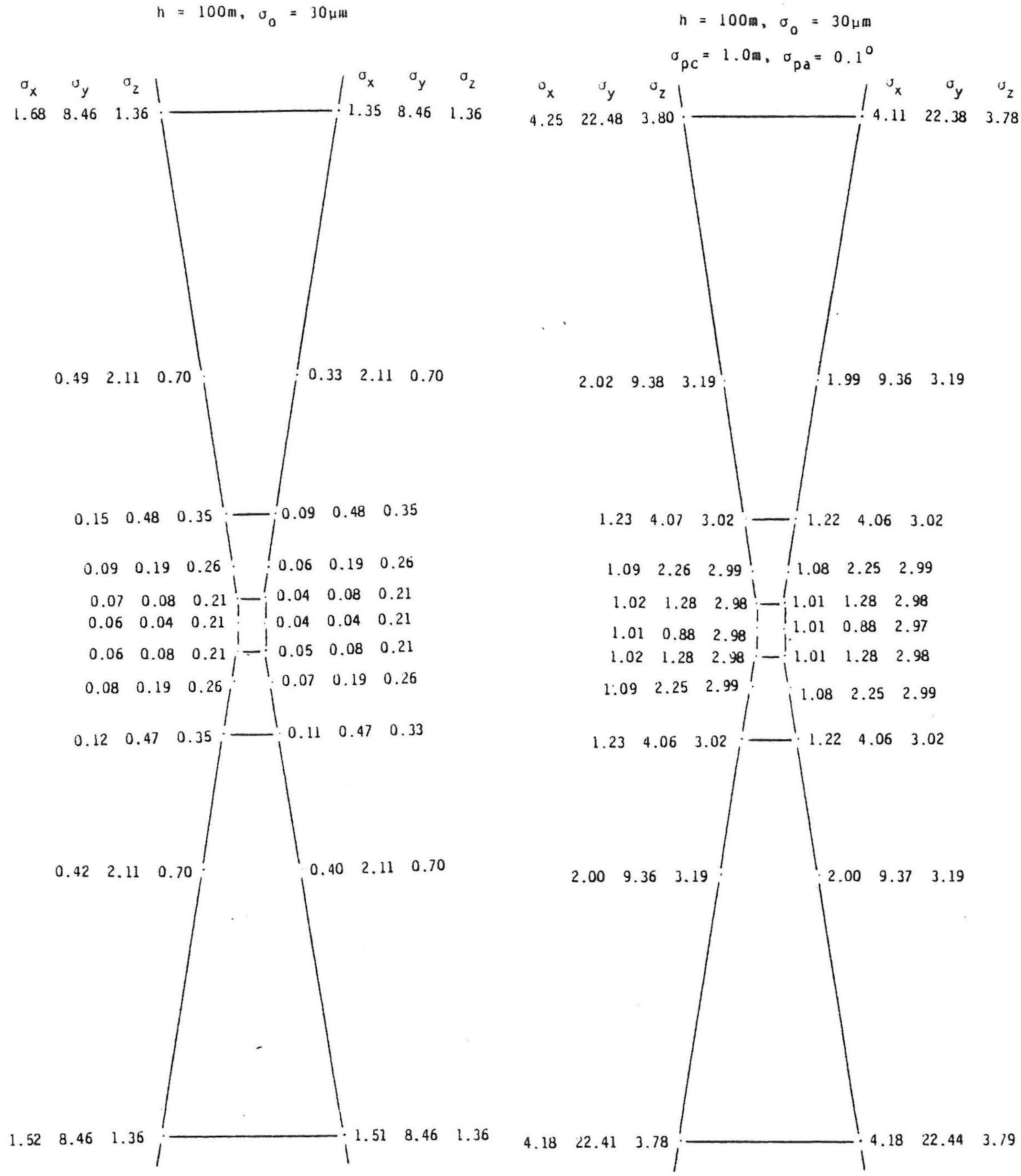


fig. 7 Results from the synthetic data set under different assumptions.

σ_x σ_y σ_z standard deviations of the ground coordinates expressed in m.

6. REFERENCES

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