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BLOCKADJUSTMENT WITH ADDITIONAL PARAMETERS

SUMMARY

The correction of systematic image errors by additional parameters in selfcalibrating blockadjustments has proven to be quite effective and successful. Current research is concerned with the problems of selection, reliability, determinability, and statistical assessment of the additional parameters.

The paper reviews the theoretical status of the method and the available experience. It also attempts to draw conclusions about the field and conditions of safe application. Finally the method is critically evaluated with regard to balancing functional and stochastical mathematical models.

1. INTRODUCTION

1.1 During the past few years blockadjustment with additional parameters has seen considerable practical and theoretical development. It has been accompanied by thorough investigation of systematic image errors.

The problem of systematic image errors and its attempted solution either by testfield camera calibration or by selfcalibration through additional parameters in the blockadjustment has been in discussion for about one decade. At the 13th ISP congress 1976 in Helsinki already quite a clear view had been obtained, as is documented by a number of invited and presented papers, see for instance /1/-/5/, and others.

In the meantime, it has been shown and confirmed that systematic image errors are always present and that their complete or partial compensation by additional parameters is effective, in many cases highly effective, see / 6/ - /15/.

Whilst the method of testfield camera calibration, although effective, has for economic reasons not found its way into regular practical application, blockadjustment will additional parameters has reached the stage of routine application in some countries, in particular in Finland, USA and the Fed.Rep. of Germany. Nevertheless the development of computer programs for blockadjustment with additional parameters cannot be considered finalized at this moment. Most operational programs with additional parameters refer actually to the bundle method of blockadjustment rather than to the independent model method.

The practical application of blockadjustment with additional parameters during the past few years has provided considerable experience about the method. In addition, the period has been used for elaborate investigations and tests about the effectiveness and the risks of the method. It is especially the working group III/3 which has carried out thorough tests the results of which will be presented and discussed at the Hamburg congress, see /16/ - /18/.

1.2 In its present form blockadjustment with additional parameters is based on the hypothesis that sets of photographs and the respective image coordinate measurements always contain certain systematic image deformations which are not accounted for by the conventional a priori corrections as derived from laboratory camera calibration (lens distortion) or from the mathematical model of the imaging process (refraction). It is evident and generally understood that a priori image corrections can only compensate for part of the total error budget. There are always additional systematic errors the reality of which has been clearly confirmed by all investigations. Although the magnitudes of such additional image deformations are surprisingly small (<10 µm) their propagation through the adjustment implies the danger of potentially large block deformation.

From a principal point of view correction of systematic image errors by additional parameters is an extension of the functional model of aerial triangulation the parameters being additional unknows (for interior orientation). With the additional parameters introduced in the adjustment the type of image deformation which is to be corrected is prefixed. It is only the magnitudes of the parameters which are determined (estimated) from the observation data in the adjustment.

The adjustment model is summarized by the observational equations:

$$\underline{l} + \underline{v} = \underline{B}\hat{\underline{t}} + \underline{C}\hat{\underline{k}} + \underline{D}\hat{\underline{s}} \qquad (1a)$$

The vectors $\hat{\underline{t}}$, $\hat{\underline{k}}$, $\hat{\underline{s}}$ refer to the parameters for external orientation, the coordinates of the terrain points, and the additional parameters for systematic errors, respectively. The weight matrix associated with the observations (image coordinates) is $\underline{P}_{ll}$.

The approach of assessing systematic image errors by additional parameters is limited by the principles of the concept. The assumption of systematic image errors, which are constant for a whole set of photographs, cannot account for the total error budget which would include correlation and variation of image deformation within a series of photographs.

Related with the functional approach there arise a number of problems with additional parameters. The problems refer to the choice of parameters, whether they can be numerically determined, their statistical significance, and the application of several sets of parameters. Those problems have been identified for some time. They are still the object of current research and will be discussed separately hereafter.

2. CURRENT RESEARCH AND DEVELOPMENT

2.1 Even if we take the reality of unknown systematic image deformations for granted there is the problem by which and by how many terms such deformations are adequately described. The problem is basic as only such deformations can be corrected which are implied in the parameter model.

There are two philosophies: The first approach attempts to anticipate real and likely optical or mechanical sources of errors (such as deformation of the pressure platen of the camera) and specific a priori types of potential distortion. An example is given by D. Brown's set of 21 parameters /1/. The second approach does not try to account for physical causes but relies on strictly geometrical considerations. The simplest possible terms are sought which would correct for systematic errors at the 9 or 25 standard points of a photograph independent of whatever the physical causes might be. Examples are the sets of 12 and 44 parameters respectively as suggested by Ebner and Grün, or the parameters of Müller or Faig (see /7/, /16/, /19/, /20/), the latter using orthogonal harmonic functions. Either approach leads to relatively simple polynomial or trigonometric terms which are functions of the image coordinates.

The 2 philosophies of approach have not been reconciled so far. Each must be considered valid within its own right. However, the results of working group III/3 indicate that both sets of parameters seem to be equally effective and give quite similar results. Therefore, the principal problem seems to settle down to the question of how many parameters are to be used rather than which ones.

Parameters selected according to the first approach are often highly correlated. The second approach, however, leaves freedom for additional considerations, especially for applying parameters which are orthogonal amongst themselves and with regard to the parameters of exterior orientation (for the case of ideal geometry). The desirability of orthogonal polynomials has been questioned, for instance by Schut. Whilst it is admittet that correlated parameters can give practically the same correction results there is no doubt that the risks of numerical instability of the solution are greater. In addition, orthogonal polynomials have the advantage that individual parameters can be independently interpreted and compared. Also the testing of parameters becomes easier.

2.2 It would be desirable to utilize always an abundant number of parameters in order to be prepared for any type of image deformation which might occur. However, such an approach runs into the problem that the geometry of the adjustment problem must be capable of determining all set parameters. With too many parameters the normal equations may become singular or highly ill-conditioned, depending on the geometry of observations, overlap, and control.

It is not feasible to ask for an a priori decision about the stability of the numerical solution. Therefore in practice one of the two procedures are applied which can be characterized as operating "from below" or "from above": One can start with few parameters which are known from experience to be real and determinable all the time. From such a safe basis an additional number of parameters is tentatively updated and checked with regard to their determination (and significance). The alternate approach which is generally preferred starts "from above" with a sufficient number of parameters and tries subsequently to select the safely determinable (and significant) parameters. In either case a two-step procedure is necessary and a test on estimability is required.

Both procedures, in particular the second one, face the problem of obtaining a solution, even if some parameters are not safely determinable. This problem is solved in most programs by introducing real (from previous experience) or fictitious observed values for the parameters. Either case leads to additional observational equations:

$$\underline{s} + \underline{v}_s = \hat{\underline{s}} \qquad (1b)$$

To the observations $\underline{s}$ is attached a weight matrix $\underline{P}_{ss}$. In the case of fictitious observations the values s are set to be 0.

The least squares adjustment of the combined system (1a) and (1b) will also give corrections v_s to the "observations" s. Such corrections will be

close to 0, in case the parameters without observed values would be not at all or ill-determined. Of course, if the weights are set too high the otherwise well determined parameter estimates are pulled towards 0.
A reasonable choice might be allowing standard errors of the fictitious parameter observations of about the magnitude itself of the parameter. However, a wide range of weights is possible (see below). Although there is some caution to be observed the procedure serves the purpose of ensuring a first stable solution or rather of preventing the numerical solution from breaking down.

After having thus obtained a preliminary solution for a first tentative set of parameters their determinability must be checked individually or per groups. A first check[1] of this kind has been applied by Klein (see /21/):

$$\nabla_o s_i = \sigma_{s_i} \; \delta_o \sqrt{\frac{1-r_i}{r_i}} \tag{2a}$$

where $\nabla_o s_i$ = lower bound of determinability for s_i

$\delta_o \approx 4$ = statistical parameter

r_i = redundancy component of observation s_i

The check has been updated by Foerstner (for details see /22/) to

$$\nabla_o \underline{k} = \nabla_o f(\underline{s}) \leq \sigma_K \; \bar{\delta}_o(\underline{s}) \tag{2b}$$

in which form it is now part of the Stuttgart PAT-B program.
$\nabla_o s_i$ and $\bar{\delta}_o$ are checked whether they can be tolerated. The checks relate to the theory of gross error detection which is fully applicable here. (2a) refers to the determinability of the individual parameters s_i in terms of internal reliability. (2b) refers to the lower bounds of controllable effects of non - or poorly estimable parameters on the adjusted coordinates, known as external reliability or sensitivity of the system. It describes the maximum possible influence of non or poorly assessable individual parameters or parameter groups on the adjustment results. The checks (2) depend only on the geometry of the system. Hence the non-assessable deviation of the parameters is related to geometry which gives the wanted criterion how well parameters are determinable. The practical procedure starts with very high weight (10^{10}) for the fictitious observations s (of value 0) for the parameters which is equivalent with the adjustment without additional parameters. (At this stage also the first data snooping for gross error detection ought to be performed.) After application of the check (2b) the non-assessable parameters are deleted and the final adjustment is carried out, using very low weights (10^{-10}) for the remaining parameters, treating them essentially as free unknowns, whilst still ensuring a numerical solution. According to available experience the check is most effective.

2.3 The check on how well parameters are estimable, or rather how stable the solution is, does not refer to the estimated magnitudes of the additional parameters. This is done independently by testing the statistical significance of the parameters. The simple testing of significance of individual parameters leads to a t-test of the estimated magnitude of the parameter against its standard deviation:

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[1] We avoid here the expression "test", as it is not a statistical test.

$$\frac{\hat{s}}{\hat{\sigma}_o \cdot Q_{\hat{s}\hat{s}}} = t_{r-1} \qquad (3a)$$

The significance test can be extended to global testing of groups of parameters which is particularly required in case of correlated parameters:

$$(\hat{\underline{s}} - \underline{s}^o)^T \underline{Q}_{\hat{s}\hat{s}}^{-1} (\hat{\underline{s}} - \underline{s}^o) / \hat{\sigma}_o^{\,2} = F_{s,r-s} \qquad (3b)$$

$\hat{\underline{s}}$ = estimated parameter values, see (1b)

$\underline{s}^o$= values for s from the 0-hypothesis, independent of s.

Both tests (3a) and (3b) refer essentially to significant deviations of σ_o against the 0-hypothesis (no systematic errors).

The best or sufficient strategies of testing the statistical significance of additional parameters is still a matter of research and development. The same is true even more with regard to the total strategy of assessing gross data errors, errors of control points, and systematic image errors. There are hardly any computer programs which have operational algorithms for the total program. The aforementioned program PAT-B at least prints out the tests (3a) and (3b), allowing the operator to draw conclusions.

It is still an unsolved problem whether non-significant parameters, after estimability has been assured, must be deleted in all cases. Whilst carrying on such parameters may be dangerous, experience seems to indicate that they are not really harmful, and in some cases still lead to slight improvement of the adjustment results.

2.4 Quite another problem is the question of how many independent sets of additional parameters are to be applied for a block. It cannot be really assumed that the systematic image errors are constant for the whole population of the photographs of a block.

Sometimes there are external reasons for subdividing the photographs of a block in 2 or more subgroups to each of which a separate set of parameters may be applied. Such is the case, for instance, if a block is composed of photography from several flight missions, with different cameras, different rolls of film, or only different flight directions. Then it is reasonable to expect different systematic image errors for each subgroup and account for them by separate sets of parameters. Even if no major subdivisions are justified it has become customary to start off with a separate set of parameters for each strip of photographs.

In such cases the total number of additional parameters is considerably increased. It then becomes vital to ensure by checks like (2) the estimability of the parameters in order to prevent ill-conditioned solutions. In addition tests ought to be applied about whether the different sets of parameters amongst them differ significantly.

Experience indicates that the individual strips of a block have indeed somewhat different average image deformations. Hence the application of additional parameters per strip proves slightly advantageous as compared with one set of additional parameters which are common for all photographs of a block, see /13/.

2.5 There remains one problem for which I cannot see an algorithmic solution. It is the question whether the a priori choice of parameters is adequate. Even when all parameters are estimable and significant, there may be systematic image deformations left which are not accounted for by the chosen parameters. Admittedly the danger is not very serious, as number and types of parameters chosen by the various programs cover the range quite thoroughly. Also aerial photographs do usually not behave viciously with regard to systematic deformation.

It has been suggested (see /17/) to make first an analysis of residual errors and then select parameters accordingly. Whilst such a concept may be also questionable from the statistical principles of establishing hypotheses, most practical cases will hardly have sufficient data for such an approach. And I see considerable difficulties making it operational in practice.

3. PRACTICAL STATUS

3.1 The above mentioned problems are being studied, at present, from the scientific and operational point of view. Rigorous or approximate solutions will gradually be incorporated in operational computer programs. The aim is to have algorithmic procedures which control safe and effective application of additional parameters.

Notwithstanding such pending developments the available results and tests have already shown convincingly that carefully applied additional parameters give practically always improved accuracy. The internal discrepancies and hence the σ_0 estimates are reduced, the external accuracy of adjusted coordinates is improved, sometimes drastically.

The rate of improvement depends on overlap, redundancy, and especially on control. For well controlled blocks the standard block adjustment compensates already very well for systematic errors / 4/. In that case additional parameters achieve only moderate improvement of the accuracy of adjusted coordinates. In planimetry it may amount to perhaps only 20 or 30 %. In case of scarce control, however, the original block deformation because of systematic errors may be very large. In relation to it the adjustment with additional parameters is most effective. Improvement factors of up to 3 or more have been witnessed..

Besides control it is also overlap and redundancy which determine whether and how well systematic image errors can be assessed. With regard to planimetric effects the systematic errors are already quite well determined by blocks with 20 % side overlap. The case is different,however, with image errors which affect mainly the heights. From blocks with 20 % side overlap some systematic errors which cause vertical model deformation cannot be determined. For instance strictly cylindrical model deformation does not cause discrepancies (in case of ideal geometry) nor does it propagate into respective block deformation. Thus, such a deformation can only be well determined from blocks with 60 % side overlap or with crossed flight directions. Such considerations explain that in cases of 20 % side overlap there is often no or only marginal improvement of vertical accuracy with additional parameters. Again, in general, one can expect the improvement in heights to be more effective the fewer vertical control points are available.

3.2 It is a most remarkable result of the application of additional parameters that the σ_o estimates of random errors of wide-angle aerial photographs are brought down to about 3 µm in the negative scale, in some cases values close to 2 µm have been reached. Such σ_o estimates, as compared to about 3 - 6 µm from standard bundle adjustment, come very close to the limiting random error level of w.a. image coordinates which Schilcher /12/ found to be 1,7 µm in a sample of 60 photographs. The σ_o values are also close to the noise level reached some time ago with ballistic cameras /24/.

Such results confirm not only that the geometric accuracy potential of photogrammetry is very high, much higher than ever anticipated. They also demonstrate that the additional parameters approach the precision limits which photographs seem to have at present.

It must be pointed out, however, that the precision limits of aerial photographs can only be approached if all additional errors of the measuring process are kept negligibly small. Practically all tests with additional parameters therefore refer to comparator measurements and to signalized points. It would be of great practical importance to extend the tests on data referring to artificially marked points. There are indications that additional parameters are highly effective also in such cases, see /25/. It may be mentioned too that additional parameters are successfully applied in Finland also with independent model adjustments with medium scale photography for topographic mapping, see /10/.

3.3 The economic aspect of the application of additional parameters is governed, of course, by the accuracy aspect. However, attention must be drawn to the fact that the computational effort of adjustment with additional parameters is quite high, contrary to former estimation. With sets of parameters per strip the computing times double about as compared with standard bundle adjustment. Although the total number of unknowns is only marginally increased it is especially the formation of the reduced normal equations which is considerably extended.

The practical handling of block adjustment with additional parameters requires specialized knowledge and experience, up to now. Also, the adjustment is to be interfaced with gross error detection. Thus quite some program development remains to be done until the computer programs have reached a safe state of operational application.

4. LIMITATION AND EXTENSION OF THE METHOD

There is no doubt that block adjustment with additional parameters has most successfully pushed the resulting accuracy to a most astonishing level which has by no means been anticipated. Nevertheless, it is time for a reflexion about the principal position of the method. Critical remarks that one is groping in the dark by tentatively applying arbitrary sets of parameters, do not really justice to the method. But in a way one is reminded of the old problems of polynomial adjustment. And the elaborate testing and checking of parameters which is necessary in order to ensure safe application may be taken as symptomatic that the method has its inherent risks.

The basic limitation of the method lies in the fact that additional parameters belong to the functional part of the mathematical model of adjustment. They are thought to refer to systematic image deformation. This approach disagrees with the knowledge that image deformations cannot be considered constant for a large group of photographs. Using several sets of parameters takes care of that in a practical way but it cannot be considered an adequate basic solution of the problem. One would like to have finally the deformation of each individual photograph separately accounted for.

In a thorough experimental investigation /12/ Schilcher has shown that the behaviour of image errors of a complete set of (in this case 60) photographs is quite in agreement with the principal properties of a stochastic process. The random noise of image coordinates is 1,7 µm (2,6 µm for super-wide-angle photographs). The systematic image deformation common to all photographs of the complete set amounts to a r.m.s. value of 1,6 µm (3.3 µm). The r.m.s. magnitudes of image deformations increase consistently from 1,6 µm (3,3 µm) to 3,1 µm (4,7 µm) if smaller and smaller subsets of photographs out of the total population are considered up to the individual photographs, see table 1.

size of group	magnitude of average image deformation m_x (wide angle)	m_y (wide angle)	m_x (super wide angle)	m_y (super wide angle)
60 photographs	1,7 µm	1,6 µm	4,0 µm	2,3 µm
15 "	1,5	1,5	4,2	2,6
12 "	1,5	1,8	4,2	2,6
6 "	2,1	2,1	4,3	2,8
3 "	2,5	2,5	4,7	3,3
1 "	3,2	3,0	5,2	4,1

Table 1. R.m.s. magnitudes of average image deformation of groups of photographs, as function of group-size (from /12 /).

Evidently the image deformations which are common for groups of subsequent photographs decrease the larger the groups of photographs are. This behaviour means that the dominant components of image deformation are not constant. They vary and are only linked by (positive) correlation. Thus they correspond to the correlation- (signal-) part of a stochastic process and belong to the stochastic part of the total error budget. We understand now why parameter sets per strip give usually better results than blockinvariant parameters. However, it is also clear that such procedure can only be a substitute for the more proper treatment which would take the correlation of subsequent image deformations into account.

The ultimate error model of block adjustment will have to consider such correlation. How much improvement will be gained, and whether the operational treatment will best use complete correlation matrices, covariance functions, or transfer probabilities remains to be investigated. The field of further research is clearly staked out. It might successfully wind up and complete the long struggle about photogrammetric image errors.

References

/1/ D.C. Brown, The bundle adjustment - progress and prospects; invited paper Commission III, ISP Congress Helsinki 1976.

/2/ H. Ebner, Self calibrating block adjustment; invited paper, Comm.III, ISP Congress Helsinki 1976.

/3/ K. Jacobsen, Test Königshügel - Control by bundle adjustment; presented paper, Commission III, ISP Congress Helsinki 1976

/4/ G. Kupfer, On accuracy achieved by different triangulation procedures; presented paper, Comm.III, ISP Congress Helsinki 1976.

/5/ W. Tegeler, Densification of trigonometric networks by bundle adjustment; presented paper, Comm.III, ISP Congress Helsinki 1976.

/6/ J. Heikkilä, K. Inkilä, Self calibration bundle adjustment; The Photogrammetric Journal of Finland, Vol. 7 No. 2, 1978.

/7/ A. Grün, Progress in photogrammetric point determination by compensation of systematic errors and detection of gross errors; ISP Comm. III Symposium Moscow 1978.

/8/ C.C. Slama, Results of an evaluation of high precision analytical photogrammetry for geodetic control densification; Proceedings ACSM-ASP Convention, Washington D.C. 1978.

/9/ S.F. El Hakim, W. Faig, Combined geodetic and photogrammetric adjustment for densification of control networks; Proceedings ASP Convention Washington D.C. 1979.

/10/ T. Sarjakoski, Analytical photogrammetric block adjustment by indenpendent models with additional parameters; The photogrammetric Journal of Finland, Vol. 7 No. 2, 1978.

/11/ G. Haug, Bestimmung und Korrektur systematischer Bild- und Modelldeformationen in der Aerotriangulation am Beispiel des Testfeldes "Oberschwaben"; Dissertation Universität Stuttgart 1979.

/12/ M. Schilcher, Empirisch-statistische Untersuchungen zur Genauigkeitsstruktur des photogrammetrischen Luftbildes; Dissertation Universität Stuttgart, 1980.

/13/ H. Klein, New results of bundle block adjustments with additional parameters; Proceedings 37th Photogrammetric Week Stuttgart 1979.

/14/ F. Ackermann, M. Schilcher, Auto- and cross-correlation of image coordinates; ISP Commission III Symposium Moscow 1978.

/15/ A. Grün, Accuracy reliability and statistics in close range photogrammetry; ISP Comm. V Symposium Stockholm 1978.

/16/ K. Jacobsen, Detection and consideration of systematic image errors using the Hannover bundle block adjustment; Proceedings ISP Working Group III/3 Seminar Aalborg 1979.

/17/ J. Juhl, Results from Jämijärvi, Proceedings ISP Working Group III/3 Seminar Aalborg 1979.

/18/ A. Grün, Self-calibration versus test field calibration; Proceedings ISP Working Group III/3 Seminar Aalborg 1979.

/19/ G.H. Schut, Selection of additional parameters for bundle adjustment; ISP Commission III Symposium Moscow, 1978.

/20/ S.F. El Hakim, W. Faig, Compensation of systematic image errors using spherical harmonics; Proceedings ASP Fall Techn. Meeting Little Rock, 1977.

/21/ H. Klein, Bundle adjustments with additional parameters of the testblock Appenweier; Presented paper, Comm.III ISP Congress Hamburg 1980.

/22/ W. Förstner, Evaluation of block adjustment results; Presented paper, Comm. III ISP Congress Hamburg 1980.

/23/ H. Ellenbeck, G. Kupfer, Bundle adjustment with field- and self calibration - recent results; ISP Comm.III Symposium Moscow 1978.

/24/ L.W. Fritz, H.H. Schmid, Stellar calibration of the Orbigon lens; Photogrammetric Engineering, 2, 1974

/25/ F. Ackermann, R. Bettin, An empirical test on point transfer in aerial triangulation; Presented paper, Comm. III ISP Congress Hamburg 1980.