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W. Förstner, Stuttgart University

EVALUATION OF BLOCK ADJUSTMENT RESULTS

Summary

Statistical tools can serve as an aid for computer assisted evaluation of block adjustment results. This concerns gross errors, systematic errors and errors in the a priori weights. The paper shows how far a computer program could be able (1) to detect, compensate, or eliminate those errors, (2) to give information about non detectable errors and their influence on the result, i. e. the coordinates. This can serve as an objective measure for the reliability of the blockadjustment.

1. Introduction

1.1 The evaluation of block adjustment results aims at the acceptability of the determined coordinates. The result is acceptable, if the data and the block geometry are checked by adequate tests. This concerns on the one side the detection of gross errors and the perception of systematic errors. On the other side one needs information about the precision and the reliability of the coordinates.

In practice this evaluation, which usually is made by human inspection (by checking the residuals, examining the "network diagram" etc.), is very combersome, for there is often a great amount of data. This is why computer assistance is wanted. The paper is supposed to show how far this is possible by using statistical tools.

1.2 The underlying theory is essentially the reliability theory by Baarda (1967/8/76). An extension to tests on several gross errors or parameters, which is urgently necessary in photogrammetry, is subject of the paper. It forms a link to the theory for general linear hypotheses, known from statistics (cf. e. g. Searle, 1971).

The theory is based on the following idea: The adjustment is founded upon the assumption (in statistical terms the nullhypothesis H_o) that there are only random errors. Other errors expected, gross or systematic, are formulated in an alternative hypothesis H_a. An optimal test decides between H_o and H_a. The sensitivity of the test leads to lower bounds for gross or systematic errors, which can just be detected by the test with a preset probability. The reliability of the result can be described by the influence of non detectable errors on the coordinates.

1.3 Gross and systematic errors in this context are both supposed to be errors in the functional model only. It allows a joint treatment of both error types. This approach, though customary, obviously can not be taken for granted for two reasons. There is no clear distinction between gross and systematic errors; local systematic errors and gross errors can have the same effect. Also the assignment of the errors to the functional or to the stochastical model is a matter of opinion to a certain extent, as the true error sources are not known usually. This is confirmed by the fact, that in a wide range the refinement of the functional model can be substituted by a refinement of the stochastical model and vice versa, e. g. by taking into account the correlation between the observed image coordinates (Schilcher, 1980) or by changing the weights of bad observations instead of introducing additional parameters or eliminating the erroneous observations.

Fortunately experiments show that systematic errors are rather constant, at least a great amount of the error budget can be absorbed by additional parameters via selfcalibration, i. e. by extending the functional model. Also a great percentage of the gross errors would either occur once more, if the measurement was repeated, or at least would be in the same magnitude, if the coordinate e. g. was measured erroneously a second time. It reveals, that gross errors can be treated as locally confined systematic errors, i. e. errors in the functional model. Generally gross errors also do not influence the precision of the observed values, at least not more, than the weights do vary because of the always very much simplified stochastical model.

1.4 The theory for treating the stochastical model is developed not nearly so far. Only estimating weights or correlations is possible. One can obtain information about the variances of the estimated values. A statistical test is not available as the probability distribution of the estimated weights or correlations is not known. This drawback of evaluating block adjustment results is not too considerable from a practical point of view, as pure economical reasons prohibit an extension of the stochastical model which is pushed too far. Therefore the paper only shows the possibility of estimating weights of groups of observations or of weights, which depend on given parameters. This seems to be necessary and practicable, as the inference drawn from gross error tests highly depend on the chosen weights. The additional effort is negligible.

2. The mathematical model and its errors

2.1 The mathematical model

Let us consider the linear (or linearized) functional model

$$E(l) = l + \varepsilon = \sum_{1}^{u} a_i \tilde{x}_i = A \tilde{x} = B \tilde{t} + C \tilde{k} \tag{1}$$

with the n observations l_i forming the vector l, the given design matrix A with the columns a_i and the corresponding u unknowns x_i forming the vector x. x may be split into the vectors t and k for the u_t unknown nuisance parameters (such as scale, orientation, possibly projection centres) and the u_k unknown coordinates resp., with the corresponding partition of $A = (B\ C)$. The model generally does not fit to the observed values. The vector ε describes the model errors.

The stochastical model has the same structure. The covariance or dispersion matrix of the observations is

$$D(l) = Q \tilde{\sigma}_o^2 = \sum_{1}^{p} Q_j \tilde{\sigma}_{o,j}^2, \tag{2}$$

in which Q is the given weight coefficient matrix and $\tilde{\sigma}_o^2$ the unknown variance factor. Similarily to the expectation $E(l)$ the variance covariance matrix is split into additive components $Q_j\,\tilde{\sigma}^2_{o,j}$ with given Q_j and unknown factors $\tilde{\sigma}^2_{o,j}$.

The extension of the stochastical model needs an explanation. Eq.(2) allows to estimate e. g. the ratio between the weights of different groups of observations. In the case of two groups, e. g. photogrammetric and geodetic observations, one chooses $Q_1 = \mathrm{diag}(Q_{pp}, O)$ and $Q_2 = \mathrm{diag}(O, Q_{gg})$. A second example is the radial component of the variance of image coordinates, which might depend on the field angle α_i between ray and optical axis: $\tilde{\sigma}^2_{r_i} = \tilde{a}^2 + \tilde{b}^2\,\tilde{c}^2/\cos^4\alpha_i$. $\tilde{a}$ represents the standard deviation of the measuring process and $\tilde{b}$ is the standard deviation of the ray, i. e. of α. c is the focal length. In this case one will choose $Q_1 = a^2\,I$, $Q_2 = \mathrm{diag}(b^2\,c^2/\cos^4\alpha_i)$. a and b are approximate values, the estimators $\hat{\sigma}_{o,1}$ and $\hat{\sigma}_{o,2}$ are factors for a and b to get better approximations.

The estimation of the unknown parameters is based on the assumption, i. e. the null hypothesis H_o:

$$E(\varepsilon|H_o) = O; \qquad E(\sigma^2_{o,j}|H_o) = \tilde{\sigma}^2_{o,j}, \qquad j=1,\dots,p. \tag{3}$$

In order to set up tests one has to specify the expected model errors, i.e. formulate one (or several) alternative hypothesis H_a. We perform this separately for the functional and the stochastical model.

2.2 Errors in the functional model

The expected errors in the functional model are denoted by $\widetilde{\nabla e}$. By analogy to eq.(1) they are assumed to depend linearily on additional parameters $\widetilde{\nabla s}$. The alternative hypothesis then appears in the form

$$E(\varepsilon|H_a) = E(\varepsilon|H_o) + \widetilde{\nabla e}; \qquad \widetilde{\nabla e} = H\,\widetilde{\nabla s} \neq O. \tag{4}$$

It covers systematic errors and gross errors, if the matrix H is chosen properly. The difference is revealed by the structure of H: Systematic errors influence all observations, this leads to a (rather) full matrix H, while in the case of gross errors H is sparse.

If we assume an error in point transfer we have to deal with two gross errors, $\widetilde{\nabla s}_x$ and $\widetilde{\nabla s}_y$ resp.. They influence six observations, i. e. the coordinates of the points in three consecutive pictures. In this case H has the form $H' = (O,\dots,O,I_2,O,\dots,O,I_2,O,\dots,O,I_2,O,\dots,O)$, in which the unit matrices are placed at the coordinates of the three points concerned.

2.3 Errors in the stochastical model

The description of the expected errors in the stochastical model, e. g. errors in the a priori weights, is simpler than eq.(4)

$$E(\sigma^2_{o,j}|H_a) = E(\sigma^2_{o,j}|H_o)\cdot\widetilde{\nabla t}_j; \qquad \widetilde{\nabla t}_j \neq O. \tag{5}$$

as the factors $\widetilde{\nabla t}_j$ are not specified in detail. Similarly to the design matrix A the structure of the matrices Q_j is fixed and can not be subject of alternative hypotheses. Otherwise one has to change the model eq.(1) and (2).

3. Estimation of parameters

The estimation of the unknowns $\tilde{x}_i$ and $\tilde{\sigma}^2_{o,j}$ is accomplished by two steps. According to the theory of least squares the estimators $\hat{x}$, $\hat{t}$, $\hat{k}$ and $\hat{\varepsilon}$ for $\tilde{x}$, $\tilde{t}$, $\tilde{k}$ and $\tilde{\varepsilon}$ resp. are linear functions of the observations. The estimators $\hat{\sigma}^2_{o,j}$ for $\tilde{\sigma}^2_{o,j}$ are quadratic functions of the observations. They only depend on the residuals $v = \hat{\varepsilon}$.

3.1 Estimators in the functional model

The practical procedure takes into account the different structure of H for systematic and gross errors, only for operational reasons.

If systematic errors are expected the model eq.(1) is therefore extended to

$$E(l) = A\,\tilde{x} + H\,\tilde{s} = B\,\tilde{t} + C\,\tilde{k} + H\,\tilde{s} \tag{6}$$

The alternative H_a eq.(4) is then equivalent to $\tilde{s} = o$ (i. e. $\tilde{\nabla s} \equiv \tilde{s}$). We use the estimated parameters $\hat{s}$ with their weight coefficient matrix

$$Q_{\hat{s}\hat{s}} = (H'\,P\,(Q - A\,(A'\,Q^{-1}\,A)^{-1}\,A')\,P\,H)^{-}\,, \qquad P=Q^{-1} \tag{7}$$

for the testing procedure.

Obviously this treatment is not possible for gross errors as one does not know which observations are erroneous. We test the residuals v with their weight coefficient matrix

$$Q_{vv} = Q - A\,(A'\,Q^{-1}\,A)^{-1}\,A'. \tag{8}$$

for error detection.

Note that $Q_{\hat{s}\hat{s}} = (H'\,P\,Q_{vv}\,P\,H)^{-}$, in which $Q_{\hat{s}\hat{s}}$ is the result of the extended model eq.(6) while Q_{vv} is the result of the original model eq.(1). We will not refine the notation to designate this difference but use it throughout this paper. We will therefore not treat systematic and gross errors simultaneously but parallel. The joint evaluation of both error types is discussed in ch. 7.

3.2 Estimating in the stochastical model

There are several algorithms for estimating the variance factors $\tilde{\sigma}^2_{o,j}$ (Searle, 1971, Grafarend, 1978, Koch, 1978, Förstner, 1979). The one given here is very economical, if the observations are not correlated

$$\hat{\sigma}^2_{o,j} = \frac{v'\,P\,Q_j\,P\,v}{tr(Q_{vv}\,P\,Q_j\,P)} = \frac{v_j'\,P_j\,v_j}{tr(Q_{vv}\,P_j)} \tag{9}$$

It only needs the diagonal elements of the matrix Q_{vv}, thus only a small part of the inversed normal equation matrix has to be computed. The other estimators offer the variance of the variance factors for a rough evaluation. As the probability distribution of the estimators is not known (cf. introduction) we will concentrate on the analysis of the errors in the functional model.

4. Statistical tests

The evaluation of the functional model usually is done in a hierarchical way. The well known F-test on the global variance factor $\hat{\sigma}_o^2$ gives very poor information as all types of model errors accumulate in $\hat{\sigma}_o^2$. Testing the alternative hypotheses is split into a global test, examining all parameters or gross errors together and a single test for localization.

4.1 Global test

The global test uses the test statistic

$$T=\hat{\hat{s}}' Q_{\hat{s}\hat{s}}^{-} \hat{\hat{s}}/\sigma_o^2 \quad \text{or} \quad T=v'PH\ (H'PQ_{vv}PH)^{-}\ H'Pv/\sigma_o^2 \quad \text{with} \quad T\sim\chi'^2(r_s,\delta^2). \tag{10}$$

They can be obtained by an adjustment in steps, starting with the extended model eq.(6) (cf. Pelzer,1977) and introducing the null hypothesis H_o: $\tilde{s} = o$ as a condition in a second step. Testing of T is the same as testing the variance factor of the second step. In both cases T is independant of the chosen generalized inverse and follows a noncentral χ^2-distribution with r_s degrees of freedom and non-centrality parameter $\delta^2(s)$, where r_s is the rank of the matrix to be inverted and

$$\delta^2(s) = \tilde{s}'\ Q_{\tilde{s}\tilde{s}}^{-}\ \tilde{s}\ /\ \sigma_o^2 \quad \text{or} \quad \delta^2(s) = \tilde{s}'\ H'\ P\ Q_{vv}\ P\ H\ \tilde{s}\ /\ \sigma_o^2 \tag{11}$$

If one does not want to use a given value σ_o^2 for the variance factor one can also take the test statistic

$$R = \frac{T/r_s}{(\Omega_o - T)/(r - r_s)} \quad \text{with} \quad R \sim F'(r_s,\ r - r_s,\ \delta^2), \tag{12}$$

which follows a non-central F-distribution. $\Omega_o = v'\ P\ v$ is the weighted sum of the residuals from the original model eq.(1) including the systematic or gross errors and r is the redundancy of this adjustment.

If $\delta^2 = 0$, i. e. if the null-hypothesis H_o is true, the test statistics follow a central χ^2- or F-distribution. H is rejected, if the statistics exceed the critical values $\chi^2(\alpha,\ r_s)$ or $F(\alpha,\ r_s,\ r-r_s)$ depending on the significance level $1-\alpha$.

4.2 Single test

For the localization of the error sources one specializes H_a, e. g. by assuming only one parameter, gross errors which influence one single observation each or a single gross error. This leads to well known single tests as the t-test, the test of Stefanovic (1978) or the "data-snooping" of Baarda (1967) resp.. Each of these tests leads to mutually dependant test statistics, if it is used for the check of several alternativ hypotheses simultaneously.

5. Sensitivity of tests

If the tests do not indicate a significant error in the functional model, there may still remain errors undetected by the test. To get an idea about the sensitivity of the tests one can ask for lower bounds $\nabla_o e$ of the errors ∇e, which can just be detected with a given probability β_o. These errors lead to a lower bound $\delta_o^2 = \delta_o^2(\alpha,\ \beta_o)$ of the non-centrality parameter dependant in addition to β_o on the significance level $1-\alpha$. α and β_o are assumed to be given, thus δ_o is fixed.

Instead of ∇e we analyse the parameters ∇s. From eq.(11) we obtain a relation for the lower bounds $\nabla_o s$

$$\delta_o = \sqrt{\nabla_o s' \, Q^{-}_{\hat{s}\hat{s}} \, \nabla_o s} \, / \, \sigma_o \quad \text{or} \quad \delta_o = \sqrt{\nabla_o s' \, H' \, P \, Q_{vv} \, P \, H \, \nabla_o s} \, / \, \sigma_o \qquad (13)$$

which desribes an ellipsoid like figure. Parameters within this figure can not be detected by the test with a probability greater than β_o (cf. Förstner, 1976, v. Mierlo, 1977, Pelzer, 1980). The standardized length $\delta'_o(s)$ of the vector $\nabla_o e$ or $\nabla_o s$ is

$$\delta'_o(s) = \sqrt{\nabla_o e' \, P \, \nabla_o e} \, / \, \sigma_o = \sqrt{\nabla_o s' \, H' \, P \, H \, \nabla_o s} \, / \, \sigma_o = \sqrt{\nabla_o s \, Q^{-}_{ss} \, \nabla_o s} \, / \, \sigma$$

in which $Q^{-}_{ss} = H' \, P \, H$ is the weight matrix of those paramters s, if the vector x is fixed. Dividing by eq.(13) we obtain

$$\delta'_o(s) = \delta_o \sqrt{\frac{s' \, Q^{-}_{ss} \, s}{s' \, Q^{-}_{\hat{s}\hat{s}} \, s}} \quad \text{or} \quad \delta'_o(s) = \delta_o \sqrt{\frac{s' \, H' \, P \, H \, s}{s' \, H' \, P \, Q_{vv} \, P \, H \, s}} \, . \qquad (14)$$

With $\delta'_o(s)$ we get a practical formula for the lower bound

$$\nabla_o s = s \cdot \sigma_s \cdot \delta'_o(s) \qquad (15)$$

of the parameters s. For the test of the additional parameters eq.(15) reduces to

$$\nabla_o s = s \cdot \sigma_o \cdot \delta_o \, / \, \sqrt{s' \, Q^{-}_{\hat{s}\hat{s}} \, s} \qquad (15a)$$

The lower bounds depend on 1. the direction of the assumed error (s), 2. the precision (σ_s), 3. the statistical parameters α and β_o (δ_o) and 4. the geometry ($Q^{-}_{\hat{s}\hat{s}}$ or Q_{vv}). In eq.(15) the lower bound $\nabla_o s$ is split into a vector part (s) and a scalar part ($\nabla_o(s) = \sigma_s \, \delta'_o(s)$, i. e. $\nabla_o s = s \, \nabla_o(s)$). If the vector s is standardized to 1 the scalar part describes the ellipsoid like figure via polar coordinates.

Eq.(14) and (15) show that the sensitivity of the tests is the greater (δ'_o small) the smaller the variance of the parameters after the adjustment or the greater the variance of the residuals, i. e. the greater the redundancy.

If we specialize and assume only one parameter, we obtain the measure for the determinability $\nabla_o s = \delta_o \, \sigma_{\hat{s}}$ of the parameter, which is proportional to its standard error a posteriory (cf. Förstner, 1980, Pelzer, 1980). If we assume only one gross error we obtain the measure for the controllability of the observation concerned $\nabla_o l_i = \sigma_{l_i} \, \delta_o / \sqrt{r_i}$, in which the redundancy number $r_i = (Q_{vv} \, P)_{ii}$ is the contribution of the observation l_i to the total redundancy (Förstner, 1979). In both cases δ_o is easy to be computed: We obtain for the test with T $\delta_o = \phi^{-1}(1-\alpha/2) + \phi^{-1}(1-\beta_o)$ with the cumulative normal distribution ϕ. E. g. $\alpha = 0.1$ % and $\beta_o = 80$ % leads to $\delta_o \approx 4.2$. I. e. parameters less than $4 \cdot \sigma_{\hat{s}}$ can not be detected by the test with a probability greater than 80 %.

6. Reliability of result

The reliability of the result can be described by the influence of non detectable errors in the mathematical model on the coordinates. Non detectable systematic or gross errors $\nabla_o e = H \, \nabla_o s$ lead to a deformation $\nabla_o k(s)$ of the coordinates

$$\nabla_o k(s) = Q_{kk}\, \bar{C}'\, P\, \nabla_o e(s), \quad \bar{C} = (I - B\, (B'\, P\, B)^{-1}\, B'\, P)\, C. \tag{16}$$

This deformation vector is very illustrative, but should be calculated for all possible directions of s, which is impossible. Therefore we only use the standardized length $\bar{\delta}_o(s)$ of this vector (similarly to the sensitivity value δ_o'). It is

$$\bar{\delta}_o(s) = \sqrt{\nabla_o k'(s)\, Q_{kk}\, \nabla_o k(s)}\; / \;\sigma_o$$

Dividing by eq.(13) we obtain

$$\bar{\delta}_o(s) = \delta_o \sqrt{\frac{s'\, H'\, P\, \bar{C}'\, Q_{kk}\, \bar{C}\, P\, H\, s}{s'\, Q_{\hat{\bar{s}}\hat{\bar{s}}}\, s}} \quad \text{or} \quad \bar{\delta}_o(s) = \delta_o \sqrt{\frac{s'\, H'\, P\, \bar{C}'\, Q_{kk}\, \bar{C}\, P\, H\, s}{s'\, H'\, P\, Q_{vv}\, P\, H\, s}} \tag{17}$$

$\bar{\delta}_o(s)$ also describes an ellipsoid like figur which directly can compared with $\delta_o'(s)$ and shows, how much an error $\nabla_o s$ influences the result dependant on the direction s. In order to get a more practical formula also in this case, we consider the influence $\nabla_o f(s)$ of $\nabla_o s$ on a linear (or linearized) function $f = g'k$ of the coordinates. By using Cauchy-Schwarz's inequality it can be shown that this influence

$$\nabla_o f(s) \leq \sigma_f \cdot \bar{\delta}_o(s) \tag{18}$$

is less than the $\bar{\delta}_o$-fold standard deviation σ_f of the function.

If we specialize and assume only a single gross error we obtain $\bar{\delta}_{o,i} = \delta_o \sqrt{u_{k_i}/r_i}$, the measure for the (external) reliability, i. e. the maximum influence of a non detectable gross error onto the coordinates (Baarda, 1976. u_{k_i} is the contribution of the observation l_i to the determination of the unknown coordinates, cf. Förstner, 1979).

7. Discussion

The extension of the reliability theory towards multidimensional tests gives answer to several practical and theoretical questions.

7.1 It is possible to test all kinds of gross errors, which can be formulated as errors in the (linear) functional model. It includes nearly all types of gross errors occuring in photogrammetric blocks, especially pointing errors, errors of point transfer, misidentification of groups of points, exchange of points etc..

This is valid, as long as the gross errors are not too large, because the tests are developped within a linear functional model. In practice this is a drawback, as very large errors have to be found by other means, e.g. by rough checks of the image coordinates, of the connections between the images or models, or by checks during the calculation of the approximate values. Special complicated errors as a wrong coordinate system will cause trouble in any case.

Nevertheless, the theory gives an objective indication when to stop the error detection process.

7.2 The evaluation of the systematic errors and of their influence on the result can be made transparent. There are three criteria: the significance and the determinability of the parameters and the reliability of the coordinates with respect to non determinable parameters. All three have a right on their own. (cf. Ackermann (1980)):

- Significance tests can be used to check whether the presumed systematic errors are inherent in the data. The parameters will have a physical meaning in this case.
- The check on determinability ($\delta'_o(s)$, eq. (14) can be used to select parameters out of a given set in order to gain a stable solution. The parameters may have a physical meaning or not in this case.
- The check on reliability ($\bar{\delta}_o(s)$, eq. (17)) can be used for the same purpose. Especially, if the parameters are not supposed to have a physical meaning, this check leads to the best result (as far as the selection is concerned), because the influence of non determinable parameters on the coordiantes is bounded.

In all three cases the evaluation is possible for single parameters or groups of parameters. This is advisable, if the parameters are highly correlated, but also if the groups are rather large (>10) and one has to expect moderate correlations (about perfect correlation, see below).

The single test for localization should be linked to the global test e.g. by choosing the significance level in a way that the corresponding sensitivity of the tests is equal, i.e. by fixing the lower bound δ_o^2 for the non centrality parameter and the power β_o of the test (cf.Baarda,1968).

7.3 The criteria of determinability and of reliability can be used for an optimization of the block geometry, as they only depend on the mathematical model, i.e. the geometry, but not on the observed values. Here one will be interested in those systematic errors, which can be determined most weakly or have the highest influence on the result, if they stay undetected, in order to strengthen the block. This means that one has to find the vector s which leads to the highest values $\delta'_o(s)$ or $\bar{\delta}_o(s)$. The solution can be obtained by solving e.g. $\delta'_o(s)\rightarrow$max, which is equivalent to finding the largest eigen value $\lambda=(\delta'_o/\delta_o)^2$ in $|Q_{\hat{s}\hat{s}}\lambda - Q_{ss}| = 0$ and determining the corresponding eigenvector. (This procedure also can be used in deformation analysis).

7.4 As systematic and gross errors allow a joint treatment, a joint testing procedure is advisable. It has the advantage that both types of errors can clearly be identified.

If the geometry does not allow a distinction between systematic and gross errors, it is indicated by identical test statistics T or R resp.. This is important, especially in the case of poor control, where gross errors in control points can pretend systematic errors in the photogrammetric images. In this case the parameters and the residuals are perfectly correlated.

The check of the observations in the original model allows to test those observations,which in the extended model would be not controllable.

Of course a refined error detection on the basis of the extended model eq. (6) is necessary as the systematic errors are in the magnitude of small gross errors (ca. 10 μm).

7.5 The test statistics T and R are independent of the used generalized inverse. This is the reason why perfect correlation of parameters and residuals leads to the same test statistic.

It also justifies the practical procedure that one of two parameters, which are perfectly correlated, or a parameter, which is determined by a single observation has to be eliminated. If the solution is stabilized by additional observations for the parameter values, perfect or very high correlation of parameters with other unknowns (not residuals) will have not to be feared, as the solution cannot tend to a wrong result.

The criteria are valid for arbitrary weight matrix. Thus also correlated observations can be tested. This implies that the test proposed by Stefanovic (1978) is not restricted to uncorrelated observations or groups of observations.

7.6 The evaluation of the chosen weights is not possible to such an extent as the evaluation of the functional model. As the error detection procedures sensitively react on errors in weights, they should at least be checked. Of course, for single observations a distinction between a gross error and an error in weight is not possible. Thus one may prefer diminishing the weight rather than eliminating an observation if the estimated error is small and the change of weight can be justified e.g. by the image quality.

7.7 Though computer assisted evaluation can be driven rather far there are still several problems to solve.

- Strategies for joint gross error detection and perception of systematic errors have to be worked out. A rigorous test on several (e.g. >10) errors is to costly and the mutual influence of both error types has to be taken into consideration. Special attention should therfore be given to preadjustment error detection (cf. Molenaar and Bouloucos, 1978) and also to sequential testing procedures with respect to the increasing use of analytic plotters.
- Most gross errors only have a locally limited influence. The inequality eq. (17), which defines the reliability of the coordinates, contains the standard error of an arbitrary function of the coordinates. This might be very large though the real influence is low, e.g. because the observation and the function concerned are in different parts of the block. A better approximation than eq. (17) is desirable. This would be a closer link between reliability and precision.
- The evaluation of the precision itself (cf. Baarda, 1973) should be made operational, possibly with approximations, in order to be able to guarantee the reliability of the result.
- An extension of the theory towards an evaluation of the stochastical model would be useful in order to gain complete information about the acceptability of the block adjustment results on the basis of the chosen model.

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