XIV Congress of the International Society for Photogrammetry
Hamburg 1980
Commission III
Working Group 3
Presented Paper

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ATTEMPT AT OBTAINING THE BEST POSSIBLE ACCURACY IN BUNDLE BLOCK ADJUSTMENT

Abstract

Using the data of the test areas Jämijärvi and Willunga supplied by the Working Group III/3 of the International Society for Photogrammetry, different methods of identifying and considering systematic image errors are investigated. This report is restricted to the extraction of information from block data. For handling a particular bundle block adjustment, a particular method is recommended.

Übersicht

Anhand der von der Arbeitsgruppe III/3 der Internationalen Gesellschaft für Photogrammetrie zur Verfügung gestellten Daten der Testgebiete Jämijärvi und Willunga werden verschiedene Methoden zur Erfassung und Berücksichtigung systematischer Bildfehler untersucht. Diese Abhandlung beschränkt sich auf Gewinnung der Informationen aus den Blockdaten. Es wird eine Verfahrensweise zur Behandlung beliebiger Bündelblockausgleichungen vorgeschlagen und untersucht.

1. Description of computer programs involved

The Hannover Bundle Block adjustment consists of a program system (see Jacobsen (4), Appendix 3) which offers the following possibilities of systematic image error compensation:

a priori: Transformation of the comparator coordinates to the fiducial or reseau coordinate system by an affine transformation (6 parameters), consideration of the symmetric or assymmetric laboratory camera calibration, consideration of earth curvature and refraction, consideration of individual systematic errors represented by the reseau crosses.

in the block adjustment: Computation and consideration of systematic errors by additional parameters (see Appendix 1) which could be computed for the whole block as a unit or for several photo groups as separate units.

iterative: Identification of systematic image errors from the residuals of a block adjustment and refinement of photo coordinates by spline interpolation before a further adjustment.

a posteriori: Systematic block errors or also the tension in the terrestrial net can be considered by a prediction. Individual point errors are considered by doing an adjustment of distances.

Solution of the bundle block adjustment

The bundle block adjustment is computed using the colinearity equations according to the solution of H.H.Schmid (8) with simultaneous refinement of photo coordinates by additional parameters (see Müller (7)). The coordinate unknowns of the terrestrial points are eliminated by applying the Schreiber's equations. Finally, the block-oriented Gauss method is applied (see Müller (6)).

The systematic image errors could also iteratively according to Masson d'Autume's method (1) be identified and considered along with, or in combination with, the additional parameters. The mean for each part-area of the photo format is found of the photo coordinate residuals in a whole block adjustment. The mean value for each photo part-area represents an estimate of the systematic image error of this area. On the basis of these mean values, corrections for the photo coordinates could be computed using a two-dimensional spline interpolation. These corrected photo coordinates then flow into a new adjustment. The number of the photo-part-areas may be 9, 25 or 121. Large part-areas cannot take care of systematic errors in small areas. For small part-areas, however, sufficient observations are required to be able to determine accurate mean values. A test value TS makes it possible to know, with the aid of available observations, whether detectable systematic effects still exist.

$$TS = \frac{\sum_{j=1}^{K} \frac{|A_j| \cdot \sqrt{n_j}}{S_0}}{K}$$

where
- K = number of part-areas
- n_j = number of points in the corresponding part-area
- A_j = mean value for the corresponding part-area
- S_0 = mean square of all photo coordinate residuals.

In the case of an iterative adjustment with spline interpolation, the iterations are continued until the test values TS are smaller than 1.0. The test values TS are also good for use in discovering systematic effects that are still present.

2. Compter used

The computations were run on the Control Data Cyber 76 of the Regional Computer Center for Niedersachsen in Hannover. Available is a core storage of about 150 K 60 bit-words. The effective computation speed is about 12 to 15 · 10^6 commands/sec. For the computation of a bundle block adjustment with the Hannover Program in 3 to 4 iterations, the following execution time is required:

$$t\,[sec] = F \cdot BL \cdot BQ^2 \cdot \sqrt[4]{\frac{PB}{18}}$$

where
- BL = photos in block-length direction
- BQ = photos in block-width direction
- PB = average number of photopoints per photo
- F = 0.056 sec for computations without additional parameters
- F = 0.065 sec for computations with 20 additional parameters

This means that the computation time is about 4 seconds for a simple Jämijärvi block with 3x8 photos, about 16 seconds for a double block with 6x8 photos and about 43 seconds for a four-fold block with 12x8 photos.

3. Description of test areas

3.1 Test area Jämijärvi

The photos of the test block Jämijärvi of the University of Helsinki have a mean photo-scale of 1 : 4 044. The area was flown with 6 strips in the East-West dirction and 6 strips in the North-South direction, each strip having 8 photographs. 3 strips, however, give a complete coverage of the area. The arrangement of the strips is somewhat irregular, which defeats its own purpose considering the distribution of the photo points in the photographs (Appendix 3). Every photograph has on the average 18.5 photo points.

3.2 Test area Willunga

The photographs of the Willunga test block from Roberts, Australia, have a mean photo-scale of 1:12,610. The area was flown in the East-West direction in 6 strips with a side-lap of about 60 %. This area is very uniformly planned and flown (Appendix 4). Equally uniform are the photo point positions which are concentrated in 21 very limited locations. There are 14.0 points on the average in every photograph.

4. Methods of identifying, and of taking into consideration, the systematic image errors

The bundle block adjustments were run with a set of 13 parameters (see Jacobsen 3), a set of 16 parameters and a set of 20 parameters (see Jacobsen (4)). The number of parameters could be dangerously high, causing a considerable worsening of the accuracy. For this reason the following procedure is used: The block adjustments are computed with the complete set of parameters, and then the additional parameters are subjected to a student test. The correlations are, however, not considered in the student test, and for this reason the Hotteling test is also applied.

$$T = \frac{V^T \cdot Q_{ap}^{-1} \cdot V}{N_{ap} \cdot \sigma_o^2}$$

where $V = V_1 - V_2$ difference vector of the additional parameters of two adjustments

N_{ap} = number of additional parameters

T = Fisher distributed test value with the degrees of freedom N_{ap} and n-u

The test is applied in two ways. The significance of the entire set as opposed to a computation without additional parameters is checked by setting the whole vector V_2 to zero. Then, the group of significant additional parameters is tested against the whole set, with the hypothesis that the values and correlations of the significant parameters would not change when an adjustment is done with only them. The number of additional parameters of this group is then raised one parameter at a time according to the size order of the ratio: absolute value/mean square error of the additional parameter, until this group is not any more significantly different from the whole set. From now on, this group will be referred to as the group of significant parameters.
The block adjustment along with the group of significant parameters is then checked with the test values TS (see paragraph 1) to know whether systematic image errors still exist. If all test values TS for the three different photo part-areas are smaller than 1.0, then the result of this adjustment is considered as the final result. If, however, this is not the case, then the number of the parameters about to be considered is raised by all parameters whose ratio: value/mean square error is larger or equal to 1.0.

From now on, this paramter group will be referred to as "extended group". If the block adjustment with the extended group still produces test values TS larger than 1.0, then the block adjustment with the complete set of additional parameters is considered as better. If test values TS larger than 1.0 still show up, then these remaining systematic effects are taken into consideration by an iterative spline interpolation. From now on, the results obtained by this strategy will be referred to as "optimum according to strategy".

5. Image coordinate refinement with the aid of reseau crosses

The photographs of Willunga test area were taken with a reseau camera. Reseau crosses, which lie in areas of heavy concentration of points, were measured. The mean distance between the photo points and the closest reseau cross is on the average about 4 mm. This is a very good condition for the consideration of local individual systematic errors using reseau crosses.
After an affine transformation with 6 degrees of freedom, the measurements of the reseau crosses produce mean deviations from their true positions of $\pm$ 3.1 μm. The differences are strongly correlated to those in similar locations on other photographs, with r = 0.53 for x-components and r = 0.76 for the y-components. This means that they represent individual systematic errors only slightly. A correlation of the residuals after the affine transformation to the photo coordinate residuals of a bundle block adjustment without additional parameters was not evident.
Corrections for the photo coordinates were derived from the residual errors after the affine transformation. The residual errors are multiplied by a factor and interpolated using an area spline function. It was clear that the factor required for the treatment of the reseau residual errors may not be too large. A factor of 1.0, that is a full treatment of the reseau residual errors in the spline interpolation, yielded in every case a worsening of the results of the block adjustment as opposed to the similar block adjustment without consideration of the reseau residual errors. A treatment-factor of 0.5 produced more favourable circumstances.
The mean error of the horizontal accuracy μxy for single blocks in block adjustments without additional parameters is reduced through the photo coordinate refinement on the basis of reseau residual errors with the treatment-factor of 0.5 by 3 %, and for double blocks by 1 %. If the block adjustment results are

compared with the computations using additional parameters, then the gain in accuracy for single blocks is reduced to 2 %. For double blocks, there is no accuracy difference between the computation with reseau refinement. On the other hand, the height accuracy is reduced through consideration of the reseau residual errors for single blocks (computation without additional parameters) by 2 %, and for double blocks by 19 %. In computation with additional parameters the accuracy loss for single blocks is reduced to 1 %; for double blocks on the other hand an increase in height accuracy of 12 % is produced.
On the whole the insignificant accuracy increase does not justify the expenditure in the measurement of the reseau crosses. In block adjustments with photographs, which do not have such an extremely uniform point distribution as in the case of Willunga, no accuracy increase through a consideration of reseau residual errors is obtainable. The danger even exists that a worsening of the results may occur.

6. Analysis of results obtained

6.1 Comparison of different sets of parameters

Only the Jämijärvi test block was run with the set of 13 parameters (see Jacobsen 3). With this set the systematic image errors could not fully be taken care of , which was quite clear from the test values TS > 1.0. For this reason, it was necessary to do an iterative adjustment with spline interpolation. This way the best results were also achieved. Unfortunately, the iterative adjustment with spline interpolation required a larger number of observations, so that the single blocks (sidelap 20 %) could not be computed in this way. With the set of 16 parameters (see Jacobsen (4)). the systematic image errors could essentially better be taken care of, so that no test values TS remained larger than 1.0.

Table 1: Accuracy increase due to the set of 16 parameters as opposed to the set of 13 parameters (compared is the Optimum according to Strategy OS) Test-block Jämijärvi:

	Position xy	Height z
Single blocks		
Sparse control	1.47 : 1	1.22 : 1
Dense control	1.15 : 1	1.10 : 1
Double blocks	1.05 : 1	0.99 : 1
Four-fold blocks	1.11 : 1	1.02 : 1

Table 2: Accuracy increase due to the set of 16 parameters as opposed to the iterative adjustment with spline interpolation (compared with the Optimum according to Strategy OS) Test block Jämijärvi

	Position xy	Height z
Double blocks	1.02 : 1	1.21 : 1
Four-fold blocks	1.09 : 1	1.01 : 1

Especially for sparse controlled single blocks, which are very sensitive to systematic errors, a considerable accuracy increase could be achieved through the set of 16 parameters as opposed to the set of 13 parameters. On the other hand, the accuracy gain of the set of 16 parameters over and above that of the iterative adjustment with spline interpolation, 2 % and 9 % respectively, for position and 21 % and 1 % respectively, for height, is not very striking.
The parameters of the set of 16 are strongly correlated to one another and also partially to the exterior orientation parameters. The strong correlations affect the position-accuracy only a little, but affect strongly the height-accuracy of sparse controlled single blocks. Hence in comparision with an adjustment without additional parameters, the height accuracy of the sparse controlled Willunga single blocks goes down on the average by 80 %.
The set of 20 parameters is exceptional in its slight correlations. This is, however, not the case with the Willunga block, since in it the photo points are concentrated only in 21 lacations. For the Willunga block, 20 parameters were too many. With this set of 20 only the Willunga test block was computed. Isolated computations with the Jämijärvi dat showed no important accuracy increases, compared with the set of 16 parameters.

Table 3: Accuracy increase due to the set of 20 parameters as opposed to the set of 16 parameters (compared are Optima according to Strategy OS) Testblock Willunga:

	Position xy	Height z
Single blocks	1.23 : 1	1.25 : 1
Double blocks:		
sparse and medium control	1.18 : 1	1.41 : 1
dense control	1.19 : 1	1.82 : 1

The accuracy of the results of the Willunga test block could through the set of 20 parameters in comparison with the 16 parameter set be increased on the average 1.20 : 1 in position.

In height, the situation was even better. Worthy of note is that the biggest gain is in the dense controlled double block. The possibility exists here, however, that this set is better able to clear the tensions in the terrestrial coordinates. Otherwise, this result is hard to explain, since in the complete removal of systematic image errors the largest accuracy increases must be produced for the sparse controlled single blocks.

The prerequisite for an accuracy increase is that the parameter-set, apart from the already described eliminations, is continually reduced until no correlation of more than 0.85 among the parameters exists. Owing to the special concentrations of photo points at only 21 locations in the photograph, correlations larger than 0.995 are produced in adjustments with 20 parameters. This leads to no numerical problems in the program, but the height accuracy is very strongly influenced.

In the extreme case there is an accuracy loss of 1 : 6.15. The negative influence on the positon-accuracy is very limited.

6.2 Identification of the best results

In order to obtain the best results, the procedure as described in paragraph 1 of "Optimum according to Strategy" (successive increase in the number of additional parameters until all test values TS are smaller than 1.0) was applied. Refined, in addition, was the radial symmetric distortion from the calibration certificates on the basis of results of a bundle block adjustment without additional parameters. For this the radial components of the residuals were meaned in the circles according to their radial distance from the principal point. The respective mean values were used for refining the radial symmetric distortion curve. The Optima-according-to-Strategy of the test area Jämijärvi were on the average neither improved nor worsened by the correction of the radial distortion. On the other hand, the accuracy of the results of the Willunga test area could be increased by 2 % in position and by 20 % in height for single blocks, and by 3 % in position and 30 % in height for double blocks. For both areas a clear improvement was produced by computation without additional parameters through the consideration of the radial symmetric corrections. It can be concluded here that radial symmetric components still exist even when an actual camera calibration is applied, and that these radial symmetric components are also not completely removable for individual cameras by a four-part radial symmetric polynomial equation.

Using the procedure of "Optimum according to Strategy" for the Jämijärvi area, an accuracy similar to that of the computation with the significant parameters was obtained. The optimum as obtained for a block configuration with varying numbers of parameters was exceeded on the average by 1 % in position and by 2 % in height. The results of the Willunga test area could, as opposed to the computation with the significant parameters, be improved on the average by 8 % in position and by 6 % in height. In a particular case an increase in position, as well as in height, accuracy in the ratio 1.31 : 1 was obtained. In one case, a worsening in position of 4 % occured. The optimum as obtained for a block configuration with varying numbers of parameters was obtained in position with the "Optimum according to Strategy"; there was, however, a worsening of 20 % in height.

If the risk of a 31 % accuracy loss should be avoided, then the result as obtained with the significant additional parameters should not be accepted as end result without further investigation.

6.3 Raising the accuracy by increasing the number of photographs

Table 4: Jämijärvi

control situation	$\frac{\mu xy \text{ single block}}{\mu xy \text{ double block}}$	$\frac{\mu z \text{ single block}}{\mu z \text{ double block}}$
sparse	1.66	1.87
dense	1.58	1.81
medium	1.71	1.81

	$\frac{\mu xy \text{ single block}}{\mu xy \text{ 4-fold block}}$	$\frac{\mu z \text{ single block}}{\mu z \text{ 4-fold block}}$	$\frac{\mu xy \text{ double block}}{\mu xy \text{ 4-fold block}}$	$\frac{z \text{ double block}}{7 \text{ 4-fold block}}$
sparse	2.12	3.38	1.27	2.00
dense	2.03	2.57	1.29	1.43
medium	2.21	2.83	1.29	1.56

Willunga

	$\frac{\mu xy \text{ single block}}{\mu xy \text{ double block}}$	$\frac{\mu z \text{ single block}}{\mu z \text{ double block}}$
sparse	1.66	3.02
dense	1.37	1.55
medium	1.44	1.82

The accuracy increases actually obtained by raising the sidelap to 60 % and by superimposing two sets of photographs, flown crosswise, each set with 60 % sidelap, are noteworthy. With the exception of the position accuracy of the Jämijärvi four-fold block, they agree perfectly with the expected theoretical accuracy increases.

6.4 Expected theortical accuracy

This bundle block program computes no inversions and hence, by itself, provides no means of accuracy investigations. Nevertheless, the following is introduced in order to be able to make statements on the nature of the accuracy. After a bundle block adjustment the photo coordinates are corrected with the residuals to produce an error-free system with known photo and ground coordinates. In the next block adjustment the error-free photo coordinates and, as the case may be, also the control coordinates are changed with normally distributed errors having a known distribution. The results of this block adjustment are compared with the previously computed ground coordinates, and hence an estimate for the accuracy of the ground coordinates is obtained. The accuracy of this estimate of accuracy can be raised through repeating the procedure several times. This way, an accuracy estimate for the individual points can also be obtained.

By this underlying principle, the expected theoretical accuracy of the blocks was computed. A deviation of the accuracy of the real blocks from the expected theoretical accuracy can be caused by uneliminated systematic image errors and errors of ground points, that is control and check points. A non-unity standard error of unit weight, σ_0 can equally cause deviations since it is the basis of the investigations. The evaluation of the standard error of unit weight is not unproblematic, and so it was calculated as follows for the respective Optimum according to Strategy:

Table 5: Standard error of unit weight of the "Optimum according to Strategy", σ_0 [μm]

	single blocks	double blocks	4-fold blocks	control situation
Jämijärvi				
	2.88	3.08	3.23	sparse
	2.95	3.10	3.22	dense
	2.90	3.07	3.21	medium
Willunga	single blocks	double blocks		
	3.88	3.97		sparse
	4.24	4.20		dense
	3.98	4.00		medium

The difference between the results of a dense controlled adjustment and those of a sparse controlled one gives an indication to the accuracy of the ground points. It is clear here that the ground points of the Jämijärvi block have extremely high accuracy, even though the ground points of the Willunga block are not error-free. The rise in the standard error of unit weight from the single block through the double block to the four-fold block is higher than theoretically expected. Based on a photo coordinate error of 3.2 μm (Jämijärvi test field), the standard error of unit weight should on the average be 3.11 μm for the single block, 3.16 m for the double block and 3.19 μm for the four-fold block. The higher rise may be explained in this way, that not only systematic but also quasi systematic errors are removed by the additional parameters, and hence the smaller the number of measurements used for the evaluation of the parameters the higher the rise.

The expected theoretical accuracy agrees very well with the practically obtained value for the Jämijärvi block. The deviations are only about 10 % on the average, which confirms the exceptional quality of this test block and the fact that all systematic image errors were removed. In particular, the position accuracy of the four-fold blocks deviates by 20 % to 30 % from the theoretical values. 30 % is however, equivalent to only 1.5 mm. The results of the Willunga block, on the other hand, deviate on the average by 40 % in position and 140 % in height from the obtainable theoretical values. There could therefore exist here, systematic image errors that have not yet been eliminated. There are, of course, several pointers to the fact that the ground coordinates are not error-free.

7. Conclusion

Using the data of the Jämijärvi and Willunga test blocks, the possibilities offered by the Hannover Bundle Block Adjustment Programs for identification and consideration of systematic image errors were investigated. In spite of the favourable initial conditions, no noteworthy accuracy increases could be obtained through an a priori refinement of the photo coordinates by means of reaseau cross measurements. The best results could be obtained in a bundle block adjustment with additional parameters on the basis of a 20-parameter set. A 13-parameter set and a 16-parameter set were found to be inadequate. The number of parameters used in the computations must be found using statistical tests, in which it was clear that not only must the significant additional parameters be applied, but their number must also, as the case may be, be raised depending on the still existing systematic residual errors, which are identifiable by a test value.
An iterative adjustment that considers systematic image errors by spline interpolation, produced better results than the bundle block adjustment on the basis of 13 additional parameters, although not as good results as the adjustment on the basis of the 16-parameter and of the 20-parameter sets. An additional accuracy increase could for the Willungs test area be obtained by an a posteriori-a priori correction of the radial symmetric distortion from the data of a previous adjustment.

With the exception of the four-fold block (8-fold photo cover), the practically obtained accuracy of the Jämijärvi test area agrees very well with the theoretically expected accuracy. The Jämijärvi four-fold block produced a position accuracy of up to 1.50 μm in the photograph, which is equivalent to 6 mm in Nature, and a height accuracy of up to 2.7 μm equivalent to 10.7 mm.

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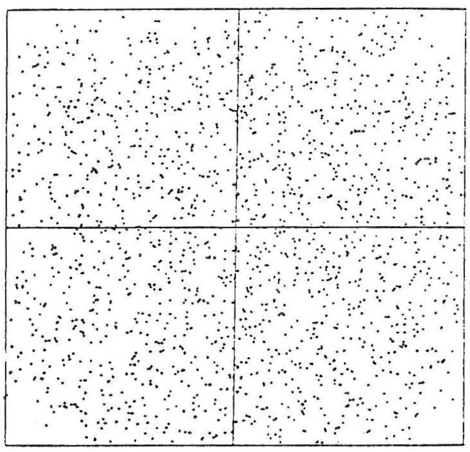
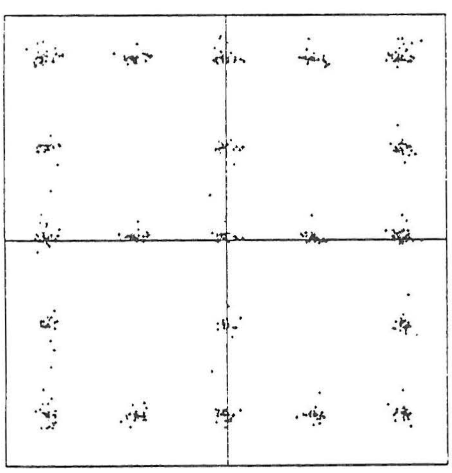
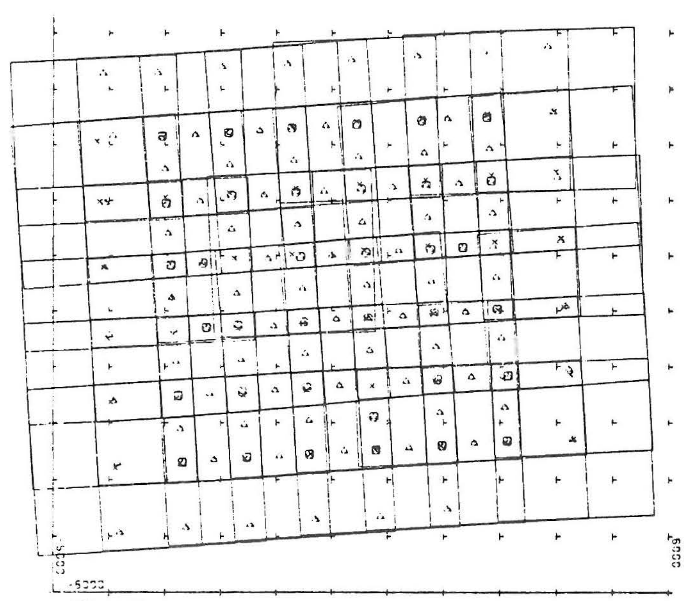


Appendix 1 Jämijärvi
point distribution in the photo

Appendix 3 photos and points double block Jämijärvi



Appendix 2 Willunga
point distribution in the photo



Appendix 4 photos and points Willunga

Block Jämijärvi 1 : 4044

Appendix 5.1

Explanation

AA: Results of bundle block adjustment without additional parameters, without refinement of radial distortion

BA: Results of bundle block adjustment with 16 additional parameters, without refinement of radial distortion

AB: Results of bundle block adjustment without additional parameters, with refinement of radial distortion computed by AA

BB: Results of bundle block adjustment with 16 additional parameters, with refinement of radial distortion computed by AA

CB: Results of bundle block adjustment with all significant additional parameters, with refinement of radial distortion cumputed by AA

DB: Results of bundle block adjustment with an extended numer of additional parameters, with refinement of radial distortion computed by AA

OS: Optimum found by strategy

OA: Actual optimum of µxy

RXY: Ratio µxy to µxy of optimum of block with refinement of radial distortion

RZ: Ratio µz to µz of optimum of block with refinement of radial distortion

Explanation of control point versions

sparse = 8 horizontal, 9 vertical control points
dense = 20 horizontal, 36 vertical control points
medium = 8 horizontal, 13 vertical control points

all results in [µm] (reduced by nominal scale 1 : 4000)

	σ_0	µx	µy	µxy	µz	Rxy	Rz	
AA	4,42	6,69	7,04	6,86	24,83	1,80	2,53	quadratic mean
BA	2,87	4,36	3,61	4,01	10,58	1,05	1,08	single blocks
AB	3,75	6,41	6,73	6,58	18,03	1,73	1,84	
BB	2,87	4,37	3,61	4,01	10,40	1,05	1,06	control point version: sparse
CB	2,90	4,21	3,47	3,86	10,49	1,01	1,07	
DB	2,89	4,45	3,75	4,12	10,34	1,08	1,06	
OS	2,88	4,22	3,35	3,81	9,80	1,00	1,00	
AA	4,84	4,22	4,36	4,29	10,95	1,43	1,62	
BA	2,94	3,08	2,96	3,02	6,90	1,01	1,02	
AB	4,09	4,03	4,04	4,03	7,98	1,34	1,18	
BB	2,94	3,07	2,94	3,00	6,76	1,00	1,00	control point version: dense
CB	2,99	3,05	3,09	3,08	6,90	1,03	1,02	
DB	2,92	3,06	2,97	3,02	6,75	1,01	1,00	
OS	2,95	3,05	3,09	3,04	6,90	1,03	1,02	
AA	4,51	6,52	6,44	6,58	17,20	1,67	2,15	
BA	2,88	4,35	3,61	3,99	8,08	1,02	1,01	
AB	3,79	6,19	6,21	6,20	13,10	1,58	1,64	
BB	2,87	4,38	3,60	4,01	8,10	1,02	1,01	control point version: medium
CB	2,96	4,33	3,53	3,95	8,64	1,01	1,08	
DB	2,87	4,63	3,76	4,23	8,01	1,08	1,00	
OS	2,90	4,35	3,45	3,93	8,42	1,00	1,05	

Appendix 5.2

quadratic mean
double blocks
Jämijärvi

control point version: sparse

	σ_0	μx	μy	μxy	μz	Rxy	Rz
AA	4,61	4,14	3,44	3,60	17,59	1,65	3,65
BA	3,07	2,30	2,33	2,32	4,63	1,01	0,96
AB	3,94	4,01	3,59	3,81	11,46	1,66	2,38
BB	3,07	2,38	2,25	2,31	4,88	1,01	1,01
CB	3,08	2,39	2,18	2,29	5,25	1,00	1,07
DB	3,07	2,40	2,19	2,30	5,25	1,01	1,07
OS	3,08	2,39	2,18	2,29	5,25	1,00	1,07

control point version: dense

	σ_0	μx	μy	μxy	μz	Rxy	Rz
AA	4,86	2,88	2,51	2,70	5,73	1,43	1,50
BA	3,11	1,87	2,02	1,95	3,88	1,03	1,01
AB	4,12	2,78	2,57	2,68	4,57	1,42	1,19
BB	3,10	1,86	2,01	1,93	3,82	1,02	1,00
CB	3,12	1,85	1,93	1,92	3,85	1,02	1,01
DB	3,11	1,86	1,93	1,89	3,84	1,00	1,01
OS	3,10	1,86	2,01	1,93	3,82	1,02	1,00

control point version: medium

	σ_0	μx	μy	μxy	μz	Rxy	Rz
AA	4,67	4,15	3,47	3,83	10,23	1,67	2,22
BA	3,07	2,36	2,22	2,29	4,62	1,01	0,99
AB	3,97	4,06	3,49	2,54	7,98	1,39	1,72
BB	3,07	2,38	2,23	2,31	4,65	1,01	1,00
CB	3,08	2,35	2,20	2,29	5,56	1,00	1,06
DB	3,07	2,39	2,22	2,32	4,88	1,02	1,04
OS	3,07	2,38	2,28	2,30	4,65	1,00	1,00

four fold blocks
Jämijärvi

control point version: sparse

	σ_0	μx	μy	μxy	μz	Rxy	Rz	
AA	4,78	2,28	2,15	2,23	13,53	1,27	4,70	
BA	3,21	1,93	1,60	1,78	2,90	1,02	1,01	
AB	4,20	2,18	2,08	2,13	7,03	1,22	2,44	
BB	3,21	1,95	1,65	1,80	2,90	1,03	1,01	
CB	3,23	1,88	1,63	1,75	2,88	1,00	1,00	OS, OA
DB	3,21	1,95	1,65	1,80	2,90	1,03	1,01	

control point version: dense

	σ_0	μx	μy	μxy	μz	Rxy	Rz	
AA	4,87	1,80	1,75	1,78	4,05	1,19	1,51	
BA	3,22	1,48	1,55	1,53	2,68	1,02	1,00	
AB	4,23	1,70	1,68	1,70	2,98	1,13	1,11	
BB	3,22	1,48	1,58	1,53	2,68	1,00	1,00	
CB	3,22	1,45	1,55	1,50	2,73	1,02	1,02	OS, OA
DB	3,22	1,48	1,55	1,50	2,68	1,00	1,00	

control point version: medium

	σ_0	μx	μy	μxy	μz	Rxy	Rz	
AA	4,82	2,35	2,15	2,25	6,90	1,26	2,32	
BA	3,21	1,95	1,60	1,78	2,98	1,00	1,00	
AB	4,21	2,23	2,05	2,15	4,33	1,21	1,45	
BB	3,21	1,95	1,65	1,80	2,98	1,01	1,00	
CB	3,20	1,95	1,63	1,80	2,98	1,01	1,00	
DB	3,21	1,93	1,63	1,78	2,98	1,00	1,00	OS, OA

results in [μm] (reduced by nominal scale 1:4000)