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BUNDLE BLOCK ADJUSTMENT WITH SMALL FORMAT PHOTOGRAPHS

Abstract

Bundle block adjustments were done with Hasselblad MK 70 photographs. The accuracies of the computed point coordinates are comparable with those obtained with large format photographs.

Kurzfassung

Mit Aufnahmen einer Hasselblad MK 70 wurden Bündelblockausgleichungen durchgeführt, die Genauigkeiten der berechneten Punktkoordinaten ergaben, die vergleichbar mit denen, erzielt mittels großformatiger Bilder sind.

1. Introduction

Aerial photography with conventional mapping cameras are expensive. Flying small areas costs in Germany at least DM 3,000.--, and as a rule DM 4,000.--. Moreover, during the major flying season in March and April, the weather permits flying only for a few days, causing an overloading in the capacity of the aerial photography companies. It is also possible that certain areas could not be flown any day before May. If there are signalized points in the locality, then these points must constantly be watched. In case the photographs are made late in the year, then one has to reckon with the fact that the number of non-identifiable points will grow, since the signals could be overgrown by grass, hidden by leaves, covered by dirt, or destroyed.

The above reasons have given cause for the flying of an area with signalized points with the institute's own camera, Hasselblad MK 70, and the subsequent aerial triangulation.

2. Mapping Camera Hasselblad MK 70

The Hasselblad MK 70 can be used as an aerial mapping camera. The production of 70 photographs with the 70 mm-wide film, or up to 200 photographs with the thin layered films, is possible. The objective, the non-focussable, calibrated and replaceable Zeiss Biogon with c = 61 mm, f/5,6 and the Zeiss Planar with c = 100 mm , f/3,5 were available. Their distortion is smaller than 10 µm. A reseau plate with 5 x 5 crosses at intervals of 1 cm is built-in as part of the camera body.

3. The Flying

The flying was carried out on April 28, 1977, between 11.45 a. m. and 12.45 p. m. with a rented one-motor Cessna Skywagon. The camera was mounted over a hole in the floor of the aircraft and was operated from inside. Unfortunately, there were no navigation auxiliary devices. The aircraft had therefore to be navigated from the seat of the copilot, from which only a very limited view of the area to be photographed was possible. The irregular sidelap and the bad parallelism of the strips can be seen in Appendix 1. Nevertheless, the required area was covered without gaps.

The rent for the aircraft with the pilot for the 3 hours flying time amounted to DM 935,-- including all side costs. Of the 3 flying hours, one was spent on the actual photography and two on the flight to and from the site: The aircraft was stationed 180 km away from the site.

4. Description of the Test Block

For the extention of the control point net in the area of land consolidation Schwienkuhlen, Schleswig-Holstein, an aerial triangulation with Zeiss-RMK 15/23 photographs were to be carried out. The points, signalized for this purpose, were also used for this attempt at an aerial triangulation with the Hasselblad MK 70 photographs.

Both the RMK 15/23 photographs and those of the MK 70 were taken on April 28, 1977. The results of both aerial triangulations may be compared directly.

Data of the RMK 15/23 photographs

Photo scale	1 : 7060
No. of strips	3
No. of photographs	20
Overlap	62 %
Sidelap	38 %
Single-Photo ground coverage	1623 m x 1623 m
Flight height above terrain	1081 m

Data of the MK 70 photographs

Photo scale:	for c = 61 mm	1 : 13 556	
	for c = 100 mm	1 : 8 256	
	mean scale	1 : 12 213	
No. of strips:	for c = 61 mm	5	
	for c = 100 mm	2	
No. of photographs:	for c = 61 mm	107	
	for c = 100 mm	39	
Average overlap		86 %	
Average sidelap		68 %	
Single-photo ground coverage:			
	for c = 61 mm	813 m x 813 m	ratio with RMK = 1 : 3.8
	for c = 100 mm	495 m x 495 m	ratio with RMK = 1 :14.7
Flight height above terrain		831 m	

In the area, there were 56 signalized points, 18 of which had known coordinates (control points). While this number of signalized points was sufficient for tying the RMK 15/23 photographs, it was, however, not sufficient for the MK 70 photographs. For the latter therefore, 270 additional, non-signalized but identifiable points (topographical points) hat to be incorporated in the block. The coordinates computed in the bundle block adjustment with the RMK 15/23 photographs served as basis for comparison with the bundle block adjustments with MK 70 photographs. Because the terrestrial net was not free of tension, the standard deviation of the check point coordinates amounted to about $\pm$ 4 cm. The relative coordinate accuracy for distances between points of under 330 m amounts to $\pm$ 3 cm.

5. Photo Measurements

All photo measurements were done on the Zeiss PSK. The incorporation of the reseau crosses was partly problematic; such was possible only for sufficiently bright photographs; and so, about 15 % of the reseau crosses were not used. Because of the better resolution of the MK 70 photographs, there was no problem finding topographical points for tying the photographs.

Standard deviations of the mean of the double measurements, computed from the differences (pointing accuracy):

Reseau crosses: $s_x = \pm 1.83$ µm, $s_y = \pm 2.00$ µm

Image points : $s_x = \pm 2.80$ µm, $s_y = \pm 2.40$ µm

Standard deviations of the mean of the image point measurements, computed from the differences of the measurements in different models (image coordinate accuracy)

$s_x = 4.11$ µm $\quad s_y = 3.03$ µm

The pointing accuracy of both, the reseau crosses and the image points, is not very high. This could be traced back partly to the bad contrast between the reseau crosses and their back-ground, and partly to the large number of topographical points, which could not be considered of the same accuracy as the signalized points.

On the PSK, only the overlap area of two photograpns is measured. A photograph is therefore fully measured in two neighboring models. In this way, points lying in the three-fold overlap area of a photograph are measured in two different models. The standarddeviation was obtained from the differences of the photo coordinates of both models after reduction to the reseau crosses, it depends on the transformation accuracy and also on the pointing accuracy. The pointing error is larger than for a double measurement in a model since between the individual measurements, a larger time interval has elapsed. With the high accuracy of the PSK, the standard deviation computed from the model measurement differences gives already a good estimate for the accuracy of the photo coordinates with no consideration given to the photo exposure errors.

The large number of topographical points accounts for the large values of the standard deviation of $\pm$ 4.1 μm and $\pm$ 3.03 μ.

6. Bundle Block Adjustment

6.1 A priori Refinements

Before the bundle block adjustment, the image coordinates were corrected with the aid of the radial symmetric distortion given in the calibration certificate. In addition, the reseau crosses make an a priori image coordinate refinement possible. The possibility exists in the Hannover bundle block program system (see Appendix 3) to interpolate for separate x- and y-components, thereseau cross residuals of an affine transformation, by means of a two-dimensional cubic spline interpolation. The corrections can be multiplied by a variable factor, FS ($0 \leq FS \leq 1$). Refraction was ignored since it is taken care of during the block adjustment by the additional parameter No. 7.

6.2 Bundel Block Adjustment with Additional Parameters

The potentials of the Hannover bundle block adjustment program are described in Jacobsen /1/. In the case of the Schwienkuhlen test area, the computations were done on the basis of the set of 24 additional parameters (see Appendix 2). The a posteriori - a priori spline interpolation was ignored.

7. Analysis of Results

7.1 Effect of Refinement by the Reseau

Image coordinate refinement before the adjustment by means of the reseau cross tranformation residuals brought no accuracy increase, but rather a worsening effect of 5 % on the average. Presented in Appendix 4 are only the results of the bundle block adjustment without and with reseau refinement with the Application Factor of 0.5. If the reseau cross transformation residuals are fully applied (Application Factor = 1.0), then the results are worsened by 15 - 20 %. This is like the tendency of the accuracy behaviour of the Willunga test area (Jacobsen /1/) of which RMK-reseau-photographs are available.
The negative influence of the reseau refinement may be traced back to the fact that the transformation residuals are not representative for a larger photo area and that they are superimposed by the inevitable pointing errors, causes for example by bad identification of reseau crosses in bad contrast. The correlation of the transformation residuals of the same reseau crosses in different photographs of r = 0.4 to r = 0.6 speaks against high random errors. On the other hand, no correlation was observed in adjacent position between the transformation residuals and the bundle block adjustment residuals. These circumstances speak for a very local character of reseau errors, which forbids interpolation over 1 cm.

7.2 Effect of Weighting of Observation

The points involved in the block are partly signalized and partly just clearly identifiable topographical points. The topographical points cannot be incorporated with the same accuracy as the signalized points. The photo coordinates of the topographical points are therefore given a smaller weight. The right weight relationships are very difficult to determine, since the standard deviation of the image coordinates depend, not only on the usability and identifiability of the points, but also on the accidental image errors. For this reason, block adjustments were computed using different weight relationships.
The standard deviations obtained show no clear dependence on the weight relationships. When horizontal control points are sparse as well as when they are dense, the optimum standard deviations are obtained if the photo coordinates are equally weighted; where as, when there is a medium number of horizontal control

points, the best results are obtained if the photo coordinates of the topographical points are given the weight 0.3. Through weight manipulation, the obtained standard deviations could the varied by about 10 %. Under no circumstances do the results justify the use of lager weight differences.

7.3 Dependence on other Factors

Ignoring the systematic image errors by non-application of the additional parameters worsens the results by 20 % to 40 %.

The accuracy behaviour of the wide angle photographs (c = 61 mm) appears generally not to differ from that of the normal angle photographs (c = 100 mm). In the two adjustments, one with the wide angle photographs and the other with all photographs together, the better results of only the wide angle photographs could be explained only by the block geometrie. For the test area, different filters and films as well as thin-layer films were used. Neither the whole -, nor any individual part -, block adjustment showed any striking differences in the results. It may be noted however that a high resolution and hence less sensitive film proved to be unfavorable, since in the inevitably darker pictures, some points were not identifiable.

Dependence of the accuracy of the bundle block computed points on the number of photographs in which the points were measured, is obvious, but insignificant because of the small number of check points.

7.4 Accuracy Obtained

For the bundle block adjustment with 18 horizontal control points, a standard coordinate deviation for the independent check points of $S_{xy} = \pm 6.8$ cm, or reduced to the corresponding photo scale, of $S_{xy} = \pm 5.55$ µm, could be obtained. These values contain the check point errors whose standard deviation amount to about ± 3 cm to ± 4 cm, so that in the bundle block adjustment with MK 70 photographs an accuracy of about $S_{xy} = \pm 4.5$ µm $\hat{=} \pm 5.5$ cm could be obtained. Considering the large overlap and hence the large number of photographs per point, this is not a very good result, which can, however, be explained by the large proportion of topographical points incorporated as tie points. The relatively strong accuracy decrease in the results of the adjustments with 13, and with 8, horizontal control points, is explained by the unfavourable block geometry and the irregular distribution of the control points.

8. Expected Theoretical Accuracy

In order to get a clearer view of the accuracy behaviour of the block, an investigation was done using the method described in Jacobsen /1/, paragraph 6.4. On the basis of the applied standard deviations, $\pm$ 3 cm for the horizontal control point coordinates, $\pm$5cm for the vertical control point coordinates and $\pm$ 6.8 µm for the photo coordinates, the following standard deviations were obtained for the computed coordinates:

for 18 horizontal control points $\mu x = 8.3$, $\mu y = 9.3$, $\mu xy = 8.8$ [µm] reduced to the photo

At the first glance, this result is obviously surprising, since the expected theoretical standard deviation is layer by the factor 1.6 than that actually obtained with the check points. The plot of the error ellipses of the individual points (Appendix 6) gives, however, a clearer insight. The standard deviation of points computed by aerial triangulation varies considerably and does not depend only on the location of the point in the block. The number of photographs in which the point was measured has a stronger influence than the point location. The check points available in the test block have, on the average, been measured in more photographs than the rest of the points.
Bringing the respective numbers of photographs per point into consideration, and on the basis of the above adjusted function for the check points, an expected theoretical accuracy of

$$S_x = 4.5 \text{ µm} \qquad S_y = 4.5 \text{ µm} \qquad S_{xy} = 4.5 \text{ µm}$$

is obtained. This is smaller by the factor 1.2 than the actually obtained standard deviation. The reason for this difference may still lie in the non-errorfree check points. By applying the control point coordinate standard deviation of 4 cm instead of 3 cm, the ratio of the accuracy of the actually obtained results to the accuracy of the theoretically expected is reduced by the factor 1.1.
The very bad accuracy of the points which were measured in only two photographs, is caused by the large overlap of the photographs and hence the small base. The functional behaviour of the standard deviation in relationship with the number of photographs per point compares well with that of the other blocks using

large format photographs; for the latter, however, the ratio S_x (N = 2) / S_x (N = 18) never exceeded the value 2.7.

Standard deviation of point coordinates computed by bundle block adjustment as function of number of photographs per point [μm in photo]

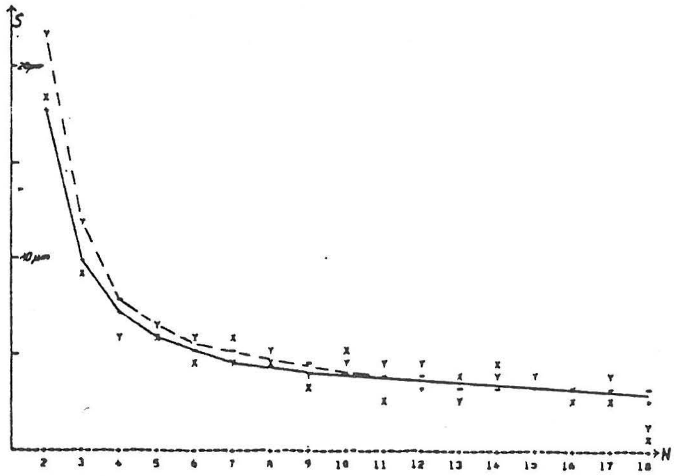
N	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Sx	18,1	9,4	6,0	5,6	4,6	5,6	4,7	3,1	5,3	2,8	4,8	3,7	4,7	4,1	2,7	2,6	0,9
Sy	21,9	11,9	6,2	6,8	5,8	4,5	5,4	4,3	4,3	4,6	4,3	2,8	3,8	3,6	3,4	3,7	1,5

Functional Relationship:

$Sx = 4{,}05 - 0{,}074 \cdot N + 54{,}7 / N^2 \quad \mu m$

$Sy = 3{,}97 - 0{,}064 \cdot N + 71{,}3 / N^2 \quad \mu m$

N = No. of photos / point



Sx, Sy as F (number of photos / point)

9. Conclusion

The Hasselblad MK 70 is considered as a fully geometrically correct aerial camera with obtainable block adjustment accuracies which are comparable with those of large format aerial cameras. Of advantage are the smaller costs of the flying, which, because of its special nature, can be more flexibly arranged. Disadvantageous, on the other hand, is the small photo format which demands a smaller photo scale and / or a denser population of tie points. If the tie points are signalized, then the financial advantage of the flying may be evened out by the additional field work. With non-signalized tie points, on the other hand, the obtainable accuracy falls.

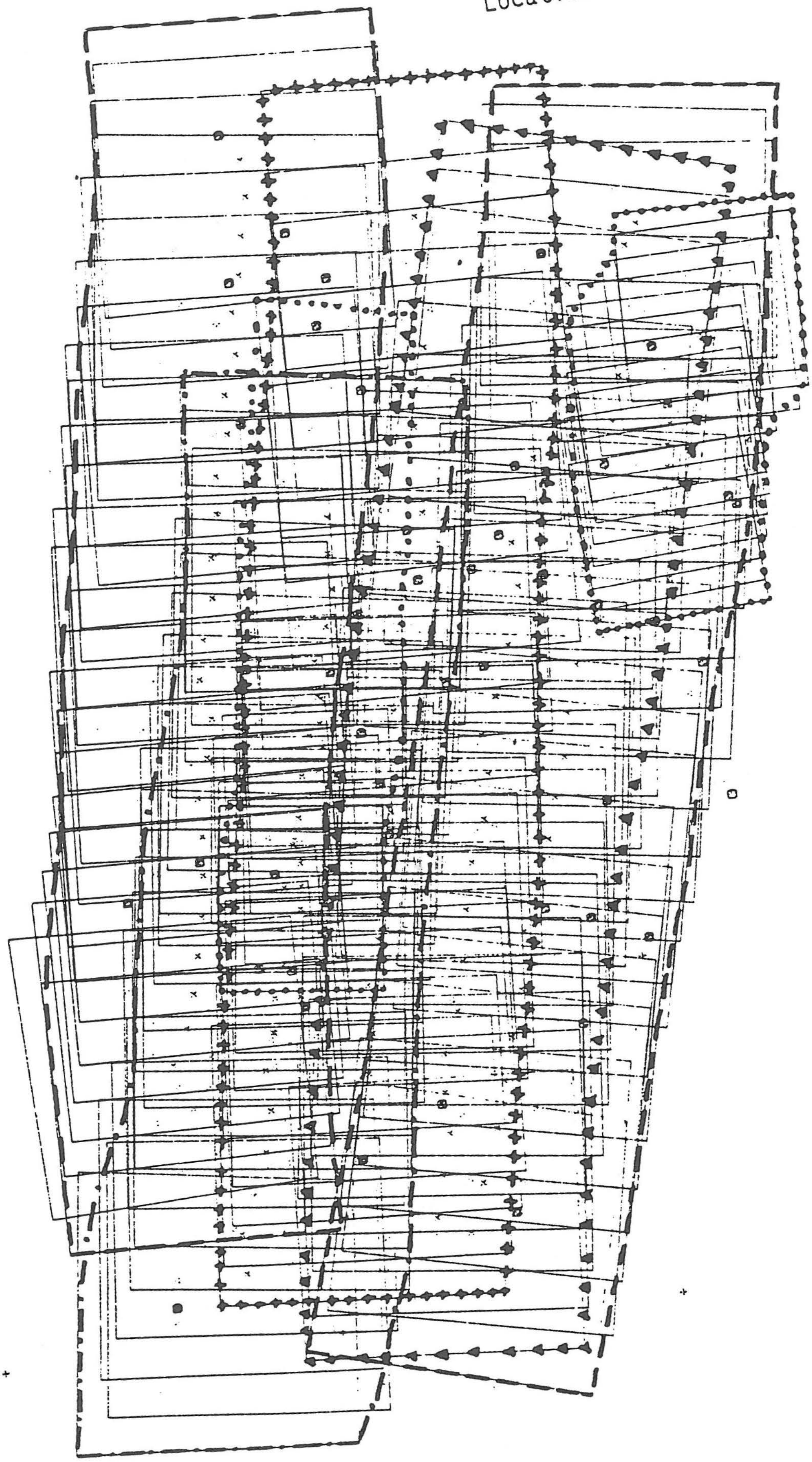
The measurement expenditure of the MK 70 photographs is, for similar photoscale and similar ground coverage, obviously higher than that of the large format photographs, since very many points are to be computed in the area. The Hasselblad MK 70 may be prefered, economically, to a large format camera, when an aerial triangulation is required for very small areas with high point density.

References

1. K. Jacobsen: Attempt at Obtaining the Best Possible Accuracy in Bundle Block Adjustment, ISP Hamburg 1980, Com III WG 3
2. K. Jacobsen: Experiences with Bundle Block Adjustments on a Mini-Computer, ISP Hamburg 1980, Com. III
3. J. Müller: Blocktriangulation mit Verbesserung der inneren Orientierung, Bildmessung und Luftbildwesen 1971 S. 107

Appendix 1

Location of photographs



x, y photo coordinates [mm] $\quad r = \sqrt{x^2 + y^2}$ [mm]

1.) $x' = x - x \cdot \cos(2 \cdot \arctan(\frac{y}{x})) \cdot P\,1$

$y' = y - y \cdot \cos(2 \cdot \arctan(\frac{y}{x})) \cdot P\,1$

2.) $x' = x - x \cdot \sin(2 \cdot \arctan(\frac{y}{x})) \cdot P\,2$

$y' = y - y \cdot \sin(2 \cdot \arctan(\frac{y}{x})) \cdot P\,2$

3.) $x' = x - x \cdot \cos(\arctan(\frac{y}{x})) \cdot P\,3$

$y' = y - y \cdot \cos(\arctan(\frac{y}{x})) \cdot P\,3$

4.) $x' = x - x \cdot \sin(\arctan(\frac{y}{x})) \cdot P\,4$

$y' = y - y \cdot \sin(\arctan(\frac{y}{x})) \cdot P\,4$

5.) $x' = x - y \cdot r \cdot \cos(\arctan(\frac{y}{x})) \cdot P\,5$

$y' = y - x \cdot r \cdot \cos(\arctan(\frac{y}{x})) \cdot P\,5$

6.) $x' = x - y \cdot r \cdot \sin(\arctan(\frac{y}{x})) \cdot P\,6$

$y' = y - x \cdot r \cdot \sin(\arctan(\frac{y}{x})) \cdot P\,6$

7.) $x' = x - x \cdot (r^2 - A) \cdot P\,7$

$y' = y - y \cdot (r^2 - A) \cdot P\,7$

8.) $x' = x - x \cdot (r^2 - B \cdot r - c) \cdot P\,8$

$y' = y - y \cdot (r^2 - B \cdot r - c) \cdot P\,8$

9.) $x' = x - x \cdot (r^4 - D \cdot r^3 - E \cdot r^2 - F \cdot r - G) \cdot P\,9$

$y' = y - y \cdot (r^4 - D \cdot r^3 - E \cdot r^2 - F \cdot r - G) \cdot P\,9$

10.) $x' = x - x \cdot (r^8 - H \cdot r^7 - \ldots - N \cdot r - P) \cdot P\,10$

$y' = y - y \cdot (r^8 - H \cdot r^7 - \ldots - N \cdot r - P) \cdot P\,10$

11.) $x' = x - x \cdot \cos(4 \cdot \arctan\frac{y}{x}) \cdot P\,11$

$y' = y - y \cdot \cos(4 \cdot \arctan\frac{y}{x}) \cdot P\,11$

12.) $x' = x - x \cdot \sin(4 \cdot \arctan\frac{y}{x}) \cdot P\,12$

$y' = y - y \cdot \sin(4 \cdot \arctan\frac{y}{x}) \cdot P\,12$

13.) $x' = x - x \cdot (r^2 - 12100) \cdot \cos(2 \cdot \arctan\frac{y}{x}) \cdot P\,13$

$y' = y - y \cdot (r^2 - 12100) \cdot \cos(2 \cdot \arctan\frac{y}{x}) \cdot P\,13$

14.) $x' = x - x \cdot (r^2 - 12100) \cdot \sin(2 \cdot \arctan\frac{y}{x}) \cdot P\,14$

$y' = y - y \cdot (r^2 - 12100) \cdot \sin(2 \cdot \arctan\frac{y}{x}) \cdot P\,14$

15.) $x' = x - x \cdot (r^2 - 12100) \cdot \cos(4 \cdot \arctan\frac{y}{x}) \cdot P\,15$

$y' = y - y \cdot (r^2 - 12100) \cdot \cos(4 \cdot \arctan\frac{y}{x}) \cdot P\,15$

16.) $x' = x - x \cdot (r^2 - 12100) \cdot \sin(4 \cdot \arctan\frac{y}{x}) \cdot P\,16$

$y' = y - y \cdot (r^2 - 12100) \cdot \sin(4 \cdot \arctan\frac{y}{x}) \cdot P\,16$

17.) $x' = x - x \cdot P\,17$

$y' = y + y \cdot P\,17$

18.) $x' = x - y \cdot P\,18$

$y' = y - x \cdot P\,18$

19.) $x' = x - x \cdot y \cdot P\,19$

$y' = y$

20.) $x' = x$

$y' = y - x \cdot y \cdot P\,20$

21.) $x' = x - x \cdot y^2 \cdot P\,21$

$y' = y$

22.) $x' = x$

$y' = y - x^2 \cdot y \cdot P\,22$

23.) $x' = x' - x \cdot (y^2 - 8100) \cdot y \cdot P\,23$

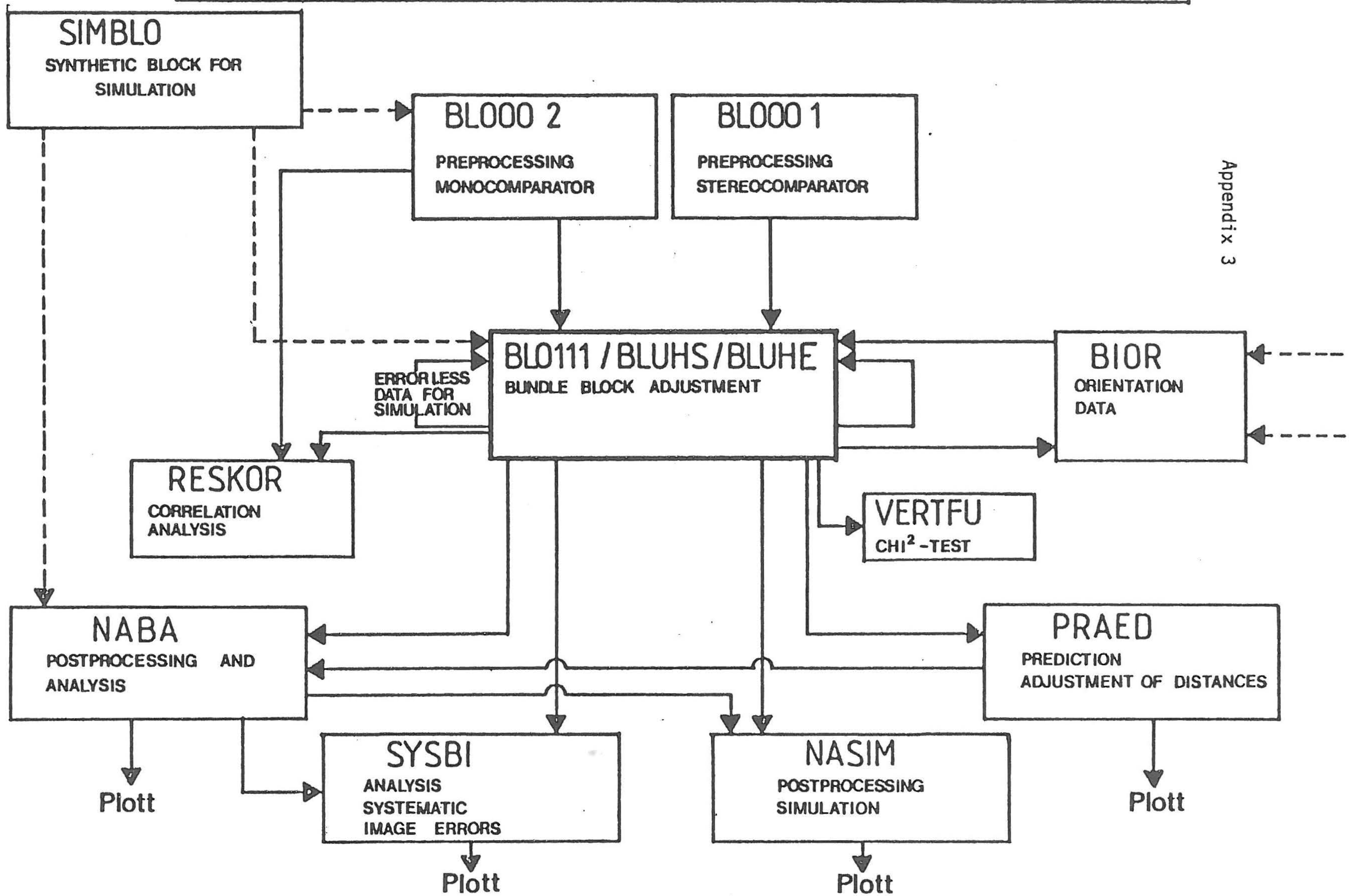
$y' = y$

24.) $x' = x$

$y' = y - y \cdot (x^2 - 8100) \cdot x \cdot P\,24$

Appendix 3

HANNOVER PROGRAMM SYSTEM FOR BUNDLE BLOCK ADJUSTMENT



Appendix 4 Results Test Schwienkuhlen

Explanation:

σ_0 = standard deviation of unit weight [µm]

ptop = weight of photocoordinates of the topgraphic points

SF = factor for multiplication of the corrections by the reseau errors

AP = total number of respected additional parameter

µx, µy = standarddeviation of coordinates of the independent check points [µm in the photo]

$$xy = \sqrt{\frac{\mu x^2 + \mu y^2}{2}}$$

µrx, µry = relative standard deviation of coordinates of the independent check points for distances up to 270 m

$$\mu rx^2 = \frac{\sum_{i-1}^{n} (DX_1 - DX_2)}{2\,n} \qquad DX = \text{check point error}$$

[µm in the photo]

1.1 Bundle block adjustment with 8 horizontal and 46 vertical controlpoints (corresponding a distance of the horizontal control points of 4 base length at the periphery (in the case of endlap = 60 %))

107 photos with c = 61 mm + 39 photos with c = 100 mm

ptop	AP	SF	σ_0	µx	µy	µxy	µrx	µry
0,3	24	0	4,51	9,51	14,86	12,48	5,14	6,40
0,3	24	0,5	4,41	10,31	15,45	13,13	5,19	6,62
0,5	24	0	5,28	8,88	14,82	12,22	5,04	6,32
0,5	24	0,5	5,16	9,59	14,96	12,56	5,08	6,46
1,0	24	0	6,76	9,56	13,44	11,67	5,06	6,16

1.2 like 1.1 but with 107 photos with c = 61 mm

ptop	AP	SF	σ_0	µx	µy	µxy	µrx	µry
0,5	12	0	5,34	9,22	10,50	9,88	7,18	6,62

2.1 Bundle block adjustment with 13 horizontal and 46 vertical controlpoints (corresponding a distance of the horizontal control points of 2,5 base length at the periphery (in the case of endlap = 60 %))

107 photos with c = 61 mm + 39 photos with c = 100 mm

ptop	AP	SF	σ_0	µx	µy	µxy	µrx	µry
0,3	24	0	4,53	9,17	8,41	8,80	4,68	5,48
0,3	24	0,5	4,43	9,88	9,00	9,45	4,96	5,63
0,5	24	0	5,31	9,19	8,66	8,92	4,61	5,23
0,5	22	0,5	5,19	9,79	9,15	9,47	4,87	5,37
1,0	24	0	6,71	9,47	8,83	9,15	4,54	5,31

2.2 like 2.1 but with 107 photos with c = 61 mm

ptop	AP	SF	σ_0	µx	µy	µxy	µrx	µry
0,5	12	0	5,38	11,06	8,08	9,69	6,93	5,85

3.1 Bundle block adjustment with 18 horizontal and 46 vertical controlpoints (corresponding a distance of the horizontal control points of 1,4 base length at the periphery (in the case of endlap = 60 %))

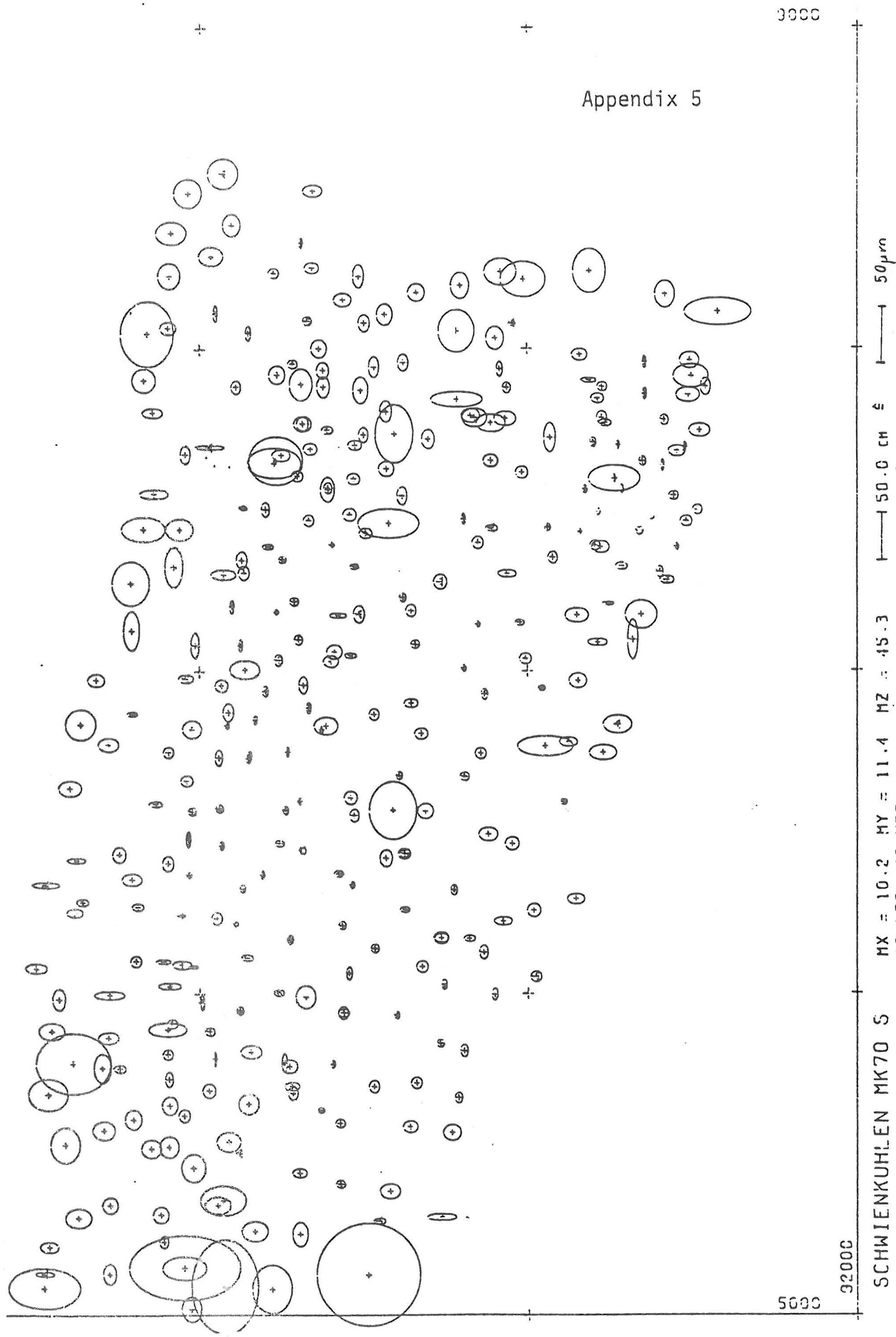
107 photos with c = 61 mm + 39 photos with c = 100 mm

ptop	AP	SF	σ_0	µx	µy	µxy	µrx	µry
0,1	24	0	3,52	5,82	6,68	6,26	3,57	5,39
0,1	24	0,5	3,43	5,64	7,12	6,17	3,30	5,30
0,3	23	0	4,62	5,42	6,26	5,85	3,28	4,72
0,3	23	0,5	4,51	5,48	6,69	6,12	3,14	4,60
0,5	23	0	5,39	5,50	5,95	5,73	3,27	4,39
0,5	23	0,5	5,27	5,63	6,37	6,01	3,15	4,27
1,0	24	0	6,80	5,79	5,31	5,55	3,25	3,95
1,0	24	0,5	6,68	6,22	5,76	6,00	3,23	3,84

3.2 like 3.1 but with 107 photos with c = 61 mm

ptop	AP	SF	σ_0	µx	µy	µxy	µrx	µry
0,3	11	0	4,70	5,42	6,14	5,79	6,15	5,58
0,5	12	0	5,54	5,54	5,88	5,71	6,25	5,58
1,0	12	0	7,03	5,89	5,64	5,77	6,51	5,67

Appendix 5



9000
5000
SCHWIENKUHLEN MK70 S
MX = 10.2 MY = 11.4 MZ = 45.3
18 LPP 46 HPP
50.0 CM
4
50μm