PRESENT STATUS OF ON-LINE ANALYTICAL TRIANGULATION

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ABSTRACT

Basic principles of solutions suitable for analytical aerial triangulation are outlined and analyzed in the context of existing on-line photogrammetric systems. Two basic categories of procedures are considered: those which primarily aim for a quality controlled data acquisition followed by an off-line adjustment, and those which make the adjustment a part of the on-line procedure proper. The discussion covers geometrical as well as numerical considerations and emphasizes practical and methodological aspects which may bring about a higher efficiency of work and a reduction of preparatory operations.

INTRODUCTION

The recent expansion of on-line photogrammetric systems paved way to a variety of new analytical applications which take advantage of the available minicomputer support and control. On-line aerial triangulation is probably the most significant of these current developments which challenge conventional off-line procedures by surpassing their potential and performance. The on-line capability to measure and immediately process data increases the speed and reliability of operations, changes some of the techniques and also has an impact on the overall organization of routinely performed aerotriangulation work.

Essentially, an analytical plotter or a computer assisted comparator ensure the high accuracy and flexibility of an analytical triangulation. The system's operator steadily receives a feedback information about the progress and quality of his work, and interacts with the system by modifying the measurement process as necessary.

This paper will provide a review of the present development of on-line triangulations, discuss their basic concepts, assess possible formulations and analyze their main features. Most of this information is based on the development of programs by either manufacturers or a few advanced user groups, as reflected in available literature or from the activities of the ISP Working Group III-4 on on-line analytical triangulation.

COMPARISON WITH CONVENTIONAL OFF-LINE PROCEDURES

The lack of immediate information on the quality of measurements in off-line analytical triangulations represents a serious drawback of these methods. One cannot avoid the possibility of errors and blunders in

measurements, and especially in identifications, when handling a large volume of data. The first feedback information is received from computations usually performed in a batch mode, only after a larger volume of data is available for processing. Thus, remeasurements and reruns which are usually needed to clean data from gross errors, are costly. In larger blocks, it is also rather difficult to pinpoint the source of error and find the measurement to be rejected, because the adjustment reduces discrepancies and distributestheir effect over a larger area. If remeasurements are necessary then the off-line systems are very impractical in the organization of the limited data additions to individual photographs or stereopairs. For these reasons, an analytical triangulation of large blocks is often treated in two steps. A series of preliminary simplified computations check the inner quality of data, which is then upgraded by rejections and remeasurements before the expensive rigorous phase of simultaneous adjustment is entered. To assist in decision making various statistical methods can be applied under computer control as reported, for example, by Rosenfield (1968), Allam (1975) and Grün (1979). However, this type of analysis is rather complex and expensive, while a simple analysis is not always efficient.

A primary goal of an on-line system is to achieve a high degree of reliability of measured data. The computations, initiated under computer control, or at operator's discretion, have an auxiliary character and do not have to be fully rigorous in using all available observations and conditions in a complete, simultaneous solution, as in off-line procedures. Rather a sequential mode of computations is preferred to be carried out in sections, small enough to allow for an immediate action based on the operator's judgement. This way the quality of data is steadily monitored, with no appreciable delays in the measurement process.

The operator plays a key role in on-line triangulations by maintaining full control of the process. More than in any other analytical application of photogrammetry, he functions as an editor of data by reacting to computer feedback in an interactive editing procedure. In addition to a direct improvement of the mensuration process, there is another benefit of the on-line mode of operations. Since the operator is continuously involved in decision making and sees his own power in refining the results, he becomes psychologically relaxed and more interested in the process which he successfully controls.

GENERAL CONCEPTS OF ON-LINE SOLUTIONS

To take full advantage of a computer available on-line, one should utilize it in two ways: to run computations whenever needed, and to use computer also for a physical control of the mensuration process. In either case, this should be done as soon as a minimum volume of data is available. Until then, the analytical plotter actually performs as a stereocomparator. Nevertheless, even this mode of operation is superior to that used in off-line triangulations, since the computer can provide some additional functions, e.g. introduce corrections for systematic errors, predrive photographs to approximate positions, etc. Once the minimum number of observations is reached, e.g., five points in relative orientation, the corresponding computation can be performed and sequentially repeated with each additional measurement entered. A simple stochastic evaluation of the current solution provides an estimate of the achieved accuracy. Derived parameters are immediately used for computer controlled positioning of photographs, thus yielding a parallax-free optical stereomodel. A good

on-line system should make it possible to introduce refinements to y-parallaxes even at this stage. A significant part of measurements can then be conducted in a controlled model with additional, rather minute changes of y-parallaxes.

There are two basic approaches to on-line triangulations. The system can be used either for a quality controlled data acquisition followed by an off-line adjustment, or the adjustment can be part of the on-line procedures, with final results available immediately after measurements are completed. The former approach, such as described by Kratky (1979), is generally more likely to be used in practice, since an off-line adjustment can be more efficient, particularly if larger blocks are considered. The latter approach is chosen mainly if a medium-size computer for off-line adjustment is not available or if the triangulation results are needed immediately after measurements, especially when densifying individual strips (Hobbie 1978, Klein 1978).

Solutions With Independent Off-Line Adjustment

This procedure has the character of a preliminary run in which the emphasis is placed on collection of photocoordinates thoroughly checked for the presence of gross errors. In model computations and the resulting positioning control of photographs the coordinates are corrected for systematic errors affecting the imagery (film deformation, optical distortion, earth curvature and atmospheric refraction), as well as for optical-mechanical errors of the system as known from its calibration. However, only the latter corrections are applied to photocoordinates stored for later off-line adjustment, leaving the choice of correction formulations with the off-line methods to be used. Also stored in continuously built up computer data banks are corresponding model coordinates, measurements on all fiducial marks and six orientation parameters for each individual photograph. The final adjustment can, of course, proceed either on an independent general purpose computer or on the directly supporting minicomputer.

The size of basic units or sections for intermediate on-line computations is optional. The simplest approach is equivalent to bridging in analog plotters. Individual stereo models are reconstructed by means of relative orientation in the dependent mode of alternately changed photographs, i.e. by solving a system with five unknowns, at a time. An analytical formulation is based on the coplanarity condition applied to conjugate pairs of rays. Absolute orientation is optional in the first model, or later in the so far completed portion of the strip, if a sufficient number of control points is available (Hobbie 1978). However, the value of linear transformation of this type is rather limited, considering the usual level of error propagation in bridging. The quality of individual models is judged from residual y-parallaxes. Subsequent models are scaled with respect to preceding ones with the aid of common tie points. Derived coordinate discrepancies then reflect the combined effect of the two relative orientations involved and of the scale transfer.

This is not a fully satisfactory solution. In general, it is more efficient and also accurate to combine the scale transfer with the current relative orientation in a computation containing all six orientation unknowns for the new photograph. To achieve this, the coplanarity formulation must be supplemented by additional conditions of collinearity (Kratky 1980).

One can also proceed another step further in formulating the process of bridging in a more general way. Both previous formulations are based on separate intersections of two pairs of rays for each tie point located in a common overlap of three photographs. For a triple intersection of rays, corresponding three photographs represent the minimum group within which all internal relations are respected. This gives rise to formulations applying triplets of photographs as units for repeated solutions in a strip triangulation. Measurements from the preceding model are combined with those in the current model and the computation yields 11 parameters for two of the three given photographs. The collinearity conditions should underlie the theory of a triplet formulation.

There is not much reason to include more than three photographs into a single solution step, when on-line triangulation is basically meant to control the quality of measurements followed by an off-line adjustment. The orientation transfer between stereomodels is fully checked by triplets, and on-line corrections can be applied only to measurements in the currently used pair of photographs. Consequently, any extension of the solution beyond triplets does not seem to be effective.

So much for checking of inner relations and of the inner accuracy of a triangulated strip. Outer relations can be checked with respect to given control points and tie points from adjacent strips and, in general, this is a task more difficult to tackle. As mentioned above, a simple linear transformation is not quite powerful and indicative, because of the inherent error propagation. More complex transformations, on the other hand, would represent an effort to perform an on-line adjustment.

Solutions With Concurrent On-line Adjustment

The main problem of on-line adjustments is the associated computer time consumption. To preserve the character of an on-line operation, the computations involved should never exceed the time limits defined and controlled by the set of necessary physical operations and of auxiliary manipulations. Although it is difficult to specify rigid limits, it is reasonable to expect that all individual computations be run in near-real time ranging up to a maximum of about 30 seconds, with more time available only when correction and refinement procedures are finished and the operator uses more time to prepare the next model.

If a bundle approach is adopted, the triplet solution can gradually be expanded into ministrips of increasing length. The computation time will increase drastically and it will not be possible or practical to repeat the full ministrip solution as often as for stereomodels or triplets, with each new redundant measurement. A compromise can be reached by organizing computations in two steps. First, smaller units are formed to check the data entering from the last photograph and only after these inner relations are evaluated, can one proceed with the more complex formation of strip sections. This way, the procedure is continued until the whole strip is covered. Full block adjustment with bundles does not seem to benefit from an on-line operation, and should be done off-line.

There are also numerous possibilities of conducting the on-line triangulation with the use of model coordinates. This type of adjustment, as developed, e.g., by Inghilleri (1978), is easier to expand over blocks of strips, than an adjustment with bundles. One can start from the simplest case of independent models or combine them into larger sections fitted

together within strips or even within blocks. The transformation form should be maintained simple, preferably linear, otherwise the computation time may be excessive.

METHODOLOGY OF ON-LINE SOLUTIONS

Operational Aspects

One of the great assets of on-line triangulation is its capacity to edit the collected data during the process by using rejections, remeasurements and additions (Hobbie 1978, Kratky 1979). However, in using these editing features one should exercise caution and not try to enforce an unrealistic fit by excessive or biased corrections. The process must be reserved only for the elimination of gross errors.

Very useful is the computer capability to speed up operations by automatic slewing of photographs into expected or given positions. The operator then selects the points or redefines their positions usually within the reach of a few millimetres. The recorded coordinates are always refined by applying suitable corrections for all a priori known systematic errors. In turn, when the photographs are positioned to display a given point in the optical system, the sequence and algebraic signs of corrections are reversed.

A statistical improvement of measurements is achieved by the averaging of multiple observations in images rotated by the computer control of Dove prism positions in the optical train, as described by Helmering (1979). The measurements are also facilitated by using an early computer control of parallaxes in the optical stereomodel, which is automatically upgraded from each new observation (Kratky, 1979).

All on-line procedures are software supported by numerous ancillary operations which make operator's task easier, improve the physical control of the instrument and provide useful operational safeguards (Kratky, 1980).

Dynamic Form of Computations

The dynamic character of operations undoubtedly affects the mathematical formulation used in the computer control of the model. New observations are continuously added, and, in the editing phase, some observations are rejected. This means that in the least squares adjustment the normal matrix and the associated free-term vector must be continuously modified. In small systems, e.g. for relative orientation, this fact does not pose any problems. The normal equations computed from contributions of individual points, can be reassembled at any time from the available set of measurements, and the computation, even though iterative, is still completed in a fraction of a second. For larger equation systems, however, the time is more important. One can economize by storing the normal matrix and vector, and by updating them through additions or subtractions of needed contributions from the editing process. The solution of the system, or its matrix inversion, must be repeated afterwards. With the size of solutions increased beyond certain limits, even this approach may not be satisfactory, because a full inversion is still needed.

In this situation, significant time savings are achieved by the implementation of a truly dynamic type of recursive adjustment as

discussed, e.g., by Mikhail (1973), Helmering (1977) and Dorrer (1978). These are methods of dynamic filtering in which new observations are directly connected with the adjusted results of previous measurements. Instead of updating the normal system of equations and solving them independently, one updates directly the known inverse matrix and the current values of unknowns. In this instance, the inversion of an extensive normal matrix is rigorously replaced by a series of simple matrix operations, within which an auxiliary inversion is limited to substantially smaller submatrix derived only from new observations. Given the matrix expression

$$M = N \pm UWV \tag{1}$$

the inverse of M can be derived as shown, e.g., by Bodewig (1959)

$$M^{-1} = N^{-1} \mp N^{-1}U(W^{-1} \pm VN^{-1}U)^{-1}VN^{-1} \quad , \tag{2}$$

if the auxiliary inverses exist. Thus, the volume of needed computations is greatly reduced and the solution can be found in a short time even if a large number of unknowns is involved. This is crucial for all triangulation methods in which the sequential formulation exceeds the size of a single model.

Mikhail (1973) applied Equations (1) and (2) to the least squares adjustment based on condition equations with unknown parameters g

$$Av + Bg = f \quad , \tag{3}$$

which is the most commonly used formulation in analytical photogrammetry. The resulting system of normal equations is then

$$Ng = F$$

with the substitutions

$$N = B^T P_f B \quad , \quad F = B^T P_f f \quad \text{and} \quad P_f = (AQA^T)^{-1} \quad , \tag{4}$$

where Q is the variance-covariance matrix associated with original observations. Using horizontal partitioning to distinguish between two independent groups of already used and newly obtained observations

$$B = \begin{bmatrix} \dot{B} \\ \ddot{B} \end{bmatrix} \quad , \quad f = \begin{pmatrix} \dot{f} \\ \ddot{f} \end{pmatrix} \quad , \quad P_f = \begin{bmatrix} \dot{P}_f & 0 \\ 0 & \ddot{P}_f \end{bmatrix}$$

one defines

$$\dot{N} = \dot{B}^T \dot{P}_f \dot{B} \quad , \quad N = \dot{N} \pm \ddot{B}^T \ddot{P}_f \ddot{B} \quad , \quad F = \dot{B}^T \dot{P}_f \dot{f} \pm \ddot{B}^T \ddot{P}_f \ddot{f}$$

and can apply Equations (1) and (2) to get

$$\boxed{N^{-1} = \dot{N}^{-1} \mp \dot{N}^{-1}\ddot{B}^T (\ddot{P}_f^{-1} \pm \ddot{B}\dot{N}^{-1}\ddot{B}^T)^{-1}\ddot{B}\dot{N}^{-1}} \quad . \tag{5}$$

The expanded constant vector F is used to compute the new updated vector of parameters by

$$g = N^{-1}F \quad . \tag{6}$$

Helmering (1977) applied this type of sequential algorithm in the general photogrammetric solution together with vertical partitioning used to eliminate the unknown ground coordinates. Dorrer (1978) considers a purely conditioned case and relates the sequential algorithm of the least squares solution with Kalman filtering.

Even higher efficiency is achieved by utilizing another formulation known from the block partitioning of matrix inverses

$$(N \pm UWV)^{-1}UW = N^{-1}U(W^{-1} \pm VN^{-1}U)^{-1} \quad . \tag{7}$$

In the combination of Equations (2) and (7) the update of unknown parameters $\dot{g}$ is then derived directly, as shown by Nasretdinov (1972)

$$\boxed{g = \dot{g} \pm \dot{N}^{-1}\ddot{B}^T(\ddot{P}_f^{-1} \pm \ddot{B}\dot{N}^{-1}\ddot{B}^T)^{-1}(\ddot{f} - \ddot{B}\dot{g})} \quad . \tag{8}$$

In this formulation most of the operations involved are common to Equation (5) for the refinement of the inverse and the computation of unknowns from Equation (8) is thus more economical than from Equation (6).

Error Detection

The editing process which is usually based on operator's experience and good judgement, should be supported by a more objective statistical evaluation of results. Any mathematical model which is subject to the least squares adjustment can automatically provide some basic assessment of the achieved fit and accuracy. In most instances, a simple computation of the standard error of unit weight yields a satisfactory measure of the overall quality of the reconstructed model and of the degree of its improvement during the editing process. However, the variance-covariance matrix characterizing the quality and correlation of derived orientation elements is not too useful in on-line checking.

An efficient procedure to detect gross errors is based on statistical concepts developed by Baarda (1967) in a reliability theory, applied to the problem of on-line bridging by Förstner (1978, 1979) and to the bundle block adjustment by Grün (1979). These "data snooping" techniques show how the geometric configuration of measurements affects the possibility to detect gross errors at suitable confidence levels. Since it is also possible to estimate the lowest possible magnitude of practically just detectable gross errors, the procedures can be computer controlled and applied automatically.

ORGANIZATIONAL ASPECTS

Some of the operational features of an on-line triangulation are not only convenient per se, in upgrading the conditions and quality of work, but they have an important impact on the overall organization and preparation of triangulation projects.

Pass Points

Probably the most promising feature of on-line triangulation, which may bring about a major change to the traditional approach, lies in the fact that the concept of pass points invariably supporting each

photogrammetric model to be cartographically compiled, could be abandoned
if this compilation is expected to follow in an analytical instrument.
This could again be an analytical plotter, Gestalt Photo Mapper, or any
similar analytical system. Pass points are primarily important for analog
photogrammetric plotters, as they provide the means to derive the exterior
orientation of both photographs involved. Orientation parameters derived
from triangulations cannot be used directly to preset the orientation
elements in analog systems, because of their limited mechanical accuracy.
However, no such limitation exists in purely analytical systems, in which
the numerical accuracy can be controlled and maintained at any required
level. The orientation parameters are related to a set of fiducial marks
whose definition is superior to that of natural details or targets repre-
senting pass points. An accurate model can then be repeatedly re-
constructed in an analytical system from known orientation parameters,
following new measurements on fiducial marks, which establish the appro-
priate inner orientation of both photographs. As shown by Kratky (1972),
inevitable random errors of measurements on fiducials are suppressed by a
proper transformation, so that their statistically expected RMS effect
within the frame is less than the standard unit error of pointing on
fiducials. Consequently, one can rely on an accurate restoration of a
stereomodel any time later from known orientation parameters.

If on-line triangulation is performed in preparation for plotting in
analog instruments, the function of pass points cannot be eliminated.
However, even in this instance, their determination can be simplified and
carried out in a more economical way than by using traditional means.
First of all, pass points can be selected, identified, sketched and marked
in paper prints during the on-line triangulation. In this instance, they
may, but do not have to be common to adjacent models, so that independent
groups of pass points will serve later to reconstruct individual models in
an analog plotter. This arrangement speeds up the selection of pass points,
since one can judge the quality of a pass point in the current model and is
not concerned at all with the definition of corresponding points in other
photographs. The number of pass points, however, is thus increased. It
is also possible to take advantage of tie points and use some of them
directly as pass points, if they are well defined in planimetry. Even if
this is not the case, any tie point can later be used as an additional
vertical check point in the analog plotting. Their adjusted ground co-
ordinates are eventually known, and, therefore, the tie points can be
included in the plotting manuscript together with pass points without any
need to have them sketched or marked. When positioned on the plotting
table, one can read out their corresponding model heights and use them in
the orientation process.

Yet another possibility lends itself to determine pass points in a
very efficient way, when réseau photography is used. Some of the réseau
crosses in a single photograph could serve as pass points, as long as they
are suitable for measurements of x- and y-parallaxes in the stereomodel.
This idea could be generalized into the concept of triangulation, in which
all tie and pass points are exclusively defined by suitable réseau crosses.
One can also envisage the production of diapositives printed together with
a simplified pattern of auxiliary marks usable in the combined technique
of on-line analytical triangulation and analog plotting.

Tie Points

In any type of triangulation, tie points have a great importance in

ensuring a good fit and continuity between consecutive stereomodels. In on-line triangulations the tie points within the strip can be selected during the operations, without any preliminary identification, marking and numbering. The tie points in consecutive models are then identified by their photo coordinates in the common photograph. Tie points do not even have to be well-defined in planimetry, as long as the parallaxes can be measured accurately. In order to measure a tie point in the current model the computer positions the appropriate photo carriage using the coordinates recorded from the previous model, and holds it there by blocking any manual control action to the contrary until the recording is completed. As long as the number and configuration of required tie points between models is standard, the computer also takes care of automatically assigning them unique identification numbers.

The pre-triangulation identification and marking of lateral tie points, may also be unnecessary. A pair of photographs can be tied together by common points which are not necessarily measured in other photographs. Following the idea suggested by Albertz (1972), one can form stereopairs for this purpose from pictures belonging to adjacent strips, with a rotated stereobase. This approach results in an introduction of additional cross-pairs to be measured within the limited side overlap of strips. The number of measured points per crosspair, and their configuration are not adequate to support any immediate computations and this measuring operation is carried out in a stereocomparator mode. Since the cross measurements represent additional burden and are isolated from the otherwise continuous on-line process, one should carefully compare the time, quality and cost figures of this procedure with those of conventional preparatory operations involving identification and marking of tie points between strips.

Preparatory Work

By eliminating the need for pass points and by choosing tie points during the measurement process, one avoids problems otherwise associated with their identifications and drastically reduces the time necessary for preparatory triangulation work. Tedious and time-consuming selection of pass points in picture overlaps, which may involve comparison of up to six adjacent photographs at a time, becomes unnecessary. Also redundant is the associated point transfer, point marking by using special devices, numbering and other coding. These are all operations which represent significant sources of errors and blunders. The only points to be identified in advance are the control points that are eventually used for the triangulation adjustment and a limited number of check points.

CONCLUSIONS

On-line analytical methods provide a fast and reliable data collection in aerial triangulation. Regardless of the type of the ultimate block adjustment which can proceed on-line or follow in a completely separated, independent procedure, the organization of analytical triangulation can be significantly simplified in comparison with conventional off-line versions. The volume of preparatory work is greatly reduced and the occurrence of gross errors in observed data substantially restricted. With the current expansion of analytical plotters and computer assisted comparators the on-line triangulation which already proved to be viable, is likely to gain more grounds and eventually dominate the field of photogrammetric control extensions.

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