INTERPOLATION OF RASTER HEIGHTS FROM DIGITIZED CONTOUR LINES

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ABSTRACT

Digital height models (DHM's) from contour lines have particular signifi-cance due to the wide availability of contour sheets for topographic maps. The automatic conversion of contour lines to a rectangular raster of height values presents specific problems since artifical morphological structures are generated in the process of interpolation. Using 15 test terrain forms, we present and evaluate an algorithm designed to minimize this problem and we find overall interpolation results to compare well with manual inter-polation, with differences amounting to 5 - 10 % of the contourline interval.

ZUSAMMENFASSUNG

Digitale Höhenmodelle (DHM) aus Schichtenlinen haben wegen der weiten Verfügbarkeit von Schichtenlinienfolien zu topographischen Karten besondere Bedeutung. Die automatische Umwandlung der Schichtlinienpolygone in ein rasterförmiges DHM zeitigt gewisse Probleme wegen der Erzeugung der künst-lichen Geländeformen im interpolierten DHM. Unter Benutzung von 15 ausge-wählten Geländeformen wird hier ein besonders geeigneter Algorithmus zur gestellten Interpolationsaufgabe erläutert und beurteilt. Im Vergleich zur händischen Interpolation ergeben sich Unterschiede von 5 - 10 % des Schichten-linienintervalls.

RESUME

Les modèles numériques du terrain derivés à partir de contours de niveau tiennent une importance spéciale à cause des vastes quantités de données qui sont disponibles sous forme de fiches de courbes de niveau. La conversion automatique des courbes de niveau sous forme d'une grille régulière n'est possible qu'avec des problèmes spécifiques de la morphologie du terrain interpolé. En utilisant 15 examples de formes de terrain on va présenter une méthode d'interpolation et on va évaluer sa performance. On trouve que les résultats ne different plusque 5 à 10 % des résultats d'une inter-polation manuelle.

1. INTRODUCTION

Conversion of digitized contour lines to a rectangular digital height model (DHM) has been an important task due to the wide availability of contour sheets that exist in the context of topographic maps. DHM's have a variety of users, e.g. for orthophoto generation and as a part of landinformation systems.

It is generally acknowledged that optimum results are obtained when interpolation in contour line plots is linear along the line of steepest descent (Finsterwalder, 1975). This is typically the way one would interpolate manually. Automated methods cannot easily copy this procedure, for the simple reason that the line of steepest descent would have to be defined. Except for the work of Lauer (1972) and Mordhorst (1976) interpolation methods usually are not specifically designed for contour lines to be used as an input. Instead algorithms are used where no specific type of input must be assumed to be available (see Kraus, 1973; Schut 1976; Ebner 1979). Such methods may be excellent under a variety of conditions, but are distinctly sub-optimal for the problem of interpolating in-between contour-lines.

A typical problem that arises with contour-to-raster conversion is illustrated in Figure 1: the interpolated terrain tends to have artifical terraces which would not occur with manual interpolation. To avoid creation of this artifical morphology, one will have to support the automated methods by addition of terrain structure lines for ridges and drainage. It is obvious that such additions can cost significant extra manual efforts which one would like to minimize.
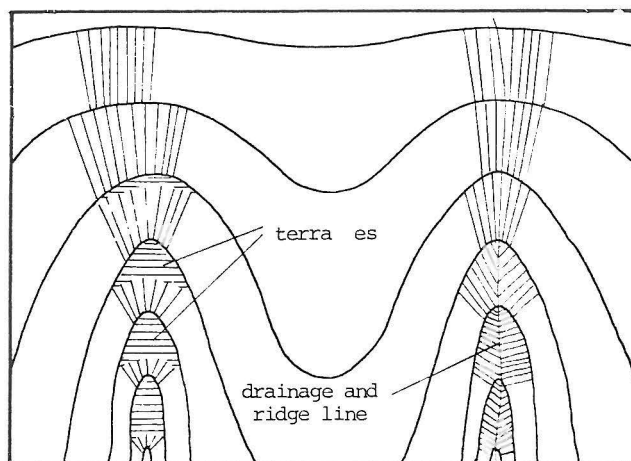

Figure 1: Interpolation in-between contourlines may create artifical terraces (left). To avoid this one needs to add terrain structure-lines. Note that terracing only occurs with particular curvature of contour lines (from DMA, 1978).

We have thus started to investigate algorithms that would reduce the need for manual support of the interpolation. There would essentially be two routes to go: (a) avoid terrain structure lines althogether, or (b) generate these lines automatically. Route (a) could be achieved in the event that one can interpolate along the line of steepest descent.

An important limitation to all considerations are the questions of storage and computing times involved. Methods must be capable of interpolating an array of 1000 x 1000 raster points in a reasonable time and without excessive use of computer memory.

We have studied several algorithms. An interesting one will be des-

cribed in the following chapter. Its evaluation will be with 15 typical
land forms to be presented prior to the discussion of the evaluation results.
The algorithm is fast and modest in computer storage needs. We have achieved
overall results only slightly different from manual interpolation, namely
5 to 10 % of the contour interval. Artifical terracing is not yet entirely
eliminated, but potential exists that this problem can be tackled success-
fully as a result of further efforts.

2. ALGORITHM

2.1. Principle

   Work with map -guided image analysis may require the generation of a so-
called "distance transform" of an image (Tenenbaum et al.,1978). Typically
one would generate a synthetic image of a feature presented originally in
a map-data bank, whereby each image pixel has associated with it not a gray
value, but a shortest distance from the feature (Figure 2). This concept of
the distance transform has led us to conceive an interpolation algorithm
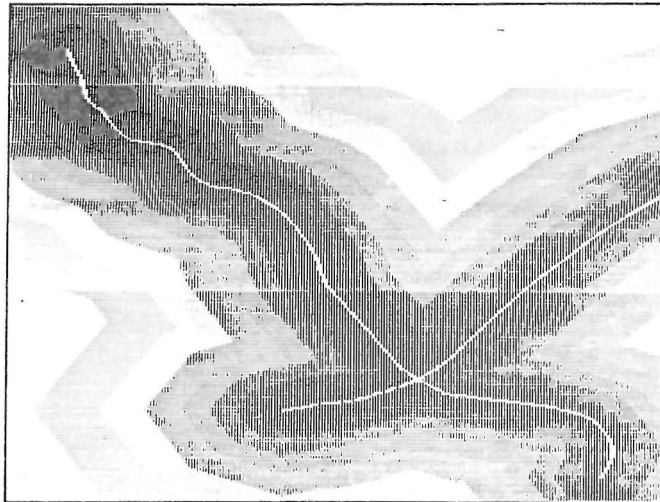
Figure 2: Distance transform of a road, where the road is taken from a map-
          data bank (from Moser 1980).

where in a raster each point has associated with it two distances, namely to
the nearest two contour lines. The two distances themselves are then used
for interpolation of a height value. This method was formulated by one of
us (WK); however, we found later that at least one other autor had also con-
sidered this concept (Mordhorst, 1976).

2.2 Formal Description

   Formal description of the algorithm must assume that the raster is present-
ed as an array A. With A and a set of contour lines in polygon-format one
obtains the following procedure.

(a) The first step is to compute all intersections of contour-lines with raster-lines and to assign a height value to the nearest raster-point in array A.

One now has an image or raster with points along the contour-lines having a height value, and all other points having no height.

(b) We now define:

$HI(i)$ Height of contour-line $i \in [1,n]$ {n Contours}, $HI(i)$ $HI(i+1)$

$D_i(x, y)$ Distance from contour-line i

$L_u(x, y)$ lower contour as seen from point (x, y)

$L_o(x, y)$ upper contour as seen from point (x, y); $HI(L_u(x,y))$ $HI(L_o(x,y))$

(c) The algorithm consists of two steps:

DISTANCE-TRANSFORM (Contour, D, L)

INTERPOLATION (D, L, HI, A)

(d) Procedure INTERPOLATION (D, L, HI, A);

for all $(x, y) \in B$ do   {$B \quad R^2$ (= image plane)}

begin

     i: = $L_u(x, y)$;

     j: = $L_o(x, y)$;

     DI: = $D_i(x, y)$;

     DJ: = $D_j(x, y)$;

     $A(x,y)$:   = $(DJ * HI(i) + DI * HI(j))/(DI + DJ)$

end

(e) Procedure DISTANCE-TRANSFORM (Contour, D, L)

begin

     for all $i \in [1,n]$ and all $(x, y) \in B$ do

     begin

         $D_i(x, y)$: = ∞

         $L_u(x, y)$: = $L_o(x, y)$: = $\emptyset$

     end

     for i: = 1 until n do

     begin   queue $S \subseteq B$; $S := \emptyset$ {S is a queue (array)}

         for all $(x,y) \in$ contour (i) do

         begin

             INSERT (S, (x, y));   {"Insert" means, that point (x,y) is added to S}

             $D_i(x, y)$: = $\emptyset$;

             $L_u(x, y)$: = $L_o(x, y)$: = i;

         end;

         while $S \neq \emptyset$ do

         begin

             (x, y): = first (S);   {eliminates autom. (x,y) from S}

             d: = $D_i(x, y)$;

             for all $(u, v) \in \Gamma(x, y) \cap B$ do   {Γ Neighbourhood}

             begin

                 if d + 1 < $D_i(u, v)$ then

                 begin

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                              D_i(u, v) : = d + 1;
                              if (u,v) ∉ ⋃_{j=1, j≠i}^{n} Contour (j) then INSERT (S, (u,v) );

                              if L_u(u, v) = ∅ then L_u(u, v): = i
                                              else L_o(u, v): = i;

                        end

                  end

              end while;

          end contours;

      end ;
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(f) Write A and present results in graphical form.

2.3   Discussion

The computed distances in each raster point are shortest to the nearest contours. Normally the interpolation (as implemented in procedure INTERPOLATION) will thus be along a (curved) line of steepest descent. However, problems arise when the situation of Figure 3 applies. A certain terracing
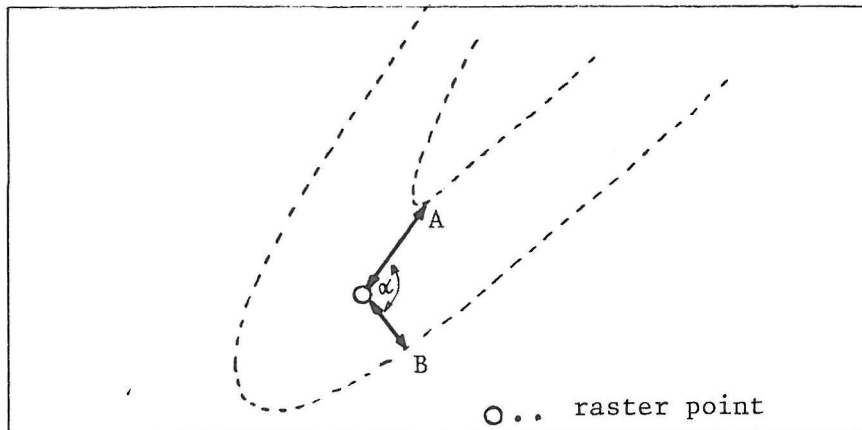

Figure 3: Shortest distance from a raster point to contour lines.

effect will occur since interpolation is in those cases not along the line of steepest descent.

An angle formed between the raster point and points A, B on the contour lines (Figure 3) may itself carry information to be used. Also, the rate of change of distances in neighbouring raster points may be of use. We have not yet investigated these aspects of the technique beyond a preliminary test (compare chapter 4).

3. EVALUATION

3.1. Test Data

Figure 5 lists all 15 given landforms used for the performance analysis of the interpolation algorithms. They are denoted by letters A to O. They were digitized manually and manipulated using our cartographic map-data bank KARTEN (Kropatsch, 1980). The data were selected to encompass typical landforms that may be available on a contour line plot. Contour-line interval is 20 m, characteristic for maps 1:25 000 or 1:50 000.

3.2. Overall Results

Table 1 presents the root mean square height -differences encountered in

Terrain	Distance -Alg. versus manual Interpol.
A	1,2
B	2.0
C	1,6
D	3,6
E	2,0
F	2,0
G	2,0
H	2,0
I	1,6
J	0,8
K	1,2
L	2,0
M	1,6
N	1,2
O	2,4

Table 1: Overall contours, accuracy in meters

25 test points between the manually interpolated heights, the heights from the distance- and search-direction-methods. As expected the differences are very small and amount to 5 % to 10 % of the contour-line interval.

No "true" heights are available. However, height accuracy standards acc. to Koppe (1905) specify that for a contour-line interval of 20 m one has standard height errors m_H as follows:

$$1: 25\ 000 \qquad m_H = \pm\ (6.08 + 7.7\ \tan\alpha)$$

$$1: 50\ 000 \qquad m_H = \pm\ (6.08 + 15.4\ \tan\alpha)$$

where α is terrain slope

We see thus that the overall differences between both automated and manual interpolation are negligibly small.
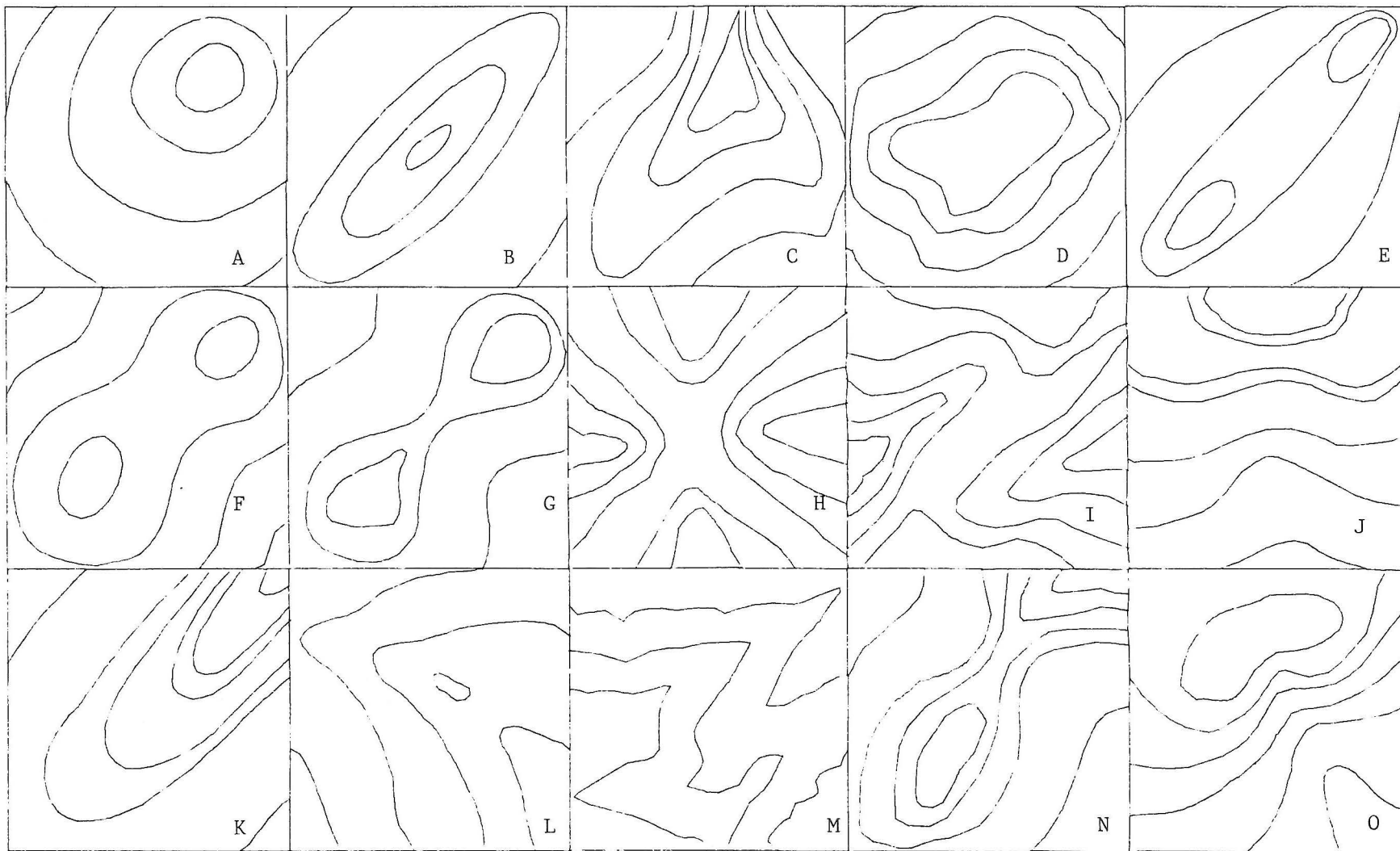
Figure 5: Test data named A through O, as selected from literature.

3.3 Morphology

A performance test requires the evaluation of the raster heights using re-interpolated given *and* intermediate contour lines. The given contour-lines are not very informative, since they are clearly reproduced. It are the intermediate contours which illustrate terracing problems.

Re-interpolation of contours for the purpose of quality control is found effective *without* smoothing of the contour-lines. Figure 6 illustrates
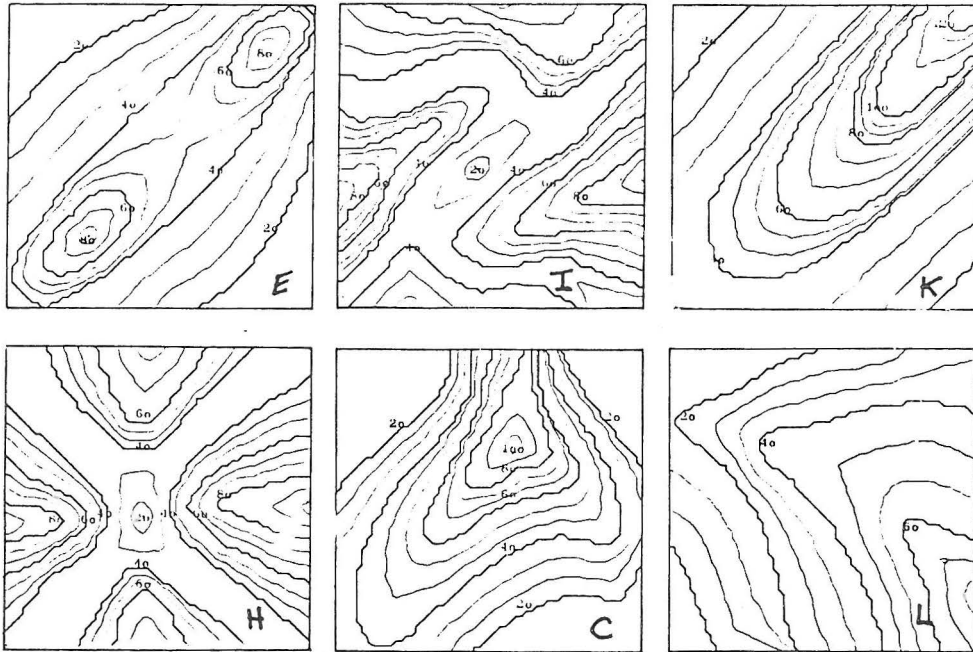

Figure 6: Results of some test areas as obtained from distance algorithm.

interpolation results where no such smoothing is evident. The plots were made automatically.
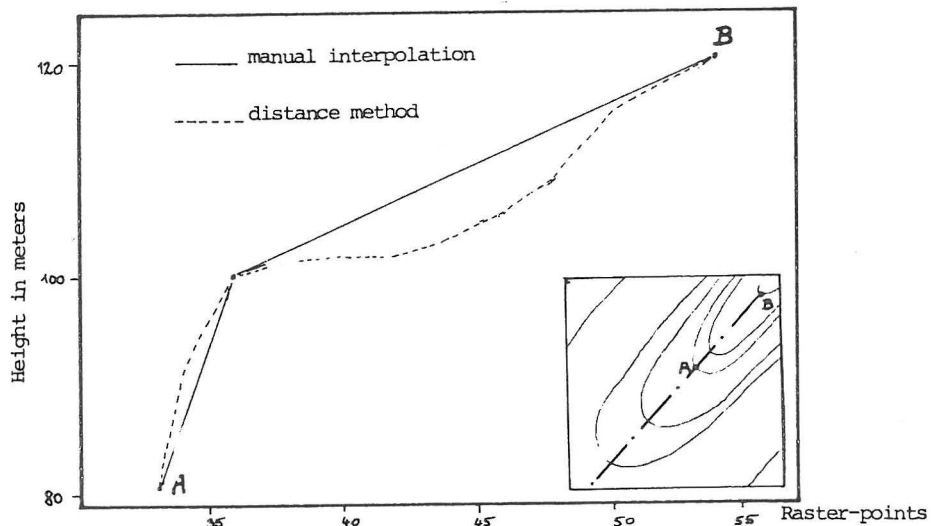

Figure 7: Profile plot along the ridge of case B (Figure 5).

Only contour lines were available as an input, with the exception of a few spot heights on mountain tops. From the analysis of the contour plots one finds that terracing is not totally avoided. This becomes particularly evident when profiles are plotted along a symmetric ridge as illustrated in Figure 7.

4. CONCLUSIONS

The evaluation of a concept for height interpolation from contour lines has been performed with experimental computer programs and the purpose to highlight strenghts and weaknesses of the basic concept. It has become obvious that interpolation from contour lines calls for specific rather than general purpose interpolation schemes.

We have thus considered an algorithm using shortest distances from a point to be interpolated to the given contour lines. This performed satisfactory from the point of view of overall accuracy: deviations from manual interpolation were only of the order of magnitude of 5 to 10% of the contour interval. The morphology of the interpolated terrain is also rather satisfactory: no totally flat areas exist in between contour lines and all raster points are assigned heights that are correctly valued in between contour lines. Some slight terracing persists.

The algorithm needs refinement to further reduce the terracing effect. For this purpose terrain structure lines such as drainage and ridge lines could support the interpolation. These lines would have to be generated by hand and added to the contour line plots. Further refinement also is needed to address contour plots with interrupted lines and spot heights.
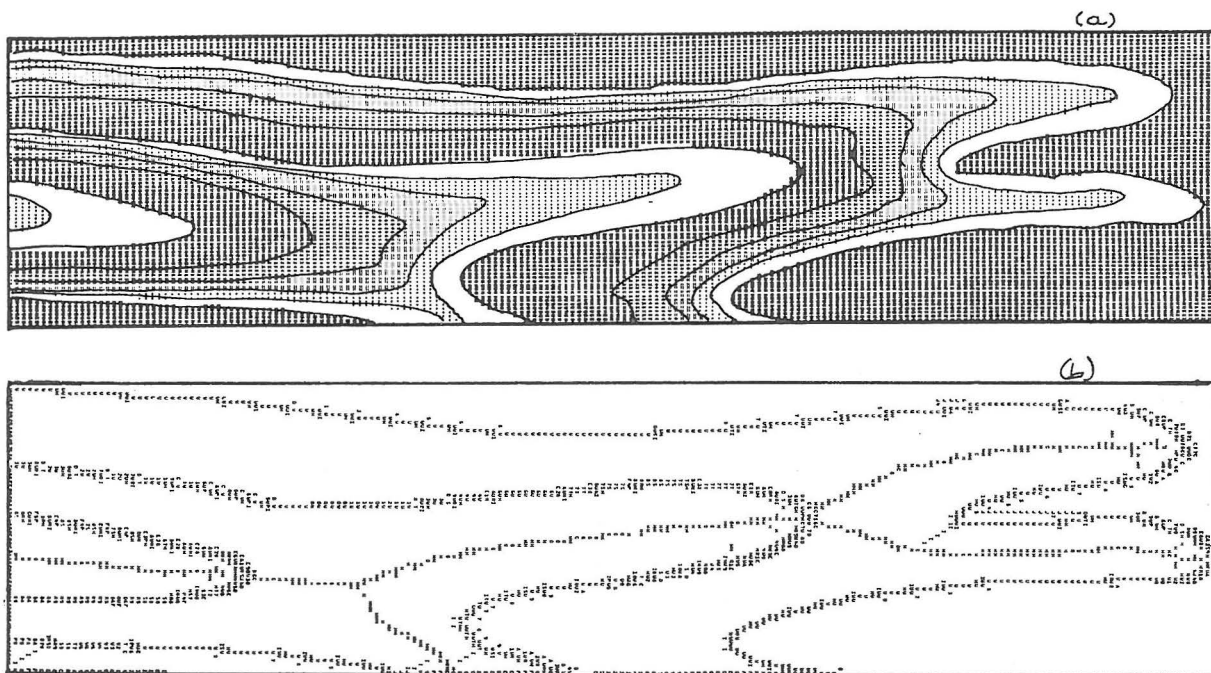

Figure 8: Example of computing a Laplacian transformation and treshold to define terrain structure lines, using distance images. Above si a grey scale peintout of a contour plot (a), and below (b) is the result of the operation.

With the distance algorithm there exists information against terracing in the form of angle α of Figure 3. Also, the rate of change of distances may be useful. One may think of a Laplacian transformation of the distance images, or of computation of a medial axis for the area in-between contour lines, to obtain data on structure lines. Gogineni et al. (1979) have proposed an analog approach to create structure lines using a type of medial axis. We have experimented with this idea and are encouraged from early results as illustrated by Figure 8 . Masson D'Autumne (1979) has also addressed the problem of automatically identifying terrain structure lines. However, that approach is essentially based on an already available raster of heights.

As is true with many interpolation problems one is faced here with a task that requires intuition and experimentation for development of a satisfactory algorithm. In any event the importance of the task seems to justify further efforts.

ACKNOWLEDGEMENT

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