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AN EXTENDED BLUNDER ELIMINATION PROCEDURE

ABSTRACT

Further elaboration on applying and interpreting the Q_{vv} matrix, on statistical distribution of different error indicators, and on understanding the scaled residuals. Introducing an extended blunder elimination procedure taking into account the functional dependence of the most suspicious observations and of the corresponding residuals. The procedure is applied in the orthophoto software SORA-OPS (absolute orientation, spatial resection), in the DTM software SCOP (absolute orientation), in the software for analytical aerial triangulation MODEL (relative orientation, model connection), and in the universal subroutine for absolute orientation MODOR.

1. PRELIMINARY REMARKS

Details necessary for the practice of blunder processing are given in section 2: formulae, a diagramm, and some explanations. The theory underlying this procedure is summarized then in section 3.

Blunder processing includes blunder location and blunder elimination. In the case of linear observation equations (not just linearized ones), it is possible to locate and to eliminate more than one erroneous observation, and then correcting the results of the adjustment /7/ or, instead of this, repeating it (section 2.1). Non-linear observation equations, on the other hand, result in an iterative adjustment, and the elimination of an erroneous observation necessitates further iterations changing the unknowns, and the vector of residuals (section 2.2). Correspondingly, procedures are different for these two cases (table 1).

Notations

A - coefficient matrix of observations (adjustment with observations)
B - coefficient matrix of unknowns (adjustment of indirect observations)
b - number of the largest errors (of the suspicious observations) checked; in this, a submatrix of Q_{vv} of size bxb is inverted. Therefore:
$b_a \leq b \leq r$
b_a - number of blunders allowed
b_o - see table 1
ε_o - vector of (true) errors
k - factor to an estimated variance to determine bounds in form of
$Tol=k\cdot\hat{\sigma}$ (e.g., $k=\sqrt{F_{1-\alpha_o,1,\infty}}$)

l = vector of observations
$\hat{l} = Bx = l + v^+$
m - the minimal number of statistically consistent observations: $m=u+b_a+1$ (for just detecting the presence of b_a blunders $m=u+b_a$)
n - number of observations
N - matrix of normal equations
$P = Q_{vv}W$
Q - co-factor matrix
Q_{vv} - the co-factor matrix of residuals (the "geometric weight coefficient matrix", /6/); for an adjustment of indirect observations:

$$Q_{vv} = W^{-1} - B\,N^{-1}B^T$$

and for an adjustment with conditions only:

$$Q_{vv} = W^{-1}A^TN^{-1}A\,W^{-1}$$

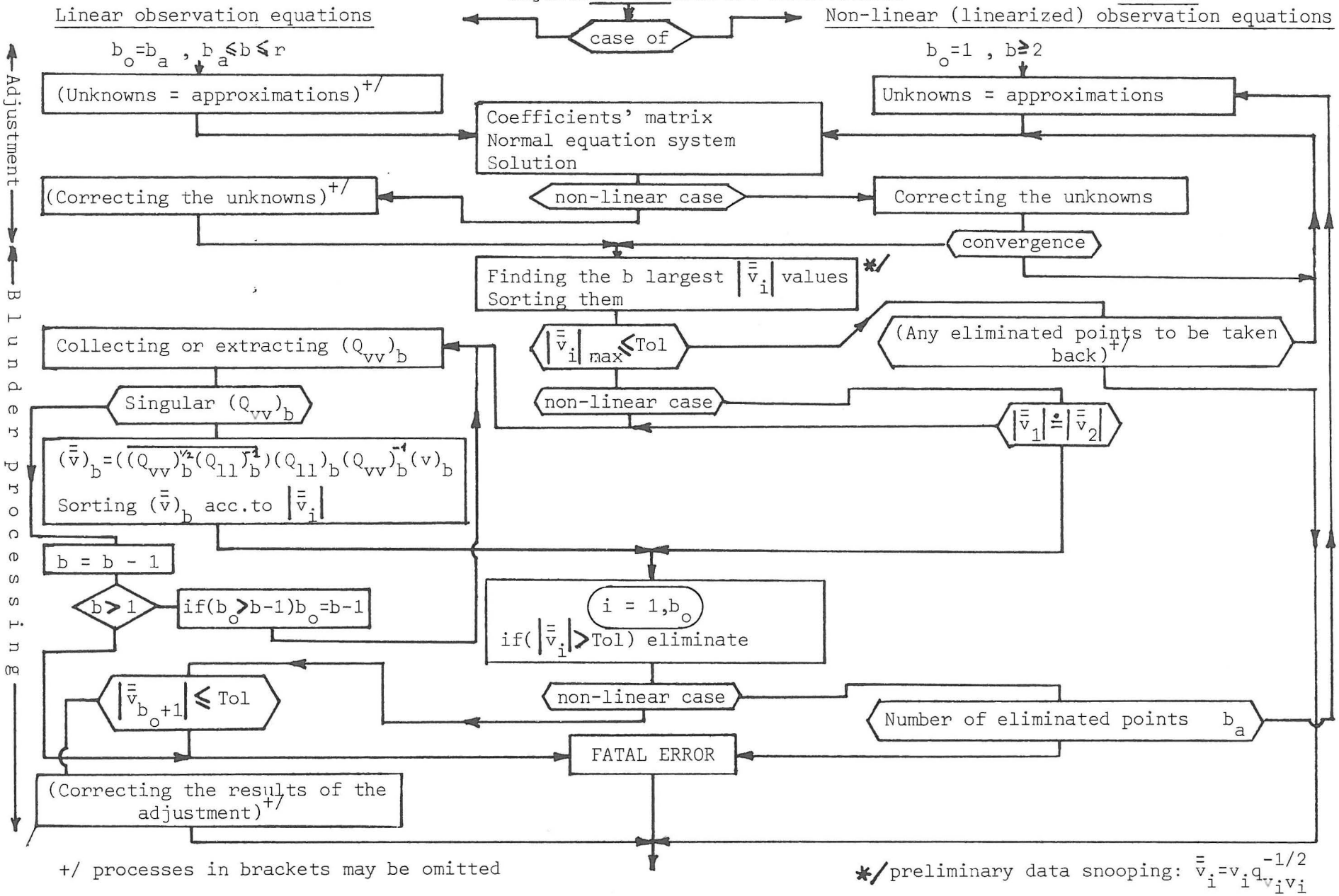
Q_{xx} - co-factor matrix of unknowns: $Q_{xx} = N^{-1}$
q - element of a co-factor matrix
r - redundancy: $r = n - u$
σ^2 - some variance
σ_f^2 - the variance of some value counted with the function as determined by the adjustment
σ_o^2 - the reference variance, corresponding to the weight unity; the variance of a scaled residual
$\tilde{\sigma}^2$ - a-priori estimate of σ_o^2
$\sigma_{v_i}^2$ - variance describing residual reliability: $\sigma_{v_i}^2 = \sigma_o^2 q_{v_iv_i}$
Σ - some variance-covariance matrix: $\Sigma = \sigma_o^2 Q$
Tol - tolerance
u - number of unknowns
v - vector of residuals
v^+ - identical with the classical resuduals $v = \hat{l} - l$. The superscript emphasizes that the corresponding observations have been involved in the adjustment
v^- - corresponds to the same discrepancy $\hat{l} - l$ for points which did not participate in the adjustment - or as if they did not participate in it;
$\bar{\bar{v}}$ - statistically defined quantities with a meaning that is not as concrete as that of v^+ or of v^-. $\bar{\bar{v}}$ contains the "scaled residuals" (as we prefer to use this term), the controllable part of v^-, the "contribution to the (reduced) sum of squared residuals" /7/, the residuals with the variance σ_o all over the network.
W - weight matrix of observations ($W = Q_{ll}^{-1}$)
x - vector of unknowns

Signs:

$\hat{}$ - expectations, estimates
$(\ )_a$ - submatrix of size a x a (or sub-vector of size a)
+ as superscript - matrix corresponding to an adjustment with the point(s) involved
- as superscript - matrix corresponding to an adjustment with the point(s) eliminated
$\bar{}$ (bar) - matrix taken for diagonal

Adjustment of indirect observations

Table 1



+/ processes in brackets may be omitted

*/preliminary data snooping: $\bar{\bar{v}}_i = v_i q_{v_i v_i}^{-1/2}$

2. THE PROCESS

2.1 Linear (not just linearized) model

In this case, the location and elimination of blunders is a straightforward process. It can be traced in table 1 (its left side and middle columns). The heart of the process is the expression

$$(\bar{\bar{v}})_b = -\overline{((Q_{vv})_b^{1/2}(Q_{ll})_b^{-1})}\ (Q_{ll})_b (Q_{vv})_b^{-1}(v)_b \qquad (2.1)=(3.6)$$

The submatrix of Q_{vv} corresponds to the b worst observations as defined by a preliminary "data snooping" (see table 1). To gain $(Qvv)_b$ it is not necessary to store Q_{vv}. And it is possible to count elements of Q_{vv} with elements of a triangular decomposition of N (e.g. of Cholesky) /4/. Evaluating (2.1) requires the solution of a symmetrical system of b linear equations, and yields sufficiently accurate estimates of the corresponding b "scaled residuals" $\bar{\bar{v}}$ (if only the "necessary assumptions", as described in /7/, are fulfilled. For practical purposes: $m = u + b_a$ statistically consistent observations must be present and have to yield a well conditioned normal equation system). Elements of $(\bar{\bar{v}})_b$ can then be sorted according to their absolute values, and the first b_a of them individually compared with a suitable tolerance Tol. $\bar{\bar{v}}$ are "scaled residuals": residuals so scaled (within (2.1)) to have the same standard deviation σ_o all over the network. Denoting by $\hat{\sigma}$ the a-priori estimate of σ_o, the tolerance for $\bar{\bar{v}}_i$ can be written as $Tol = k\hat{\sigma}$, where k is generally taken for 2 or 3 (see (3.7)). The corresponding confidence intervals can be read from table 2 or fig. 1. - The condition $|\bar{\bar{v}}_{b_a+1}| > Tol$ indicates the presence of more than b_a blunders.

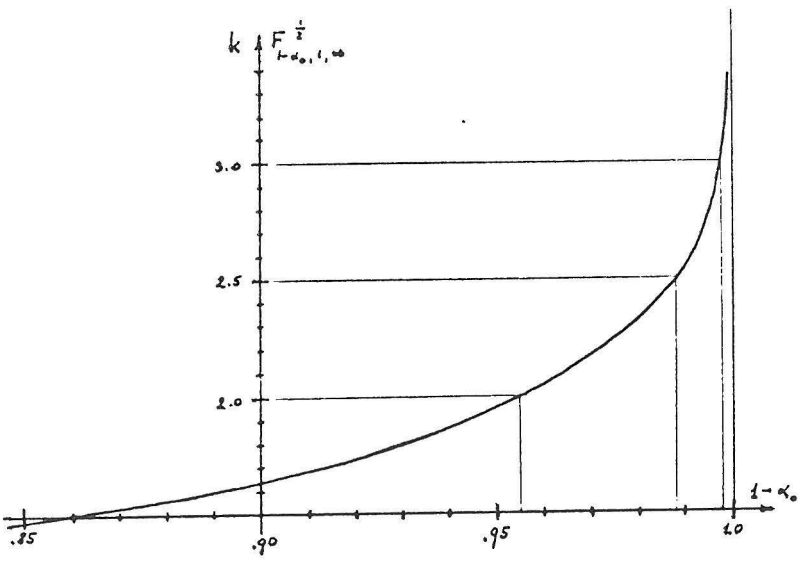


Fig. 1

Table 2

$k=F^{1/2}_{1-\alpha_o,1,\infty}$	$1-\alpha_o$
1.5	.86
2.0	.955
2.5	.98
3.0	.998

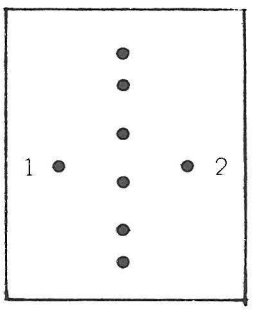


Fig. 2
Distribution of observed heights. Residuals on points 1 and 2 after a least squares plane fitting will be equal to each other. Correspondingly, $|(Q_{vv})_{1,2}| = 0$

Important details of the process are concerned with singularities in $(Q_{vv})_b$. Such singularities mirror the unacceptable geometric configuration of the b observations involved. Fig. 2 illustrates this point, and the diagram in table 1 describes the handling of such cases.

2.2 Non-linear (linearized) model

In this case points have to be eliminated one-by-one, each time jumping back into the process of iterative determination of unknowns ("error and trial" method). Generally, submatrices of Q_{vv} with the size b=1 suffice for this purpose, corresponding to the elimination of the observation with $|\bar{v}_i| = \max$. However, if two or more $|\bar{v}_i|$-values are equal among themselves (see Appendix II), the corresponding submatrix of Q_{vv} has to be checked on singularity:

a. as indicated earlier, the singularity of $(Q_{vv})_b$ corresponds to a geometrically unacceptable configuration of the participating observations. Only the presence of one (or more) blunder(s) can be detected in such cases. Eliminating all these suspicious equations leads to a singular normal equation system; correspondingly, the case is "fatal".

b. if $(Q_{vv})_b$ is non-singular, one can apply expression (2.1) to decide which one of the observations should be eliminated. There always remains the case of two or more actually equal errors; the sequential elimination of them leads to right conclusions with high probability (notwithstanding an eventually wrong order of such eliminations).

2.3 Counting reliability characteristics of residuals σ_{v^-}

σ_{v^-} is identical with σ_f counted for an imaginary adjustment with the concerned point eliminated. It can be (we would like to say: it has to be) determined for all residuals as

$$\sigma_{v_i^-} = \sigma_o \, q_{l_i l_i} \, q_{v_i v_i}^{-1/2} \qquad (2.2) \text{ related to } (3.4)$$

σ_{v^-} shows how far the corresponding measured quantity is controlled by all the other measurements having been involved in the adjustment. In this sence $\sigma_{v_i^-}$ is the variance characterizing the reliability of the i-th residual (possibly in the form $v_i^+ \pm \sigma_{v_i^-}$)

See Appendix III for examples.

2.4 Swapping branches for linear and non-linear models

As seen from the point of view of exact mathematics, processing should proceed as indicated in table 1. However, in some cases of application this may be impractical, and in certain circumstances more flexible alternatives can be employed.

2.4.1 Eliminating blunders one-by-one, as shown in table 1 by the branch for non-linear (linearized) models, can be applied to linear models, as well. This is practicable for small systems (involving relatively small sets of data). The process is similar then to that described in /5,6/ with the important addition of checking the corresponding submatrices of Q_{vv} on singularity. This is the theory underlying the subroutine LSQSUB to automatize the processing of blunders (and of checking the Q_{xx} matrix) in some of the programs developed at the Institute of Photogrammetry of the Technical University of Vienna (in all processes of a universal subroutine for absolute orientation MODOR - applied in the orthophoto package SORA-OPS,

in the DTM package SCOP, and as an autonomous system; - furthermore, in SORA for spatial resection; in the analytical aerial triangulation program MODEL for relative orientation and model connection). For linear models, LSQSUB conducts the entire adjustment of indirect observations on the basis of given matrix B, vector l, weights, and tolerances (for individual residuals, and for individual unknowns). For non-linear (linearized) models, LSQSUB performs just the last iteration, and eliminates (if necessary) just the worst observation (but in case of singularities, two or three observations).

2.4.2 On the other hand, for the sake of economy (and even of feasibility), the way of blunder processing indicated for linear models can be, under certain circumstances, applied to non-linear ones. This can be recommended for large systems provided with sufficiently accurate first approximations of the unknowns, and for eliminating within one step a series of blunders. If observations belonging to remote areas of the network are only insignificantly correlated, submatrices of Q_{vv} can be collected on a regional basis.

An example of such applications may be a bundle adjustment proceeding some differentiated adjustment procedures,when these last ones have been provided themselves with efficient blunder processing (based on the same theory). The adjustment of large blocks of independent models is another area where this technique can be recommended.

And if just "small" blunders have been eliminated, all elements and results of a final adjustment can be computed by the formulae given by Stefanovic /7/.

3. THE RELATED THEORY

Lack of space forces us to assume the knowledge of the general theory of blunder detection. As a preparation to this particular text, the reading of /5,7,6 - in this order/ is best suited. The sources of the theory go back to Baarda /1/.

As indicated in /3/, a major problem with "data snooping" is the neglection of the correlations among residuals. To solve this problem, the relationship between residuals and observational errors /7/ will be used:

$$v = - Q_{vv} Q_{ll}^{-1} \varepsilon \qquad (3.1)$$

Assuming that we have determined the b "most suspicious" observations (e.g. by a preliminary data snooping), and using the fact that blunders are larger in their absolute value than accidental errors are, the following can be written on the basis of (3.1):

$$(\hat{\varepsilon})_b \simeq -(Q_{ll})_b (Q_{vv})_b^{-1} (v)_b \qquad (3.2)$$

$(\hat{\varepsilon})_b$ contains approximate expectations of true errors of b observations ($b \leq r$). It is identical with the vector $(v^-)_b$ for these observations: the "function" will be determined by the rest of the observations, which have to yield, therefore, a consistent system. The co-factor matrix $(Q_{ff}^-)_b$ of $(\hat{\varepsilon})_b$, and the corresponding sub-matrix of Q_{vv}^+ are in reciprocal relation to each other (Appendix I):

$$(Q_{ll})_b^{-1} (Q_{ff}^-)_b = (Q_{ll})_b (Q_{vv}^+)_b^{-1} \qquad (3.3)^{+/}$$

+/ see footnote on next page

(3.2) contains the entire right side of (3.3): when using the right side of (3.3), no additional computational effort is necessary. With it, the variance-covariance matrix of $(\hat{\varepsilon})_b = (v^-)_b$ can be expressed as

$$(\Sigma_{\hat{\varepsilon}\hat{\varepsilon}})_b = (\Sigma_{v^-v^-})_b = \sigma_o^2 \, (Q_{ll})_b^2 (Q_{vv})_b^{-1} \qquad (3.4)$$

Individual $|\hat{\varepsilon}_i|$ values, determined in (3.2), can now be compared with their respective variances $\sigma_{\hat{\varepsilon}_i}$ from (3.4); the Fisher-test has to be applied to the resulting "K-factors":

$$K_i = \frac{|\hat{\varepsilon}_i|}{\sigma_{\hat{\varepsilon}_i}} \gtrless F_{1-\alpha_o,1,\infty}^{1/2} \qquad (3.5)$$

In (3.5), we utilize only diagonal elements of $\Sigma_{\hat{\varepsilon}\hat{\varepsilon}}$. "Scaling" (3.2) with the correspnding co-factor matrix, yields a vector $(\bar{\varepsilon})_b$ corresponding to $(\bar{v})_b$:

$$(\bar{\bar{v}})_b = (\bar{\bar{\varepsilon}})_b \simeq (\bar{Q}_{\hat{\varepsilon}\hat{\varepsilon}})^{-1/2} (\hat{\varepsilon})_b \simeq \overline{((Q_{vv})_b^{1/2} (Q_{ll})_b^{-1})} \; (Q_{ll})_b (Q_{vv})_b^{-1} (v)_b \qquad (3.6)$$

The co-factor matrix of $(\bar{\bar{v}})_b$ is the unit matrix. Therefore, (3.5) can be replaced by

$$|\bar{\bar{v}}_i| \gtrless \sigma_o \cdot F_{1-\alpha_o,1,\infty}^{1/2} = \mathrm{Tol} \qquad (3.7)$$

(3.7) is superior to (3.5) when practical understanding of tolerances, of probabilities (table 2, fig. 1), and of controlled quantities is essential.

The author is indebted to Dr Kraus for his help and critics.

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4. Kager,H: Documentation to the Universal Bundle Adjustment Package ORIENT. Internal manuscript, Institute of Photogrammetry, TU Vienna

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+/ (3.2) and (3.3) give rise to a vulgar but nevertheless useful understanding of the "geometric weights": the further apart the observation, the less the residual v^+, the larger the residual v^-, the stronger the influence of the corresponding observation on the unknowns, - and correspondingly the larger the "geometric weight" $q_{v_i v_i}^{-1/2}$.

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6. Molnar,L: Self Checking Analytical Relative Orientation and Strip Formation. Geowissenschaftliche Mitteilungen, TU Wien, Heft 14, 1978

7. Stefanovic,P: Blunders and Least Squares. The ITC Journal, 1978-1

APPENDIX I

Proving the reciprocal relationship between Q^-_{ff} and Q^+_{vv}

$$(Q^-_{ff})_b = (Q_{ll})_b + (B)_b Q^-_{xx} (B)_b^T \qquad (I.1)$$

$$(Q^+_{vv})_b = (Q_{ll})_b - (B)_b Q^+_{xx} (B)_b^T = (Q_{ll})_b - (Q_{\hat{l}\hat{l}})_b \qquad (I.2)$$

$$Q^+_{xx} = (B^T W B)^{-1} \qquad (I.3)$$

$$Q^-_{xx} \doteq (B^T W B - (B)_b^T (W)_b (B)_b)^{-1} \qquad \text{with } {}_b(B)_u \qquad (I.4)$$

The difference of inverses of (I.3) and (I.4):

$$(Q^+_{xx})^{-1} - (Q^-_{xx})^{-1} = (B)_b^T (W)_b (B)_b$$

Multiplying from the left by $(B)_b Q^-_{xx}$, and from the right by $Q^+_{xx}(B)_b^T$:

$$(B)_b Q^-_{xx} (B)_b^T - (B)_b Q^+_{xx} (B)_b^T = (B)_b Q^-_{xx} (B)_b^T (W)_b (B)_b Q^+_{xx} (B)_b^T$$

Multiplying by $(Q_{ll})_b$, adding $(Q_{ll})_b^2$ to both sides, and taking into account (I.2):

$$(Q_{ll})_b^2 = (Q_{ll})_b^2 + (B)_b Q^-_{xx} (B)_b^T (Q_{ll})_b - (B)_b Q^+_{xx} (B)_b^T (Q_{ll})_b - (B)_b Q^-_{xx} (B)_b^T (W)_b (Q_{\hat{l}\hat{l}})_b (Q_{ll})_b$$

Approximating the underlined matrices by $(Q_{\hat{l}\hat{l}})_b$, and rearranging:

$$(Q_{ll})_b^2 \doteq ((Q_{ll})_b + (B) Q^-_{xx} (B)_b^T)((Q_{ll})_b - (B)_b Q^+_{xx} (B)_b^T)$$

or, with (I.1) and (I.2):

$$(Q_{ll})_b^2 \doteq (Q^-_{ff})_b (Q^+_{vv})_b$$

Two other forms of this last expression:

$$(Q_{ll})_b^{-1} (Q^-_{ff})_b \doteq (Q_{ll})_b (Q^+_{vv})_b^{-1}$$

$$((Q^+_{vv})_b (W)_b)^{-1} \doteq (W)_b (Q^-_{ff})_b \qquad (I.5)$$

(I.5) does not contain any neglections for W = I (the unit matrix). In all other cases, (I.4), and consequently (I.5), are only valid if the test group of b observations is insignificantly correlated with the rest of the observations.

Some remarks on Q_{vv} and on $\bar{\bar{v}}_i$ for highly (or totally) correlated residuals

APPENDIX II

Residuals v^+ and observational errors ε are connected by the relationship /7/:

$$v^+ = -Q_{vv}Q_{ll}^{-1}\varepsilon = -P\varepsilon \qquad \text{(II.1)}$$

Both Q_{vv} and P are idempotent, and the rank of both of them equals the redundancy r. Some consequences are: for diagonal elements d_i of any of them $0 \leq |d_i| \leq 1$; the trace of both of them equals their rank r, and therefore the average diagonal element of both of them equals the "relative redundancy" r/u. Correspondingly, if r=0, both Q_{vv} and v^+ equal 0 (full absorption of ε); and if all observations are repeated infinite times, both Q_{vv} and Q_{ll} (and P) become unit matrices of infinite size, and $v^+ = \varepsilon$ (no error absorption).

Submatrices of Q_{vv} (or of P) of the size $b \leq r$ are, generally, non-singular. Singularity of such submatrices indicates the total correlation of (some) residuals corresponding to the submatrix. This is, by given mathematical model and given Q_{ll}, a result of improper geometric peculiarities of the network (e.g. some undesirable simmetricities of the distribution of observations).

If a k x k (with $k \leq r$) submatrix of Q_{vv} (or of P) has the rank 1, than the corresponding totally correlated scaled residuals $\bar{\bar{v}}_i$ counted as

$$\bar{\bar{v}}_i = q_{v_i v_i}^{-1/2} v_i^+ \qquad \text{(II.2)}$$

are equal among themselves in their absolute values. This is bacause the scaling by $q_{v_i v_i}^{-1/2}$ in (II.2) corresponds to scaling each row of the singular submatrix of Q_{vv} (or of P) in question by its own diagonal element (it is easy to verify that for any symmetrical k x k matrix with the rank 1 a division (scaling) of each row by the square root of its own diagonal element yields a matrix containing identical rows - each being a series of the corresponding square roots with the sign of the original element). And vice versa: scaled residuals close to each other in their absolute values, are suspicious of being highly correlated.

Example: Plane fitting (3-parameter transformation) APPENDIX III

The same set of data (see the data summary and point scheme below) has been counted twice: with Z-tolerance of 0.1 m (case a.), and with Z-tolerance of 0.4 m (case b.). The relevant messages, summaries and the point scheme, as provided by the routine for absolute orientation MODOR, are reprinted below.

Case a. Note the simultaneous elimination of points 20 and 10.

```
MODOR: XY-TOLERANCE=      .10     ANGULAR TOLERANCE FOR          KAPPA   .00564 RADIANS
       Z-TOLERANCE=       .10                           FOR OMEGA AND PHI   .00564 RADIANS


   3-PARAMETER TRANSFORMATION
--> ERROR EQUATION          30 ELIMINATED WITH DISCREPANCY      .49810E+00
--> ERROR EQUATION          20 ELIMINATED WITH DISCREPANCY      .34979E+00
    BECAUSE OF SINGULAR GEOMETRIC POSITION:
--> ERROR EQUATION          10 ELIMINATED WITH DISCREPANCY      .34979E+00
>>> BAND10: OVERFLOW SUPPRESSED

>>> MODOR: FAILURE IN EQUATION SOLUTION
```

Case b. With a larger tolerance, points 10 and 20 have participated (have not been eliminated) in this adjustment. Notice their location, their different v_i^+ listed in column DZ (and in the point scheme). Column DZ contains the reliability characteristic of residuals σ_{v_i} (they are to be found in the point scheme another time).

--> denotes a warning, and >>> denotes a fatal error message.

```
MODOR: XY-TOLERANCE=      .10      ANGULAR TOLERANCE FOR          KAPPA    .00564 RADIANS
       Z-TOLERANCE=       .40                        FOR OMEGA AND PHI    .02256 RADIANS

   3-PARAMETER TRANSFORMATION
--> ERROR EQUATION          30 ELIMINATED WITH DISCREPANCY      .49810E+00
```

SUMMARY OF DATA AND OF RESULTS

		COORDINATES INPUT					
		MODEL			REFERENCE SYSTEM		
	N	X	Y	Z	X	Y	Z
	1	0.00	10.00	9.90	.00	10.00	10.00
	2	2.00	6.00	10.10	2.00	6.00	10.00
	3	4.00	2.00	9.90	4.00	2.00	10.00
	4	6.00	-2.00	10.10	6.00	-2.00	10.00
	5	8.00	-6.00	9.90	8.00	-6.00	10.00
	6	10.00	-10.00	10.10	10.00	-10.00	10.00
	10	0.00	-10.00	10.30	.00	-10.00	10.00
	20	6.00	6.00	10.30	6.00	6.00	10.00
-->	30	10.00	0.00	9.40	10.00	.00	10.00

RESIDUALS AND THEIR STANDARD ERRORS "#"

	N	DX	#DX	DY	#DY	DZ	#DZ
	1	*******	*******	*******	*******	.14	.26
	2	*******	*******	*******	*******	-.05	.22
	3	*******	*******	*******	*******	.16	.29
	4	*******	*******	*******	*******	-.03	.20
	5	*******	*******	*******	*******	.18	.22
	6	*******	*******	*******	*******	-.01	.27
	10	*******	*******	*******	*******	-.11	.58
	20	*******	*******	*******	*******	-.29	.23
-->	30	*******	*******	*******	*******	.62	*******

SIGMA IN Z= .19 (REDUNDANCY= 5)

UNKNOWNS AND THEIR STANDARD ERRORS

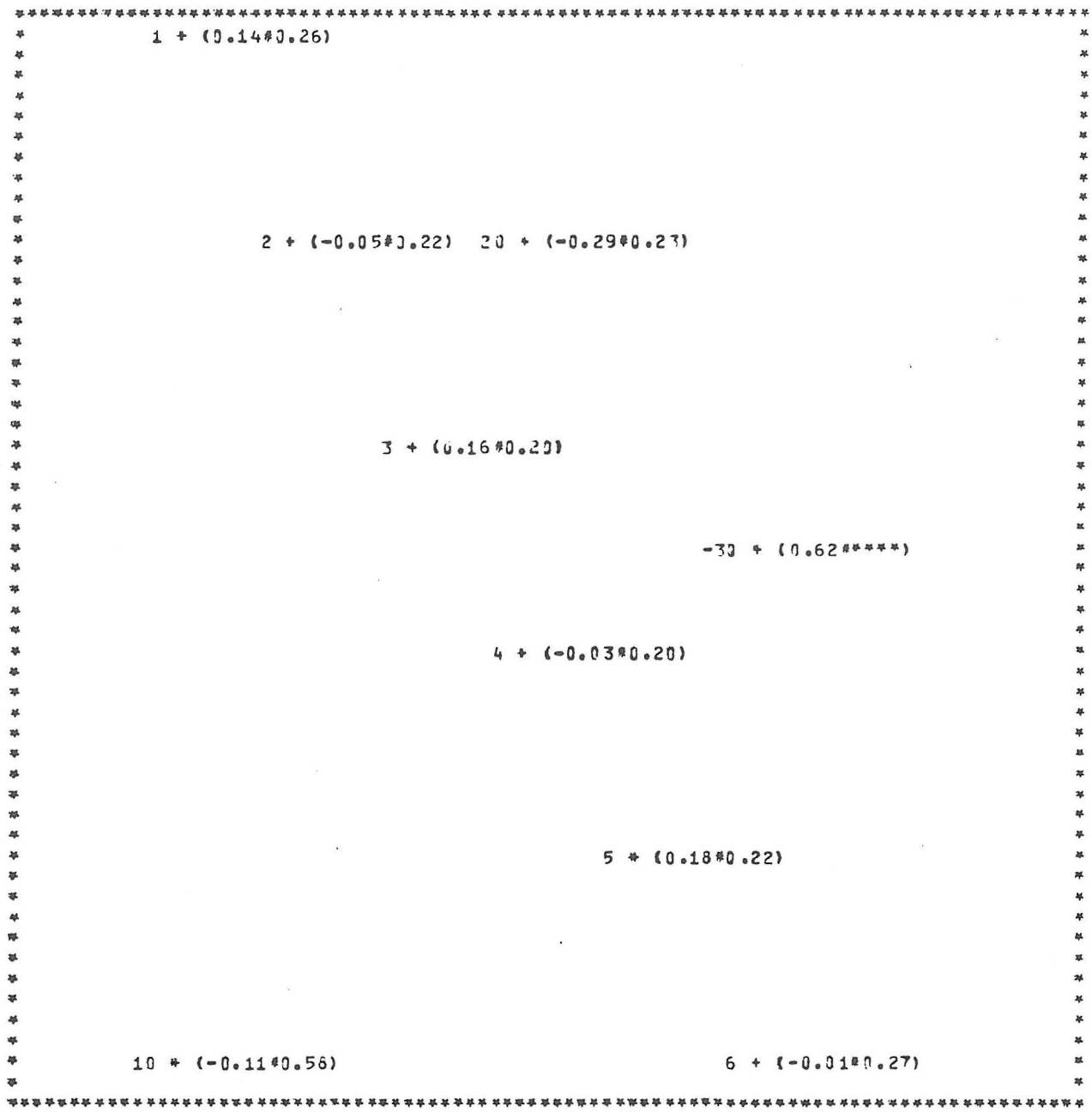
	SHIFTS IN			ANGULAR ELEMENTS			SCALE FACTOR
	X	Y	Z	OMEGA	PHI	KAPPA	
UNKNOWNS	0.00	0.00	-.11	.0070808	-.0096698	-.0000016	.1000000E+01
STANDARD ERRORS				.0102807	.0215753	**********	**********

X-FACTOR= 1.0

TRANSFORMATION FORMULA:

$$\begin{pmatrix} XR \\ YR \\ ZR \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.000 \\ -.114 \end{pmatrix} + \begin{pmatrix} .9999532E+00 & .1558511E-05 & -.9669616E-02 \\ -.7002610E-04 & .9999749E+00 & -.7083364E-02 \\ .9669356E-02 & .7080710E-02 & .9999282E+00 \end{pmatrix} * \begin{pmatrix} 1*XM \\ YM \\ ZM \end{pmatrix}$$

SCHEME OF VERTICAL RESIDUALS AND THEIR STANDARD ERRORS



IN THE ABOVE SCHEME
STANDS FOR "PLUS-MINUS"
POINTS WITH NEGATIVE NUMBERS DID NOT PARTICIPATE IN THE ADJUSTMENT