AN INVESTIGATION INTO FACTORS AFFECTING
VERTICAL EXAGGERATION

By

Dr. Ali Radwan Shawer
Assistant Professor
Civil Department, College of Engineering
University of Riyadh

Abstract

When a pair of overlapping photographs are examined under a stereoscope, a stereoscopic optical model will be observed. This model is affected by some exaggeration in the vertical direction, i.e. the ratio between the height of any object on the photograph and the horizontal dimension of this object observed under the stereoscope does not equal to the correct ratio on the field. This phenomenon is called Vertical Exaggeration.

This paper discusses the effects of the factors causing Vertical Exaggeration phenomenon.

INTRODUCTION

This paper discusses in detail the factors causing 'V.E.' phenomenon and its effects on the stereoscopic models observed under the stereoscopes. These factors can be classified as follows:

I. Photographic factors:

    a. Flying height above sea level 'H'.

    b. Air base - the ground distances between exposure stations 'B'.

    c. The focal length of the camera lens 'f'.

II. Stereoscopic viewing factors:

    a. The viewing distance under the stereoscope 'd'.

    b. The separation of the photographs under the stereoscope 'S'.

    c. Eye base of the observer 'E'.

III. There are other factors such as:

    a. The angle of inclination of the mirrors (in some stereoscopes).

    b. The shape and the colour of the objects on the photographs.

These factors III have small effects and can therefore be neglected.

The Vertical Exaggeration factor (V.E.) $= (\frac{h_m}{L_m} \div \frac{h}{L})$

where:

h_m and L_m = the height and the horizontal dimension of
an object observed under the stereoscope.

h and L = the height and the horizontal dimension of
the same object on the field.

By taking the photographic factors and the stereoscopic viewing factors into consideration the (V.E.) may take the following form:

$$V.E. = F\left(\frac{B.d.S}{F.H.E}\right)$$

The relative effects of the above factors (I and II) were studied experimentally and then analysed. An emperical expression for the Vertical Exaggeration in terms of the influencing factors was derived.

ANALYSIS

There are many factors affecting Vertical Exaggeration and by experiments in the laboratory it is found that some of these factors have significant effects. These factors are:

I. Photographic factors:

a. Flying height of the aircraft.

b. Air base - the distance between exposures stations, B.

c. The focal length of the camera lens, f.

II. Stereoscopic vision factors:

    a. Viewing distance - the distance between the photo objects through the instrument to the observer's eyes, d.

    b. The Eye base - the distance between the two eyes of the observer.

    c. The separation between the two photographs under the stereoscope.

The relative effects of the above mentioned factors on V.E., can be classified as follows:

$$\text{V.E.} = F[(\frac{B}{H}) \cdot (\frac{d}{E}) \cdot (\frac{S}{f})] \qquad \ldots\ldots \qquad (1)$$

to take a formula, *that* can be treated by experiments.

The experimental study requires overlapping photographs using different values for the following parameters:

1. Flying heights.

2. Air bases.

3. Focal lengths.

The photographs were examined stereoscopically in the laboratory by taking into consideration the stereoscopic viewing factors i.e. the experiments carried out by changing the following factors:

1. Viewing distances of the stereoscopes.

2. Separation between the photographs.

3. Eye bases.

After classifying the factors as illustrated in expression (1), the experiments were carried out separately by taking into consideration the new factor (B/H) as a variable factor. It has been observed that this factor has great effects into 'V.E.' Also, the ratio d/E and S/f were evaluated by using stereovision as variable factors.

EXPERIMENTS

Laboratory experiments were carried out on some known objects on the photographs as well as on the field. Vertical and horizontal dimensions were obtained from field observation. After examining the same objects on the photographs under a stereoscope, a viewing ratio between the vertical and horizontal dimension on the stereoscopic model was obtained. Then, a (V.E.) factor can be obtained as follows:

$$V.E. = \left[\frac{\text{A viewing ratio between the vertical and horizontal}}{\text{The field ratio between the vertical and horizontal}} \right.$$

$$\left. \frac{\text{dimension of the stereoscopic model under the stereoscope}}{\text{dimension of the same object}} \right]$$

i.e. after evaluating the viewed ratio between (h_m and L_m) for the stereoscopic model of the object, and by dividing this value on the observed ratio on the field (h and L) for the same object, the result taken as a V.E. factor.

Experiment No. 1

This experiment was carried out on a pair of overlapping vertical photographs under a stereoscope, parallax differences were measured on some objects having known heights and horizontal lengths on the field, - (parallax differences were measured here

to check only the observed heights on the field, but the V.E. ratios were evaluated as mentioned before.

By taking into consideration the ratio (B/H) for each two overlapping photographs, the (V.E.) ratios were obtained after dividing the viewed ratio on the observed ratio of the dimensions of the objects as mentioned before. Also the following table and chart were constructed. See table (I) and figure (1).

NOTE: The values of (V.E.) on the tables (I, II and III) are the average values of many laboratory observation.

(B/H)	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
(V.E.)	0.7	1.1	1.5	1.7	2.0	2.25	2.5	3	3.25

Table (I) shows the relation between (B/H) and (V.E.) ratio.

Experiment No. 2

In this experiment the ratio $(\frac{S}{f})$ is the variable factor, by using photographs taken with lenses having variable focal lengths. These photographs oriented under the stereoscope, and by changing the separating distance, the vertical exaggeration ratio were observed for each $(\frac{S}{f})$ value. The following table and chart were obtained. See table (II) and figure (2).

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(S/F)	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
(V.E.)	0.9	0.99	1.1	1.18	1.32	1.4	1.59	1.85

Table(II) shows the relation between (S/F) and
(V.E.) ratio.

NOTE: The photographs dimensions are 180 x 180 mm.

Experiment No. 3

The ratio $(\frac{d}{E})$ which is the variable factor, will be dis-
cussed and examined in this experiment.

By examining the stereoscopic model of the known objects
on the overlapping photographs, and by using various stereo-
scopes having a different viewing distance; also by changing
the viewing distance in the same apparatus, the vertical exa-
ggeration ratio can be observed for each case by the observer
who have an eye base = E_1. The same experiment was carried
out by other observers having E_2 & E_3, ... etc.

By taking the ratio $(\frac{d}{E})$ as a variable factor, the
corresponding (V.E.) ratio can be obtained as illustrated
before as showing in table (III) and figure (3).

(d/E)m	4.5	4.75	5	5.25	5.5	5.75	6
(V.E.)	0.97	1.04	1.1	1.15	1.22	1.29	1.34

Table(III) shows the relation between (d/E and
V.E. ratio)
where m = magnification factor

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MATHEMATICAL ANALYSIS

From above tables and curves, the following formulas can be derived.

From table (I) and figure (1) it is clear that the relation between (B/H) & (V.E.) is linear function. Hence we can put the relation in the following formula:

$$Y = \alpha x + C_1$$

$$\therefore V.E = \alpha \frac{B}{H} + C_1 \qquad \ldots\ldots \qquad (2)$$

$$\frac{d(V.E)}{d(B/H)} = \alpha + O$$

By substituting the data from experiment number (1) in equation (2)

$$\therefore V.E. = 6.6 \frac{B}{H} + O \qquad \ldots\ldots \qquad (3)$$

$$\therefore \frac{d(V.E.)}{d(B/H)} = 6.6$$

$$\therefore \alpha = 6.6 \quad \text{and} \quad C_1 = 0.0 \qquad \ldots\ldots \qquad (4)$$

Also, from experiment numbr (2) it is noticed that the relation between $(\frac{S}{f})$ and (V.E.) ratio is a linear relation, and we can put this relation as a linear function:

$$Y = \beta x + C_2$$

$$\therefore V.E. = \beta \frac{S}{f} + C_2 \qquad \ldots\ldots \qquad (5)$$

By substituting the values obtained from experiment (2) in equation (5)

$$\therefore \text{V.E.} = 1.0 \ \frac{S}{f} + 0.4 \qquad \ldots \qquad (6)$$

$$\therefore \frac{d(\text{V.E.})}{d(S/f)} = \beta + 0$$

$$\therefore \beta = 1.00 \quad \text{and} \quad C_2 = 0.4 \qquad \ldots \qquad (7)$$

Also, as illustrated before, the data obtained from experiment number (3), the relation between (d/E) and (V.E.) can take the following form:

$$\text{V.E.} = \gamma(\frac{d}{E})m + C_3 \qquad \ldots \qquad (8)$$

$$\therefore \gamma = \frac{d(\text{V.E.})}{d(d/E)m}$$

$$\therefore \text{V.E.} = 0.258(\frac{d}{E})m - 0.2 \qquad \ldots \qquad (9)$$

$$\therefore \gamma = 0.258 \quad \text{and} \quad C_3 = -0.2 \qquad \ldots \qquad (10)$$

From the above equations (3, 6 and 9), and from the relation between V.E. and the factors discussed before [as established in equation (1) $\text{V.E.} = f(\frac{B}{H} \ \frac{d}{E} \ \frac{S}{f})$], the vertical exaggeration can be expressed by the following formula:

$$\text{V.E.} = (\propto \frac{B}{H} + C_1) \cdot (\beta \frac{S}{f} + C_2) \cdot (\gamma(\frac{d}{E})m + C_3) \ \ldots \ (11)$$

$$\therefore \text{V.E.} = (\propto \beta \ \gamma \ \frac{B}{H} \cdot \frac{S}{f} \cdot \frac{dm}{E}) + (\beta \ \gamma \ C_1 \ \frac{S}{f} \cdot \frac{dm}{E})$$

$$+ (\propto \gamma \ C_2 \ \frac{B}{H} \cdot \frac{dm}{E}) + (\gamma \ C_1 C_2 \ \frac{dm}{E})$$

$$+ (\propto \beta \ C_3 \ \frac{B}{H} \ \frac{S}{f}) + (\beta \ C_1 C_3 \ \frac{S}{f})$$

$$+ (\propto C_2 C_3 \ \frac{B}{H}) + (C_1 C_2 C_3) \hspace{2cm} (12)$$

by substituting the values of $\propto$, β, γ, C_1, C_2 and C_3 in equation (12)

$$\therefore \text{V.E} = [(6.6 \times 1.0 \times 0.258 \times \frac{B}{H} \cdot \frac{S}{f} \cdot \frac{dm}{E})$$

$$+ (0.0) + (6.6 \times 0.258 \times 0.4 \times \frac{B}{H} \cdot \frac{dm}{E})$$

$$+ (0.0) - (6.6 \times 1.0 \times 0.2 \times \frac{B}{H} \cdot \frac{S}{f})$$

$$- (0.0) - (6.6 \times 0.4 \times 0.2 \ \frac{B}{H})$$

$$+ (0.0)]$$

$$= [(1.7 \ \frac{B}{H} \ \frac{S}{f} \ \frac{dm}{E}) + (0.68 \ \frac{B}{H} \ \frac{dm}{E})$$

$$- (1.32 \ \frac{B}{H} \ \frac{S}{f}) - (0.528 \ \frac{B}{H})]$$

$$\therefore \text{V.E} = B/H \left[\left(1.7 \frac{S}{f} \cdot \frac{\bar{d}m}{E}\right) + \left(0.68 \frac{\bar{d}m}{E}\right) \right.$$

$$\left. - \left(1.32 \frac{S}{f}\right) - (0.53) \right] \tag{13}$$

Graphical solution of equation 13

By solving equation (13) for practical values of each items and by drawing the results between effective factors and (V.E.) ratio the following Tables (4→7) and chart can be constructed. This chart Figure (4) is very easy for use and give the V.E. ratio directly after determining $\frac{B}{H}$, $\frac{S}{F}$ and $\frac{d}{E}$ values. Quick results can be obtained as demonstrated in the following example:

Assume $\frac{B}{H} = 0.5$, $\frac{S}{f} = 0.5$ and $\frac{\bar{d}m}{E} = 5$

V.E. = 3.22 from equation 13, also as shown in Figure(4) and as illustrated by dotted line from $\frac{S}{f} = 0.5 \rightarrow$ to the line of $\frac{d}{E} = 5 \rightarrow$ to the line of $\frac{B}{H} = 0.5 \rightarrow$ to V.E. values = 3.22 = the same result.

$$\therefore \text{V.E} = \frac{B}{H} \left[\left(1.7 \frac{S}{f} \frac{dm}{E}\right) + \left(0.68 \frac{dm}{E}\right) - \left(1.32 \frac{S}{f}\right) - (0.53) \right]$$

$\therefore$ The following Tables can be constructed after substituting the practical values for each factor as follows:-

$1.7\dfrac{S}{f}$ / $\dfrac{dm}{E}$	0.6	0.7	0.8	0.9	1.0	1.2	1.4
4	4.08	4.76	5.44	6.12	6.8	8.16	9.52
4.5	4.59	5.35	6.12	6.88	7.65	9.18	10.7
5.0	5.1	5.95	6.8	7.65	8.5	10.2	11.9
5.5	5.61	6.54	7.48	8.4	9.35	11.22	13.1
6.0	6.12	7.14	8.16	9.18	10.2	12.24	14.28

Table (4) shows the results of the factor

$$(1.7\ \frac{S}{f} \cdot \frac{dm}{E})$$

$(1.7\ \frac{S}{f}\ \frac{dm}{E})\ +\ (0.68\ \frac{dm}{E})$						
6.8	7.48	8.16	8.84	9.52	10.88	12.24
7.65	8.41	9.18	9.94	10.91	12.24	13.76
8.5	9.35	10.2	11.05	11.9	13.6	15.3
9.35	10.28	11.22	12.4	13.1	14.96	16.84
10.20	11.22	12.24	13.26	14.28	16.32	18.36

Table (5) shows the values of

$$(1.7\ \frac{S}{f}\ \frac{dm}{E})\ +\ (0.68\ \frac{dm}{E})$$

	The results of Table (5)						
$(-1.32 \frac{S}{f} - 0.53)$	5.47	6.0	6.6	7.15	7.7	8.76	9.89
	6.15	6.72	7.62	8.25	9.06	10.12	11.41
	6.83	7.89	8.64	9.3	10.05	11.48	12.95
	7.5	8.82	9.66	10.7	11.25	12.84	14.5
	8.19	9.76	10.68	11.57	12.43	14.2	16

Table (6) shows the results of Table (5)

$$+ \ (-1.32 \ \frac{S}{f} - 0.53)$$

H/B \ V.E.	3	4	5	6	7	8	9
0.1	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.2	0.6	0.8	1.0	1.2	1.4	1.6	1.8
0.3	0.9	1.2	1.5	1.8	2.1	2.4	2.7
0.4	1.2	1.6	2.0	2.4	2.8	3.2	3.6
0.5	1.5	2.0	2.5	3.0	3.5	4.0	4.5
0.6	1.8	2.4	3.0	3.6	4.2	4.8	5.4
0.7	2.1	2.8	3.5	4.2	4.9	5.6	6.3

DISCUSSION

From the above results (Tables - Equations - Charts) it is found that the Vertical Exaggeration factor take a positive values larger than 1 for all photographic factors. By close examination of the items of stereoscopic vision (d, S and E) the Vertical Exaggeration factor can be minimized, by adopting the appropriate values.

In photo interpretation processes, to obtain good results from examined photograph it is very useful to know the exaggerated values and to take into account.

The following expression:

$$V.E = \frac{B}{H} \left[(1.7 \frac{S}{f} \cdot \frac{dm}{E}) + (0.68 \frac{dm}{E}) - (1.32 \frac{S}{f}) - (0.53) \right]$$

give us a good result for exaggerated values, also by using the chart Figure (4), the V.E. values at any, stereopair and stereoscopic examination may be determined. After taking the V.E. values into consideration, the V.E. becomes under control and good results on photo-interpretation can be obtained.

CONCLUSIONS

From the previous analysis of the factors causing V.E. and its values, also from the charts and tables constructed between the V.E. and photographic factors and stereoscopic vision factors it may be concluded that:

1. The V.E. and its approximate value for any object can be determined if the effective factors (H, B,

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f, d, S and E) are known.

2. The formulated expression (13) can be used for determining the V.E. values at any variable condition of photographic and stereoscopic viewing processes.

3. Tables and charts Figure (4) may be used for determining the V.E. values by a graphical method.

By taking V.E. into account, the photo-interpretation process can be kept under control and hence results improved should be expected.
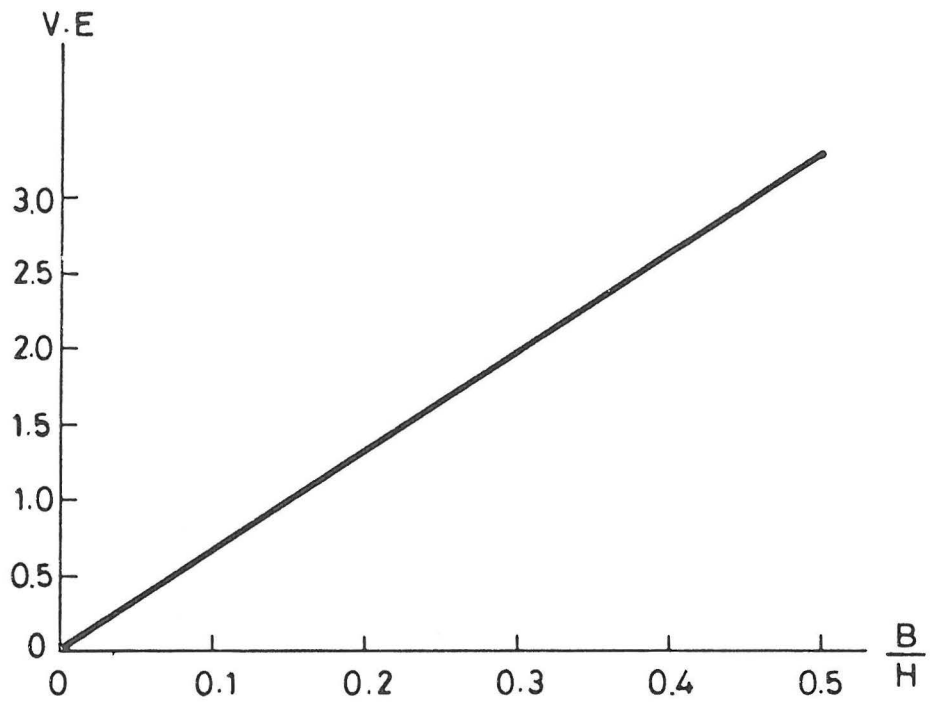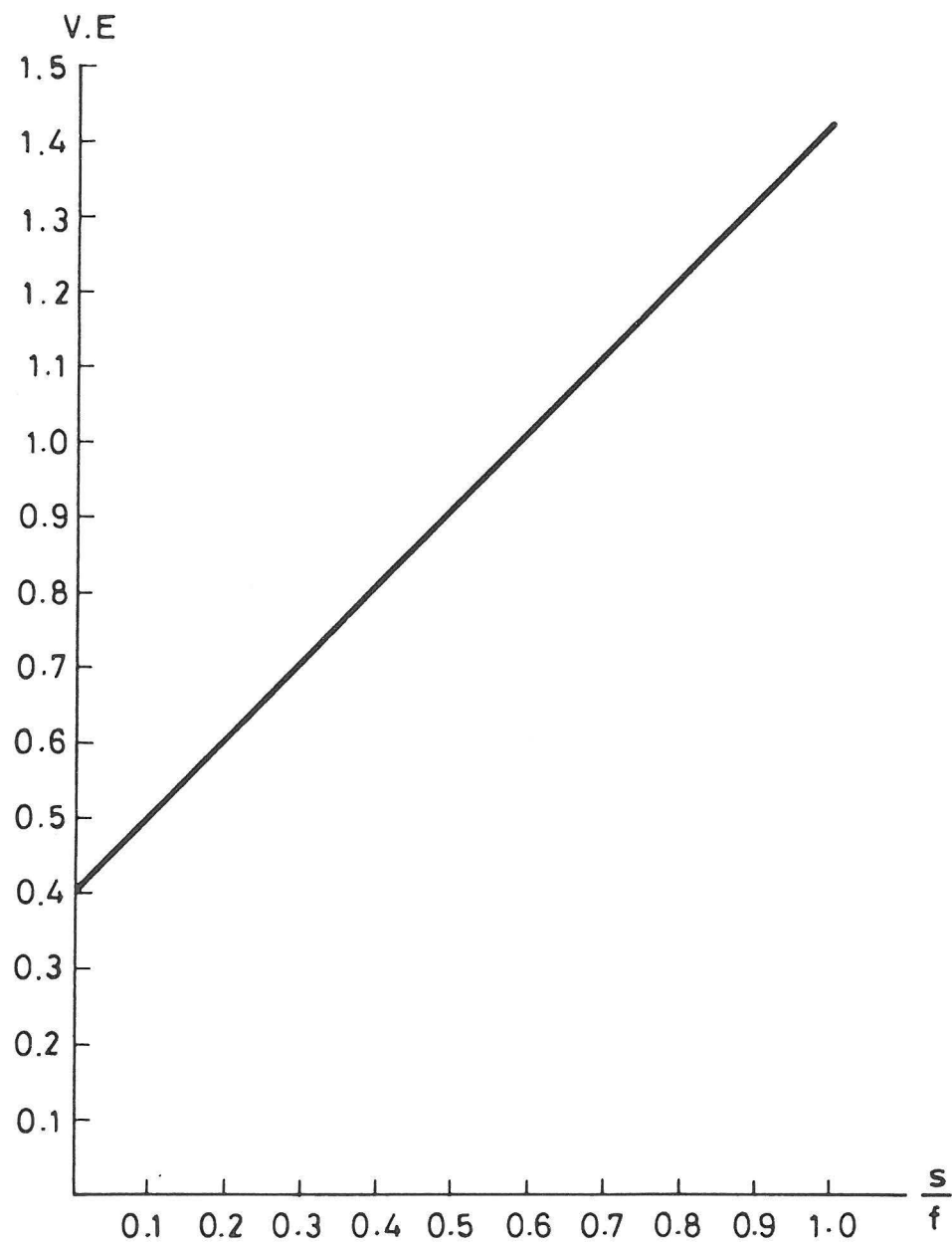
FIG.(1)

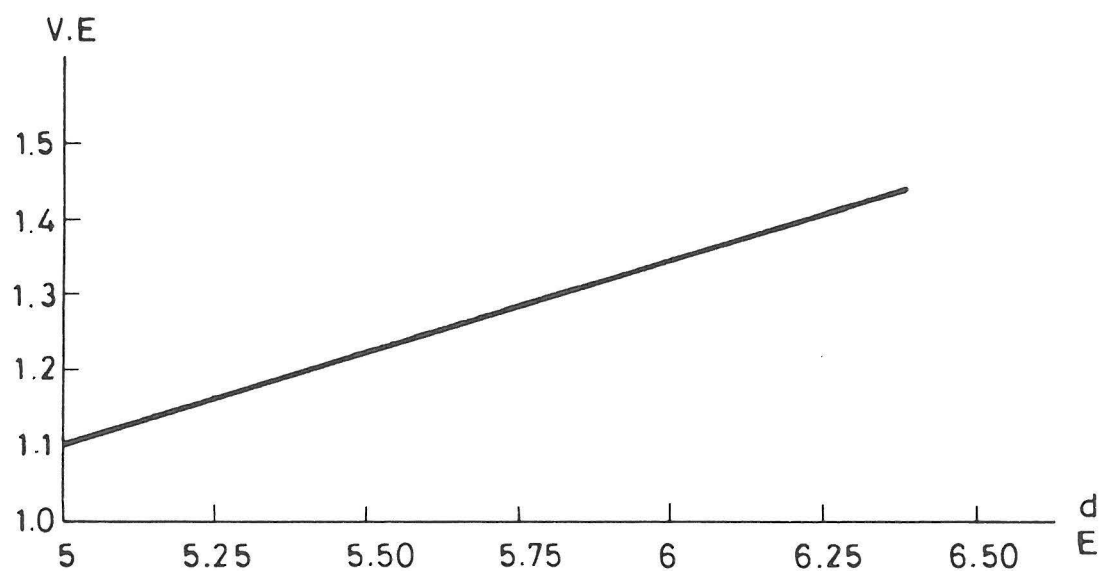$$V.E = 6.6 \frac{B}{H} + 0$$

FIG. (2)

$$V.E = 10 \frac{s}{f} + 0.4$$

FIG. (3)

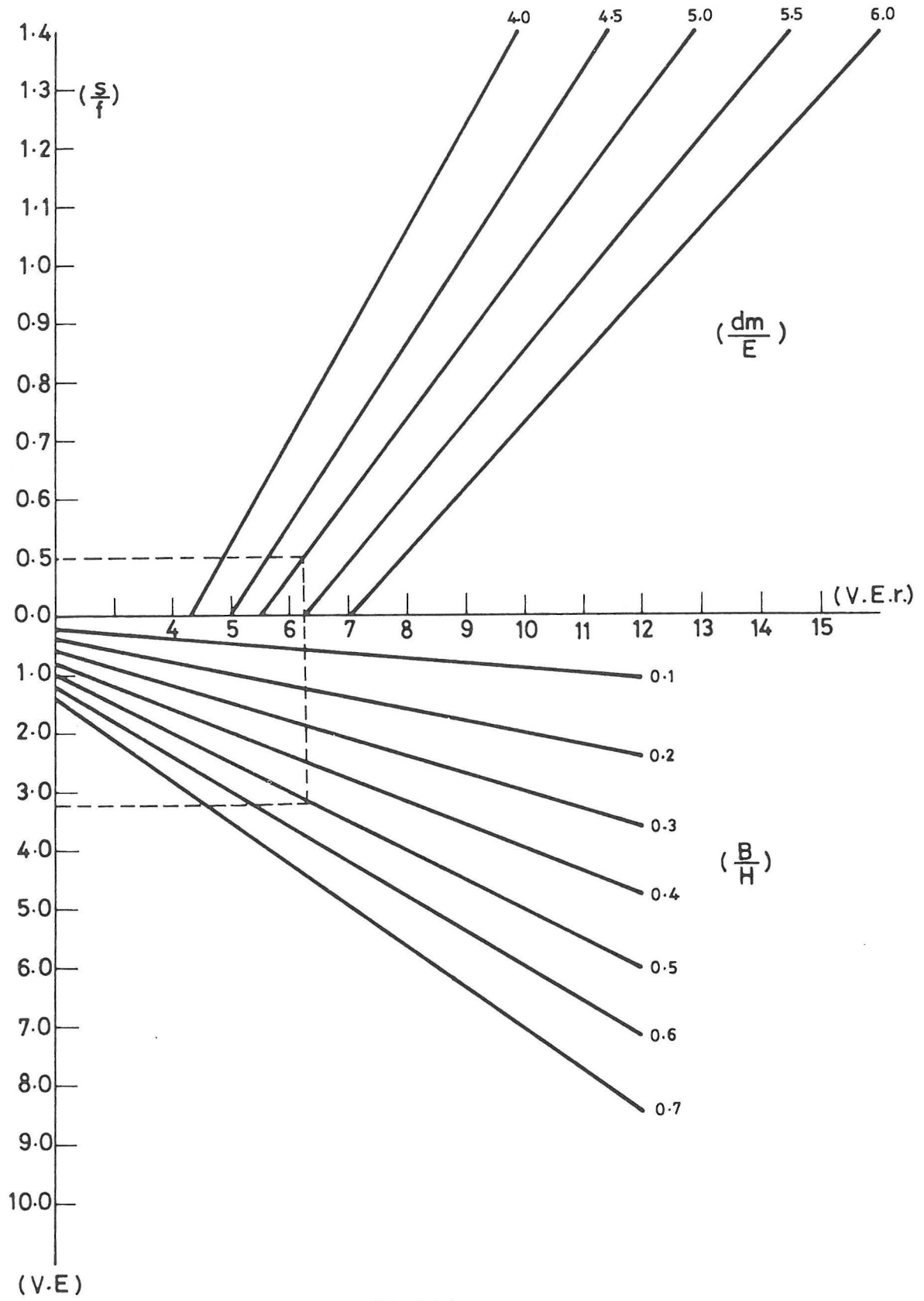$$V.E = 0.258 \frac{d}{E} - 0.2$$

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Fig.(4)

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