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1. Introduction

A stereoorthophotogram is a stereopair consisting of two stereocomponents: an orthophotogram and a q-orthophotogram /called also a stereomate or a stereopartner/. A typical q-orthophotogram can be received by a special differential rectification of that of the two pictures of a stereogram, which has not been used as the image-giver for the orthophotogram. Thanks to this the residual radial displacements of both components of a stereoorthophotogram unite, when stereoviewed, in a residual stereoscopic effect embodying such terrain accidents as trees or buildings.

The stereo-effect of topographic surface presented on a stereoorthophotogram is artificialy developed by introducing specific displacements /so called artificial q-parallaxes/ while producing a q-orthophotogram.

Artificial parallaxes are defined by such a mathematical function, that, in any chosen intervals, the height differences can be calculated using the same formula no matter whether one surveys the artificialy created parallaxes of topographical surface or differentialy transformed "natural" residual parallaxes of terrain objects. A good matching of the increments of artificial and "natural" parallaxes on a stereoorthophotogram is possible, however, only in those cases when the terrain object was recorded on the background of the flat and horizontal terrain or when a longitudinal terrain slope does not exist. In such a case the image of an overground point of a terrain object e.g. the image of a tree-top, receives similar amount of differential correction on the left and right components of a stereopare.

Not always, however, the terrain objects such as tres, buildings and constructions are recorded on the background of a terrain without longitudinal slope. In a somewhat rough country the image of a top point A of a tree /fig.1/, for example, is recorded on the background of the terrain point A_T which can be situated higher or lower than the foot T of this tree. The image of the terrain point A_T will get pertinent corrections in the differential rectification procedure, and these corrections will automatically be applied also to

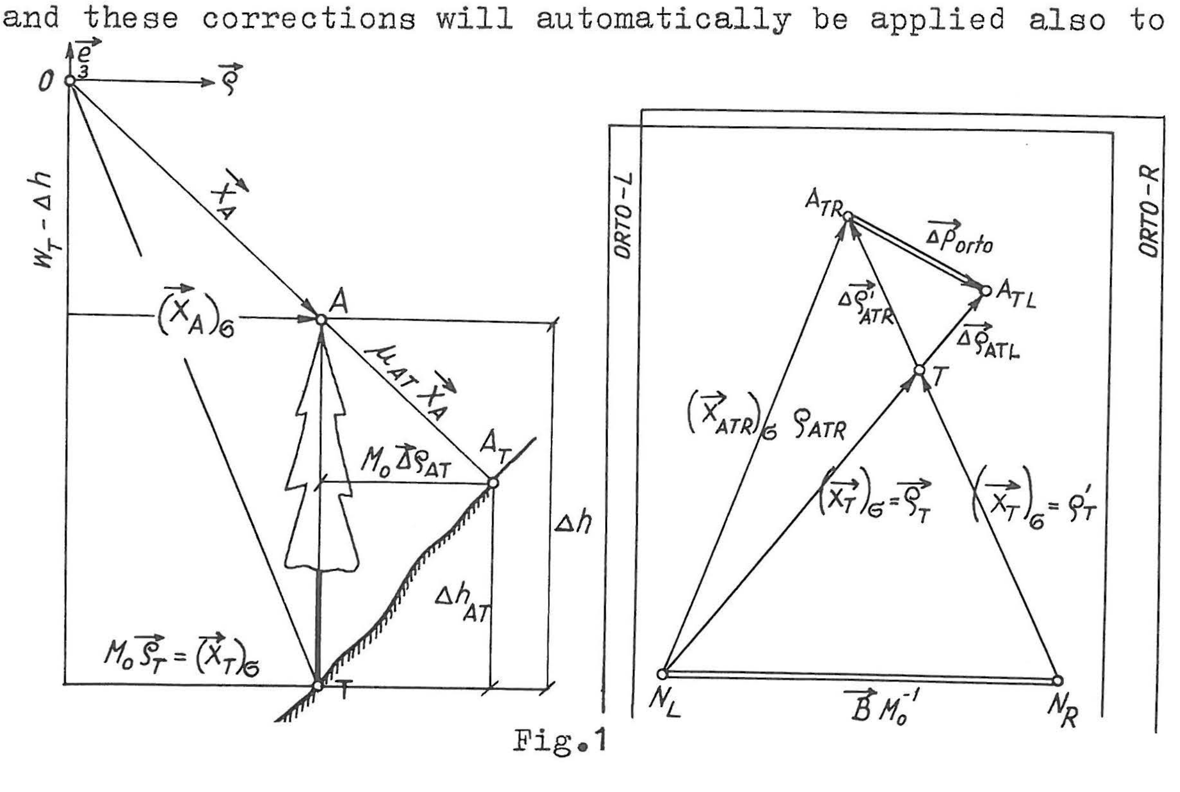


Fig.1

to the image of tree top point A, which coincides with the terrain point A_T. Of course, the vectors of these corrections depend not only on the elevation of the terrain point A_T, but also on this point position on the picture /radius $\vec{\varrho}$ /, and on the type of rectification /ortho or q-ortho/. These three factors very rarely produce similar correction vectors for the images of overground point A on the left and right components of a stereoorthophotogram. As the result a random-systematic errors appear, which relax the homogenity of geometry on a stereoorthophotogram.

2. Relief geometry

To ease the analysis of the geometry of stereoorthophotographs we can assume that an ideal orthophotogram and an ideal q-orthophotogram are discussed. In such a case we are dealing with the pictures made strictly /with no errors/ in orthogonal or q-orthogonal projection.

Let us assume further an orthogonal coordinate system $\{O_L, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ placed in the projection centre /versor $\vec{e}_3$ is vertical/. Than terrain point T can be described by a vector $\overrightarrow{OT} = \vec{X}_T = X_T \vec{e}_1 + Y_T \vec{e}_2 - w_T \vec{e}_3$. Similar way the vectors $\vec{X}_A$ and $\vec{X}_{AT}$ will describe positions of overground point A /which is placed above point T/ and the position of the point A_T being the central projection image of point A on the terrain surface. On an ideal orthophotogram of discussed terrain portion /recorded on the horizontal plane in the scale $1:M_o$/ points A and A_T will be recorded in the same place of the nadir radius $\vec{\varrho}_T$. Vector $\vec{\varrho}_T = \varrho_x \vec{e}_1 + \varrho_y \vec{e}_2$ describes a position of the image of a terrain point T on the orthophotogram in relation to its nadir point.

The radial displacement $\overrightarrow{\Delta\varrho}_{AT}$ of e.g. an image of the tree top in relation to an image of the tree foot can by given by the formula /see fig.1/

$$\overrightarrow{\Delta\varrho}_{AT} = M_o^{-1}/\vec{X}_{AT} - \vec{X}_T/_6 = M_o^{-1}\left[/1 + \mu_{AT}/\vec{X}_A - \vec{X}_T\right]_6 = M_o^{-1}\mu_{AT}/\vec{X}_T/_6 = \mu_{AT}\vec{\varrho}_T$$

The coefficient μ_{AT} depends on the elevations of points A,T,A_T and on the flight height W_T above the point T.

Its values for the left and right orthophotogram of a stereo-par respectivly is

$$\mu_{ATL} = /\Delta h - \Delta h_{ATL}/ : /w_T - \Delta h/ \qquad (2)$$

$$\mu'_{ATR} = /\Delta h - \Delta h_{ATR}/ : /w_T - \Delta h + B_z/$$

The spatial effect being observed on a stereopair of photos is caused by longitudinal parallaxes, namely by a difference of parallaxes of the two given points. The differences of longitudinal and traversal parallaxes can be derived directly from the radial displacements of two discussed points on the left and right photographs. Applying this rules to the pairs of orthophotos obtained from the left and right photographs of a stereopair respectively one can write the following equation for a difference of parallaxes /see fig. 1 and formulas 1-2/.

$$\vec{\Delta p}_{orto} = \vec{\Delta \rho}_{ATL} - \vec{\Delta \rho'}_{ATR} = M_o^{-1} \left[\mu_{ATL} /\vec{X}_T/_6 - \mu'_{ATR} /\vec{X}'_T/_6 \right] =$$

$$= M_o^{-1} \left[\frac{\Delta h - \Delta h_{ATL}}{w_T - \Delta h} \vec{X}_T - \frac{\Delta h - \Delta h_{ATR}}{w_T - \Delta h + B_z} /\vec{X}_T - \vec{B}/ \right]_6 \qquad (3)$$

One can introduce to the above formula the notion of an average flight height level w_o, such, that: $w_T = w_o - h_T$, h_T - elevation of a terrain point refered to reference level of the mean flight height w_o. Finally we can develope the full formula for the "natural" residual parallaxes on the pair of orthophotos

$$\vec{\Delta p}_{orto} = M_o^{-1} \left[\vec{B} \frac{\Delta h}{w_o - h_T - \Delta h + B_z} + \vec{X}_T \frac{\Delta h \, B_z}{/w_o - h_T - \Delta h/ \, /w_o - h_T - \Delta h + B_z/} \right.$$

$$\left. - \vec{X}_T \left(\frac{\Delta h_{ATL}}{w_o - h_T - \Delta h} - \frac{\Delta h_{ATR}}{w_o - h_T - \Delta h + B_z} \right) - \vec{B} \frac{\Delta h_{ATR}}{w_o - h_T - \Delta h + B_z} \right] \qquad (4)$$

Only the first segment of this formula is not dependent on the shape of topographical surface. For the convenience of the notion we can denote

$$\Delta p_x^{oo} = /B_x M_o^{-1} \Delta h/ : /w_o - h_T - \Delta h/ \qquad (5)$$

The secound term of formula 4 describes the influence of B_z on the value of $\overrightarrow{\Delta p}_{orto}$. Third and fourth segments show the influence of the terrain roughness on the $\overrightarrow{\Delta p}_{orto}$.

As it is known, the q-orthophotogram produced from a left or right photogram differs from the related orthophotogram only about the value of each elementary image shift, being parallel to the X-axis. In mathematical formulas this shift q must be introduced to the $\overrightarrow{\Delta p}_{orto}$ value to write a formula for residual natural parallaxes on a stereoorthophotogram. Assuming q-orthophotogram produced from the right photogram the following formula can be produced

$$\overrightarrow{\Delta p}_{s-ortho} = \overrightarrow{\Delta p}_{ortho} + \overrightarrow{\Delta q}_{ATR} \qquad (6)$$

The value of $\overrightarrow{\Delta q}_{ATR} = \overrightarrow{q}_{ATR} - \overrightarrow{q}_{T}$ means a relative artificial shift of image of the terrain point T in relation to the image A_{TR} of the tree top point A on the topographical surface.

The function generating artificial parallaxes q is chosen so that it approximates with the best possible accuracy the increments of "natural" residual parallaxes / $\overrightarrow{\Delta p}_x$/$_{ortho}$ on varoius levels w_T. As when ounce defined such a function must represent the "natural" residual parallaxes in all points of a stereoorthophotogram, then one can consider the approximation only for the part Δp_x^{oo} of formula 4 that does not change under random factors.

It was proved in the above discussion that natural residual parallaxes on a stereoorthophotogram depend not only on the hights of pertinent terrain accidents, but also on the shape of photographed terrain surface. For this reason the geometry of artificialy developed relief of rough topographical surface can never be fuly unified with the geometry of "natural" residual parallaxes.

3. Estimation of geometric homogenity relaxation factor

For various a priori error analysis it is convinient to consider rather the slopes of the terrain surface that the terrain elevations. To facilitate this it will be useful in our case to introduce a plane ϑ , tangent to the topographical surface at the terrain point T. Such a plane can be given by formula

$$\vec{X}_{AT} = /1+ u_{AT}/\vec{X}_A = \vec{X}_T + t_1/\vec{e}_1 + tg\ \vec{e}_3/ + t_2/\vec{e}_2 + tg\ \vec{e}_3/ \qquad (8)$$

where α , β - the angles of longitudinal and traversal tilt of the plane .

From three equations of type /8/ new formula for μ_{AT} can be derived. Considering $\vec{X}_A = \vec{X}_T + \Delta h\ \vec{e}_3$ we will get for the left and right projection centres respectively

$$\mu_{ATL} = \Delta h : A \qquad \mu'_{ATR} = \Delta h : /A-A_B/$$

$$\text{where} \quad A = X_T tg\,\alpha + Y_T tg\,\beta + w_T - \Delta h \qquad (9)$$

$$A_B = B_x tg\,\alpha + B_y tg\,\beta - B_z$$

Acording to equations /3/ and /6/ we can state that

$$\vec{\Delta p}_{s-ortho} = \vec{\Delta\rho}_{ATL} - \vec{\Delta\rho}_{ATR} = M_o^{-1}\left[\vec{X}_T\ \mu_{ATL} - \vec{X}'_T\ \mu'_{ATR}\right] + \vec{\Delta q}_{ATR} =$$

$$= M_o^{-1}\left[\vec{X}_T \frac{\Delta h}{A} - /\vec{X}_T - \vec{B}/\frac{\Delta h}{A - A_B}\right] + \vec{\Delta q}_{ATR} \qquad (10)$$

The value Δh_{ATR} that is necessary to calculate Δq_{ATR} acording do equation /7/ can be derived from formulas /2/ and /8/

$$\Delta h_{ATL} = \Delta h\ /1 - \frac{w_T - \Delta h}{A}/$$

$$\Delta h_{ATR} = \Delta h\ /1 - \frac{w_T + B_z - \Delta h}{A - A_B}/ \qquad (11)$$

Equation /10/ is easy to use and can be successfuly utilized for various calculations. In many cases, however, when an a priori error analysis is to be performed it would be more convenient to use rather image than field coordinates. In case of such analysis it is often also better to use relative values of the errors than the absolut ones. To facilitate the analysis let us take for granted that $B_y = B_z = 0$, and let us assume that the errors of $\vec{\Delta p}_{s-ortho}$ will be related to the corresponding Δp_x^{oo} value.

Denoting:

$$\Delta h_{ATL} = t_L \quad \Delta h \qquad \Delta q_{ATP} = t_q \, \Delta p_x^{oo} \qquad \ldots(12)$$
$$\Delta h_{ATP} = t_p \quad \Delta h$$

one can get from equation /4/

$$\overrightarrow{\Delta p}_{s\text{-}ortho}/B_y = B_z = 0/ = \left[\vec{e}_1 - \frac{\vec{X}_T}{B_x} /t_1 - t_p/ - \vec{e}_1/t_p - t_q/\right] \Delta p_x^{oo} \quad \ldots(13)$$

Let us denote additionaly: $\vec{X}_T = /w_T : c_k/ \ \vec{x}_T$

$\Delta h_{max} = k \cdot w_o$ = the height of the heighest terrain accidents,

$$\Delta h = g \, \Delta h_{max}, \qquad h = d \, w_o, \qquad b_x^o = B_x \frac{c_k}{w_o}$$

$$a = x_T \, tg\,\alpha + y_T \, tg\beta + \frac{1 - d - gk}{1 - d} c_k$$

$$\Delta p_x^{oo} = n \, b_x^o \frac{g\,k}{1 - d - gk}$$

$n = M_o^{-1} \dfrac{w_o}{c_k}$ = orthophotogram to photogram enlargement ratio

Substituting /14/ to /13/ and considering also /11/ and /12/ one can prove the following formulas for $B_y = B_z = 0$:

$$/\Delta p_x/_{s\text{-}ortho} = \Delta p_x^{oo} \left[/1-d-gk/c_k \left(\frac{1}{a/1-d/-b_x^o tg\alpha} - \frac{x_T tg\,\alpha}{a\left[a/1-d/-b_x^o tg\alpha\right]} \right) + tq \right]$$

$$/\,\Delta p_y/_{s\text{-}ortho} = \Delta p_x^{oo} \left[/1-d-gk/\ c_k \frac{y_T \ tg\,\alpha}{a\left[a/1-d/-b_x^o tg\,\alpha\right]} \right] \qquad \ldots(15)$$

$$tq_o = /1-d-gk/ - \frac{/1-d-gk/^2 \ c_k}{a/1-d/ - b_x^o tg\,\alpha} \qquad \ldots(16)$$

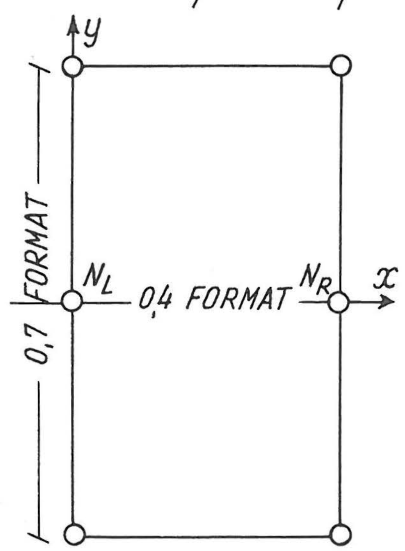
$$t_q = \frac{1-d-gk}{gk} \left[\ln \frac{1}{1-d-gkt_p} - \ln \frac{1}{1-d} \right]; \quad t_p = 1 - \frac{/1-d-gk/c_k}{a/1-d/-b_x^o tg\alpha} \qquad (17)$$

where tq_o and t_q are the relative values of linear and logarithmic artificial parallakses. Equations 15 - 17 can be used to calculate the errors of residual "natural" parallaxes on a stereoorthophotograph made in the original scale of mother photograph.

To illustrate the nature of errors on a stereoscopic pare of orthophotographs and on a stereoorthophotogram the diagrams of absolute error values were made for the rather unfavorable maps scale 1:1000. A very big height /0.05 W_0/ of terrain axidents was chosen and large angles of terrain slope / $\alpha = 20^{\circ}$, $\beta = 30^{\circ}$/ to expose the error distribution. For the same reason the area of error analysis was rather big; at the left side it goes even beyond the 60% overlap./see fig.2/.

Tab.1

Relative errors of „natural" residual parallaxes on stereo-orthophotomaps" for the terrain slope : $\alpha = 20^{\circ}$, $\beta = 30^{\circ}$



Δp_x^{oo} given for the scale of orthophotogram equal to the photo-scale

c_k [mm]	d	gk	Δp_x^{oo} [mm]	$\overrightarrow{\Delta P}_{sortho}$ errors in % Δp_x^{oo} — Stereomodel point numbers: 1	2	3	4	5	6
152	+0,1	0,05	5,41	+2,3 0,0	-19,3 0,0	-0,4 -14,5	+12,7 +72,4	-35,7 +34,5	-12,7 -10,1
	0,0	0,05	4,84	+1,7 0,0	-18,9 0,0	-0,5 -14,1	+9,6 +67,0	-34,7 +32,9	-12,5 -9,9
		0,03	2,85	+0,9 0,0	-18,6 0,0	-0,2 -13,9	+5,6 +63,4	-34,6 +31,8	-11,8 -9,7
		0,01	0,93	+0,3 0,0	-18,3 0,0	+0,1 -13,7	+1,7 +60,1	-34,6 +30,8	-11,3 -9,7
	-0,1	0,05	4,38	+1,4 0,0	-18,5 0,0	-0,5 -13,8	+7,8 +63,2	-33,9 +31,7	-12,1 -9,7
210	+0,1	0,05	4,24	+1,0 0,0	-11,8 0,0	-0,2 -9,4	+3,4 +21,1	-15,9 +15,2	-9,3 -7,5
	0,0	0,05	3,79	+0,8 0,0	-11,6 0,0	-0,3 -9,2	+2,8 +20,5	-15,7 +14,8	-9,2 -7,4
	-0,1	0,05	3,43	+0,7 0,0	-11,5 0,0	-0,3 -9,1	+2,3 +19,9	-15,6 +14,5	-9,1 -7,3

The chosen values of the geometric homogenity relaxation coefficients $\left(t_x = /\Delta p_{x\text{-sortho}} - \Delta p_x^{oo}/ : \Delta p_x^{oo} \text{ and } t_y = \Delta p_{y\text{-sortho}} : \Delta p_x^{oo}\right)$ calculated in Δp_x^{oo} percentage for the six points of useful part of stereogram using formulas 15 and 17 are shown in the table 1. The diagrams on fig.2 as well as the coefficients in table 1 are refered exclusively to the terrain slope given by the angles $\alpha = 20^{\circ}$ and $\beta = 30^{\circ}$, and for this reason can be meant as an example only. The distribution of residual "natural" parallaxes errors on a stereoorthophotogram depend signi-

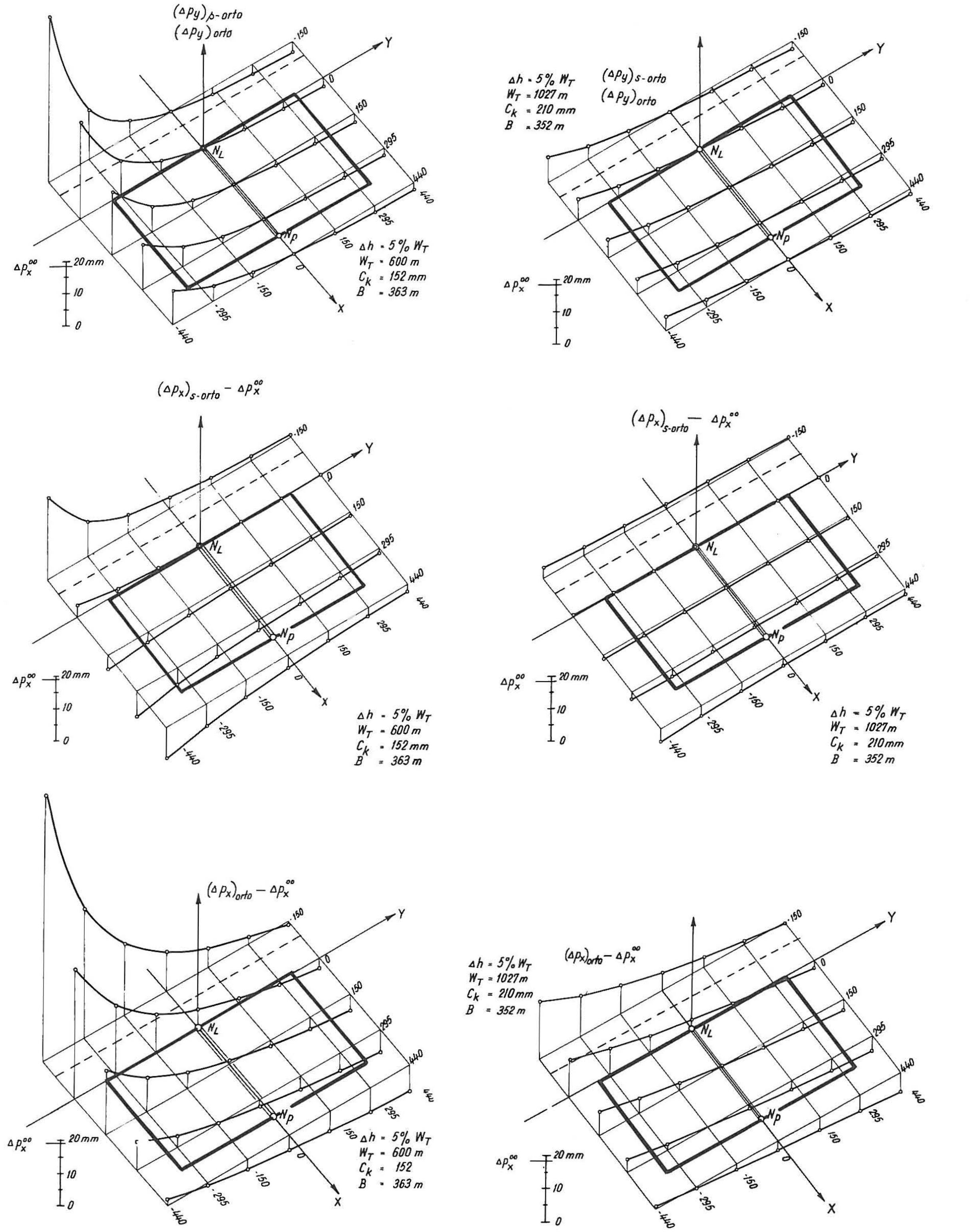


Fig.2. Diagrams of errors of residual "natural" parallaxes on a stereoorthophotogram and on a stereoscopic pair of orthophotographs. The errors were calculated exclusively for the cameras c_k= 152 mm /left/ and c_k= 210 mm/right/, for the terrain slope α= 20°, β= 30°, orthophoto scale 1:1000, and heights of terrain axidents equal 5% W_o. The graduation at the lower left corner of each diagram is to evaluate the error in comparission to Δp_x^{oo}.

ficantly on this angles sign and value, and also on the camera view angle. The maximum of error value appeare when the projection ray $\vec{X}_A$ is closs to parallelism to the plane ϑ which approximates the terrain surface around the terrain point T.

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