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PRESENTED PAPER

ON THE THEORETICAL ACCURACY OF PHOTOGRAMMETRIC VOLUME DETERMINATION

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Abstract:

Photogrammetric methods are used in volume estimations for open pit mining in the mining industry, and for volume estimation of water reservoirs in irrigation works. Height values of a set of terrain points or contour lines are determined by photogrammetric strip- and block triangulation and from these the volume enclosed by the terrain surface and a reference level is estimated.

There are two error sources in this volume determination, namely photogrammetric triangulation and the terrain roughness. The influence of the photogrammetric triangulation on volume estimation is studied in this paper. This paper thus forms a continuation of earlier studies of Richardus, where the influence of photogrammetric restitution and contouring was studied.

Besides the photogrammetric process, also the terrain roughness and unaccounted terrain details result in volume errors. This influence was studied by Botman, Dijkstra and Kubik.

ON THE THEORETICAL ACCURACY OF PHOTOGRAMMETRIC VOLUME DETERMINATION

A. Introduction

Photogrammetric methods are used in volume estimation for open pit mining in the mining industry, and for volume estimation of water reservoirs in irrigation works. Height values of a set of terrain points or contour lines are determined by photogrammetric strip- and block triangulation and from these, the volume enclosed by the terrain surface and a reference level is estimated.

There are two error sources in this volume determination, namely photogrammetric triangulation and the terrain roughness. The influence of the photogrammetric triangulation on volume estimation is studied in this paper. This paper thus forms a continuation of earlier studies (Richardus, 1973 and 1976), where the influence of photogrammetric restitution and contouring was studied.

Besides the photogrammetric process, also the terrain roughness and unaccounted terrain details result in volume errors. This influence was studied by Botman, Dijkstra and Kubik 1975.

B. Accuracy of Photogrammetric Volume Determination by Strip Triangulation

Assume the volume has to be estimated for an area covered by a photogrammetric strip. The strip is triangulated and adjusted into errorfree control by a suitable adjustment method (rigorous least squares adjustment or suitable polynomial fitting). The terrain is assumed to be flat (terrain undulations $Z \ll$ flying height H). The volume between the terrain surface Z and the reference level zero is defined by $V = \iint\limits_{AB} Z dx dy$ and may be rewritten into $V = Z_{mean}.A.B$, with $Z_{mean} = \frac{1}{A.B}\iint Z.dx.dy$ denoting the mean height value, and A and B denote the extensions of the area (A striplength equal to n.b for strip of n models and baselength b; B strip width equal to 2b).

In order to study the volume accuracy, it suffices to study the standard deviation σ_{Zmean} of Z_{mean}, which may be estimated as the weighted arithmetic mean of the measured heights, the weights being assigned in proportion to the area of influence of the individual height points within the project area.

Result 1:

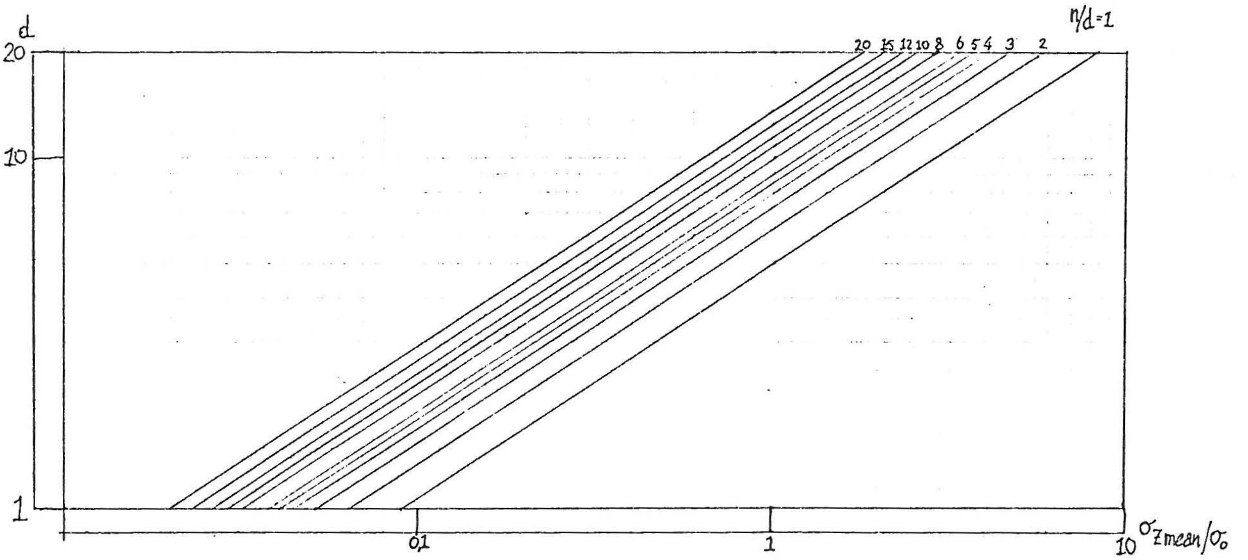
The standard deviation σ_{Zmean} of the volume error due to photogrammetric triangulation is equal to

$$\sigma_{Zmean} = \sqrt{\frac{1}{120}\,\frac{d^4}{n}}\cdot\sigma_o + \ldots \qquad (1)$$

$$d > 1$$

$$n \geq d$$

d denotes the number of models bridged and n the number of models in the strip. The quantity σ_o denotes the photogrammetric point measuring accuracy, usually $\sigma_o = 0.2\ {}^o/oo$ H. Only random errors of strip triangulation are considered.



Nomogram 1: Accuracy of volume estimation σ_{Zmean} for strip triangulation (n number of models in strip, d number of models between control).

Nomogram 1 gives the standard deviation σ_{Zmean} as function of d and n.

Outline of Proof: Limiting ourselves to the double summation of errors throughout the strip, it holds for the height values at the nadir points (cf. Jordan, Eggert, Kneissl, 1972).

$$Z_i = \sum_{k=1}^{i}\sum_{j=1}^{k} \Delta\,\varphi_j . b$$

i,j,k indices of nadir points, i,j,k = 1...n

$\Delta\,\varphi$ transfer error in strip triangulation

or

$$Z_{i-1} - 2Z_i + Z_{i+1} = b.\Delta\,\varphi_i .$$

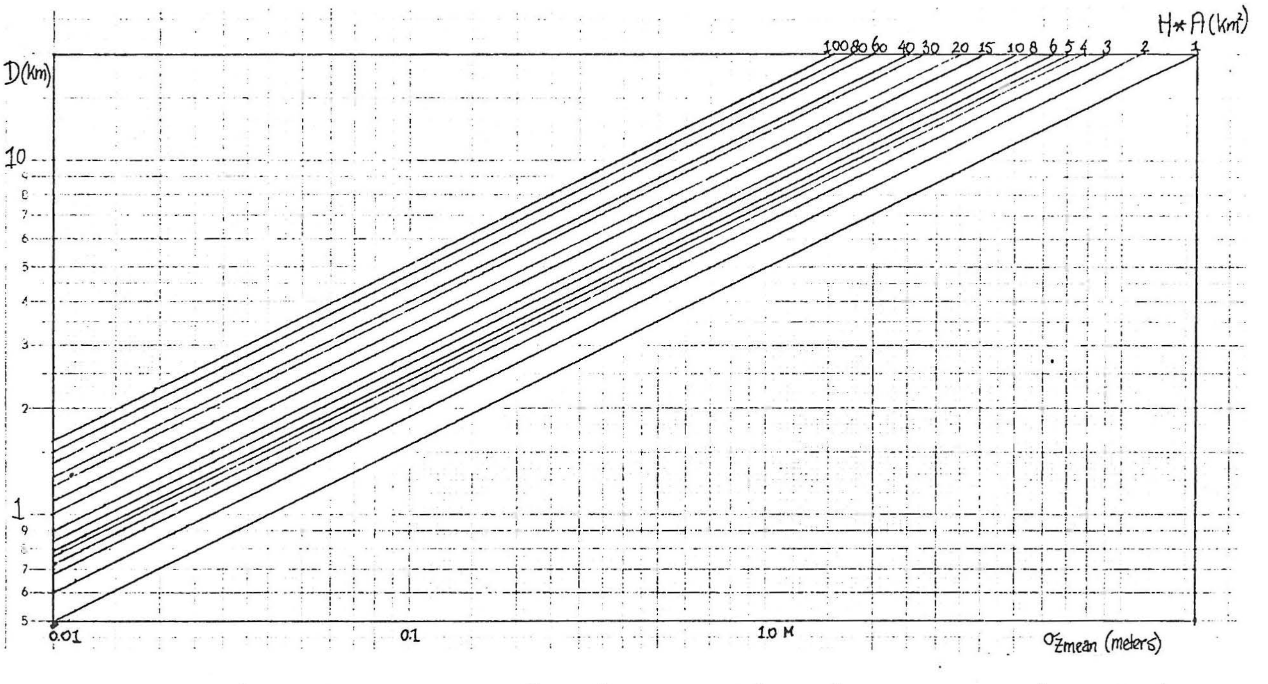
The true height value is assumed to be known at every d'th nadir point. Least squares adjustment of the strip into the control points results in a covariance matrix $Cov(Z_i, Z_j)$ of the adjusted (estimated) heights of the nadir points, which may be evaluated analytically (also cf. Ackermann, 1965). For adjustment according to standard problem II this covariance matrix is equal to the inverse normal equation matrix, N^{-1}. The variance of the value $Z_{mean} = \frac{1}{n} \sum_{i=1}^{n} Z_i$ may now be derived straightforward by computing the algebraic mean of all elements of N^{-1}. The derivation of (1) was done both analytically and numerically. In this derivation it is assumed that the errors at the nadir points are representative also for the surrounding of the points.

Result 2:

The accuracy σ_{mean} increases rapidly with decreasing bridging distance D (km) and increases slowly for the increasing flying height H (km)

$$\sigma_{Zmean} = 0.4 \times 10^{-5} . D^2 \sqrt{\frac{1}{H.A}} \qquad (2)$$

(all units in metric scale, wide angle photography)..



Nomogram 2: Accuracy of volume estimation σ_{Zmean} by strip triangulation; D bridging distance, H flying height, A strip length (A≥D; D≥H)

Nomogram 2 shows σ_{Zmean} as function of D and H.

Outline of Proof: By substitution of $\sigma_o = 0.2\ ^o/oo$ H and baselength $b = 0.6$ H the above expression results directly from (1).

Result 3:

The accuracy of volume estimation by free strip triangulation (full control of first model only) is equal to

$$\sigma_{Zmean} = \sqrt{\frac{1}{20} n^3} \; \sigma_o .$$

The proof proceeds along the same lines as the proof of result 1.

C. Accuracy of Volume Estimation by Block Triangulation

Assume that the volume has to be estimated for an area of length A and width B, covered by a photogrammetric block of strips with n models each.
Let the volume and Zmean be defined as above. It then holds

Result 4:

The standard deviation σ_{Zmean} of volume determination by block triangulation is equal to

$$\sigma_{Zmean} = \sqrt{\frac{1}{120} \frac{d^4}{n.m}} \; \sigma_o \qquad \begin{array}{l} d,m \geq 1 \\ n \geq d \end{array}$$

or, with $\sigma_o = 0.2$ ^{o}/oo H (wide angle photography)

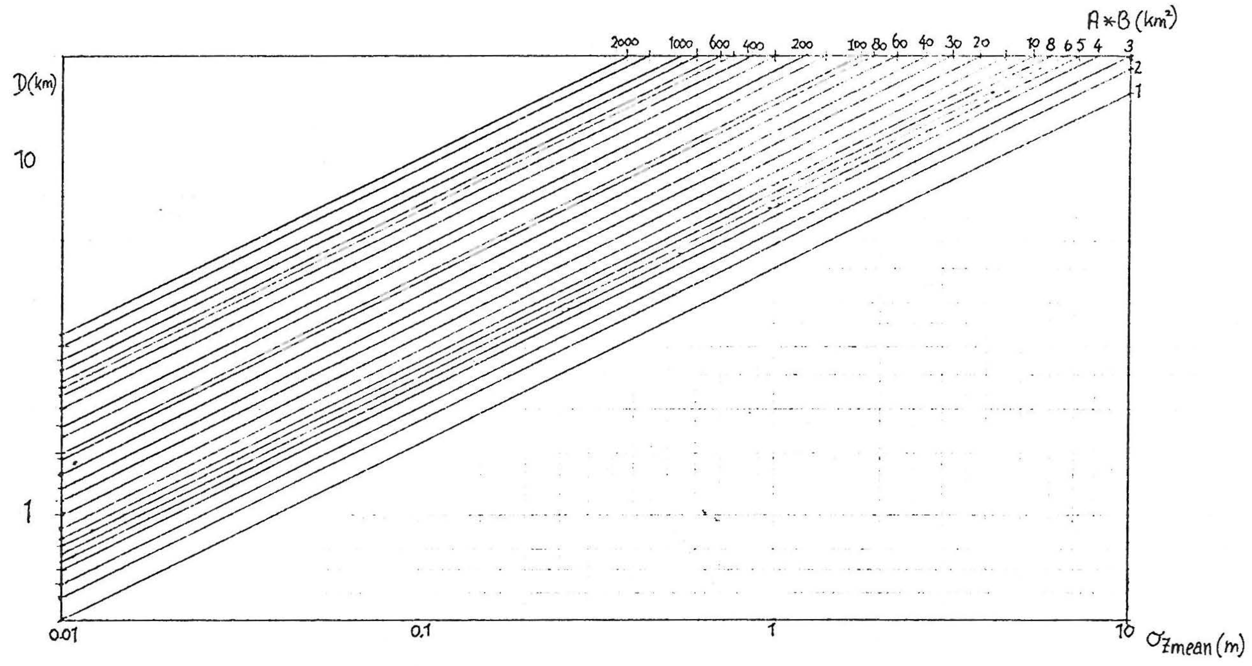
$$\sigma_{Zmean} = 4.3 \times 10^{-5} D^2 / \sqrt{A.B} \qquad \begin{array}{l} D \geq b \\ A \geq D \end{array}$$

(cf. Nomogram 3).

Outline of Proof: In order to compute Cov (Z_i, Z_j), the normal equation matrix $\bar{N}$ for least square block adjustment (standard problem II) has to be inverted. The matrix $\bar{N}$ is equal to

$$\begin{array}{cc} & \begin{array}{ccccc} \text{strip 1} & \text{strip 2} & \ldots\ldots & \text{strip n} & \end{array} \\ \bar{N} = & \begin{bmatrix} N+D & -D & & \\ -D & N+2D & -D & \\ & \ddots & \ddots & \\ & -D & N+2D & -D \\ & & -D & N+D \end{bmatrix} \end{array}$$

Here N denotes the normal equation matrix of strip adjustment and D a diagonal matrix representing tie point conditions. It proves that the arithmetic mean of the elements of $\bar{N}^{-1}$ is identical to the mean of the elements of N^{-1}, as may be analytically verified for m = 2,3 and the transition to m+1.



Nomogram 3: Accuracy of volume estimation σ_{Zmean} by block triangulation; D bridging distance, A and B extensions of area
($A \geq D$; $B \geq$ flying height H, $D \geq H$)

Thus the variance of Z_{mean} in block triangulation is equal to the variance Var(Zmean) in strip triangulation divided by n.

D. Discussion of the Results

The results prove that the volume accuracy strongly depends on the bridging distance D. Densification of control yields a significant accuracy improvement.
For strip triangulation the volume accuracy may also be improved by selecting a larger flying height, although the improvement is only proportional to $\sqrt{H}$. For a photogrammetric block, the volume accuracy becomes independent of the flying height and depends only on the bridging distance.

Note that when fitting individual strip sections into two bands of control each, and computing the average of Z over all the strip sections in the block, the same accuracy results (in absence of systematic errors) as after least squares block adjustment, viz.:
- Accuracy of linear fitting of strip into two bands of control (d=n)

$$\sigma_{Zmean} = \sqrt{\frac{1}{120} d^3} \ \sigma_o$$

- Accuracy of mean value of whole block

$$Zmean = \frac{1}{m.n/d} \sum_{\substack{strip\\sections}} Zmean$$

$$\sigma_{Zmean} = \sqrt{\frac{1}{120} \frac{d^4}{n.m}} \sigma_o$$

Thus volume accuracy is independent of the method of strip- and block adjustment, if proper methods are used. In particular, adjustment of the individual strips in a block suffice for volume estimation. This result is in agreement with earlier results (cf. Botman, Dijkstra, Kubik 1975) where also independence on the method could be proved. Rigorous (block-) adjustment methods are known to improve point accuracy, but at the same time they increase correlation so that the net result for volume determination is nil.

E. References

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