TESTFIELD AND SELF-CALIBRATION OF FIDUCIAL MARKS

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ABSTRACT

Many computer programs for adjustment of aerial triangulation and close-range photogrammetry allow for additional parameters to increase precision and accuracy. Analytical calibration is based on the collinearity equations estimates which have been extended to include parameters of interior orientation and radial and decentering distortion.In these equations image coordinates are observables. They are given in a camera coordinate system defined by fiducial marks. Comparator coordinates are transformed to the camera system before the calibration or triangulation computations begin. Fiducial marks are assumed to be known beforehand. But in this study the coordinates of the fiducial marks are estimated in the same adjustment as the estimation of esterior, interior, and distortion parameters. Equations are presented and possible singular cases are discussed. The conclusion is that testfield and self-calibration of fiducial marks would be possible.

KEY WORDS: Camera calibration, fiducial marks.

INTRODUCTION

Testfield and self-calibration of cameras serves two main purposes, namely, to check interior orientation of metric cameras under operational conditions, and to determine interior orientation parameters of non-metric cameras. Main interest has been paid to radial distortion and estimates of residual image coordinate errors such as standard error of unit weight, σ_0. It has often been of less interest to check the camera constant (principal distance) and the position of the principal point, because many photogrammetric processes are such that the effect on the result of errors of these entities is eliminated.

The position of the principal point is given in relation to the fiducial marks. Fiducial marks also check shrinkage of the emulsion base. They determine the scale of the interior orientation including the camera constant. But, as the effect of errors of principal point and of camera constant often is eliminated in the process, little attention has been paid to determination of fiducial marks in testfield and self-calibration.

Analytical calibration of cameras has now reached such a degree of refinement, and analytical multistation photogrammetry such a level of generality in network design, precision and reliability, that estimation of fiducial marks should be considered as an integral part of testfield and self-calibration. This paper is a theoretical study of the possibility to estimate fiducial marks, interior orientation, and distortion simultaneously.

ANALYTICAL CAMERA CALIBRATION

Analytical camera calibration is based on the colinearity equations which have been extended to include parameters to describe radial and decentering distortion. In case of aerial photography atmospheric refraction is also included in the mathematical model. The number of distortion parameters depend on the type of camera and how accurate one wants to model the geometric relations between image and object. The following equations are typical:

$$x' = x'_0 - c\,U / W +$$
$$+ (x' - x'_0)\,[\,k_3\,(r^2 - r^2{}_0) + k_5\,(r^4 - r^4{}_0) + \dots\,] +$$
$$+ [\,p_1 (r^2 + 2\,x'^2) + 2\,p_2\,x'\,y'\,]\,(\,1 + p_3 r^2 + p_4 r^4 + \dots\,)$$

$$y' = y'_0 - c\,V / W +$$
$$+ (y' - y'_0)\,[\,k_3\,(r^2 - r^2{}_0) + k_5\,(r^4 - r^4{}_0) + \dots\,] +$$
$$+ [\,2\,p_1 x'\,y' + p_2 (r^2 + 2\,y'^2)]\,(\,1 + p_3 r^2 + p_4 r^4 + \dots\,)$$

where

$$r^2 = (x' - x'_0)^2 + (y' - y'_0)^2$$

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \mathbf{R} \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$

X, Y, Z	object space coordinates
x', y'	image space coordinates
R	rotation matrix with three independent variables, e.g. ω, ϕ, κ
$\omega, \phi, \kappa, X_0, Y_0, Z_0$	exterior orientation of image
x'_0, y'_0	principal point
c	principal distance, or camera constant
r_0	radius for zero radial distortion
k_3, k_5	radial distortion parameters
p_1, p_2	decentering distortion parameters.

In test field calibration the object space points (X, Y, Z) are known. In self-calibration they are unknown and either estimated in the adjustment or eliminated from the equations.

THE PROBLEM OF FIDUCIAL TRANSFORMATIONS IN CALIBRATION

In these equations the image coordinates x',y' are regarded as observables, e.g. when the task is to calibrate an image, a set of images, or a camera. The axes of the coordinate system are parallel to the sides of the frame, its origin is usually in the centre of the image, and often coinciding with the principal point. Fiducial marks and their coordinates define the position (and scale) of the camera coordinate system. The location of the principal point (and the center of perspective) is given in relation to the fiducial marks. Images are measured with a comparator giving coordinates in the comparator system which then are transformed to the camera coordinate system using the fiducial marks as common points to determine the transformation parameters.

Before the calibration calculations can begin, the coordinates of measured image points are transformed from the comparator system to the camera system x',y' using "given" coordinates of the fiducial marks. Then the calibration procedure provides estimates of the unknown interior and distortion parameters. But how were the "given" values of the fiducial marks determined? They were not estimated simultaneously with the other interior orientation parameters. They were most likely determined in a separate procedure which preceded the calibration.

The coordinate transformation from comparator to camera system can be done with various transformations: congruent, similarity (Helmert), affine, projective, bilinear of varying order, conformal of varying order, or the like, depending on number and distribution of fiducial marks. Each combination of configuration of fiducial marks and transformation has its typical charateristics in terms of redundancy, precision, and inner and outer reliability. Generally speaking, a low relative redundancy gives a poor inner and outer reliability. This means that it is impossible or difficult to detect, locate and eliminate small gross errors or blunders in "given" or measured values, and that such small blunders distort the geometry of the transformed image coordinates.

An error in a "given" coordinate of a fiducial mark will distort all images in one and the same way. Such spurious distortion can be counteracted by the introduction of additional parameters in the calibration adjustment. It is e.g. common practise to apply an affine transformation so as to compensate for film shrinkage, which is assumed to cause lacking orthogonality and differences in scale along and across the film direction. In case of affine transformation on only four fiducial marks, the relative redundancy is 0.25 in each coordinate direction. For symmetric positions of the fiducials, all residuals will have the same absolute value but varying sign.

It can happen that small constant errors in the coordinates or small changes in the positions of the fiducials (blunders of "given" values) are interpreted as affin film shrinkage. In the later stages of the photogrammetric process this leads to discrepancies in the observation equations for calibration and absolute orientation. Relative orientation and self-calibration are not affected as long as the camera axes are parallel for all images concerned, but as soon as κ is not equal to 0 or π, the fiducial errors will cause discrepancies. These effects will lead to an increase of the residual variance, unless additional affine parameters are not included in the functional part of the mathematical model.

The idea of testfield and self-calibration is to estimate the parameters of interior orientation and distortion under real working conditions of the imaging system. To follow this line of thought we should estimate also the coordinates of the fiducial marks under working conditions. Another reason for testfield and self-calibraton is to check the stability of the camera and the correctness of available calibration parameters. For some types of cameras the fiducial marks can move relative to each other, e.g. aerial cameras with projected fiducials. For this type of cameras testfield and self-calibration certainly should include fiducial coordinates as unknowns. In other cases, like cameras with a reseau plate, the fiducials are not likely to move relative to each other.

In on-the-job calibration the elements of interior orientation and distortion are estimated simultaneously with the adjustment of the observations on the estimation of the unknowns in the project itself. Here it is a matter of finding the "best" fitting functional model between object and image space coordinates. This is very different from the case of pre-calibration, where parameters of the interior orientation and distortions are determined in a procedure separate from the photogrammetric mensuration. In pre-calibration it is very important that the mathematical model relates to a physical model of the imaging geometry. The parameters should be interpretable and separable. In our case above with small blunders in "given" fiducial coordinates as example, we must be able to separate the effect of fiducial errors from affine film shrinkage. Or in other words, any additional parameters in the adjustment must have a physical explanation, and the calibration method must be able to separate different physical causes from each other. Our task is to calibrate the camera for the future, not to determine the shrinkage of the film used in the calibration. The camera is stable, the shrinkage varies. Thus, we have to separate the film shrinkage form the effect of fiducial coordinates.

EQUATIONS FOR FIDUCIAL CALIBRATION

We will now introduce the fiducial coordinates as unknowns in the calibration. We will use affine transformation in the following, but it is very easy to exchange it for any transformation where the image coordinates are explicit functions of the measured comparator coordinates. The measured comparator coordinates are denoted (x, y), image coordinates are (x', y'), measured fiducials are (x_r, y_r), and "given" fiducial are (x'_r, y'_r).

For object space points we have

$$x' = x'_0 - c\,U/W +$$

$$+ (x' - x'_0)\,[\,k_3\,(r^2 - r^2{}_0) + k_5\,(r^4 - r^4{}_0) + \ldots\,] \; +$$

$$+ [\,p_1(r^2 + 2x'^2) + 2\,p_2\,x'\,y'\,]\,(1 + p_3 r^2 + p_4 r^4 + \ldots)$$

$$y' = y'_0 - c\,V/W +$$

$$+ (y' - y'_0)\,[\,k_3\,(r^2 - r^2{}_0) + k_5\,(r^4 - r^4{}_0) + \ldots\,] \; +$$

$$+ [\,2\,p_1 x'\,y' + p_2(r^2 + 2y'^2)]\,(1 + p_3 r^2 + p_4 r^4 + \ldots)$$

and

$$x' = a_1 + a_2\,x + a_3\,y$$

$$y' = b_1 + b_2\,x + b_3\,y$$

where a_1 , a_2 , a_3 , b_1 , b_2 , b_3 are transformation coefficients to be estimated. The four above equations are combined to the following functions

$$F1 = a_1 + a_2\,x + a_3\,y$$

$$- x'_0 + c\,U/W$$

$$- (x' - x'_0)\,[\,k_3\,(r^2 - r^2{}_0) + k_5\,(r^4 - r^4{}_0) + \ldots\,]$$

$$- [\,p_1(r^2 + 2x'^2) + 2\,p_2\,x'\,y'\,]\,(1 + p_3 r^2 + p_4 r^4 + \ldots)$$

$$F2 = b_1 + b_2\,x + b_3\,y$$

$$- y'_0 + c\,V/W$$

$$- (y' - y'_0)\,[\,k_3\,(r^2 - r^2{}_0) + k_5\,(r^4 - r^4{}_0) + \ldots\,]$$

$$- [\,2\,p_1 x'\,y' + p_2(r^2 + 2y'^2)]\,(1 + p_3 r^2 + p_4 r^4 + \ldots)$$

For comparator measurements of fiducials we get

$$F3 = a_1 + a_2\,x_r + a_3\,y_r - x'_r$$

$$F4 = b_1 + b_2\,x_r + b_3\,y_r - y'_r$$

where (x'_r, y'_r) are unknowns to be estimated.

As we now have introduced the fiducial coordinates as unknowns , we have to define the datum for their coordinate system. This can be done with the equations

$$F5 = \underline{x}'_r - x'_r$$

$$F6 = \underline{y}'_r - y'_r$$

where $(\underline{x}'_r, \underline{y}'_r)$ are approximate values of the fiducials to be calibrated.

To avoid singular cases in the least squares solution of the over-determined linearized equation system, we can add equations of approximate values for the unknowns which are expected to have coefficients that are linear dependent on other coefficients. These equations can represent ficticious observations on the unknowns, saying that we have guessed (measured with very low precision) that the corrections to these unknowns are zero. Aerial photographs of testfields in flat terrain would, as example, require approximations (ficticious observations) for the camera constant (principal distance) and principal point.

$$F7 = \underline{x}'_0 - x'_0$$

$$F8 = \underline{y}'_0 - y'_0$$

$$F9 = \underline{c} - c$$

where the underlined variables denote the approximations. This group of functions can and should include also ficticious observations on the radial and decentering distortion parameters.

LINEARIZATION

The functions F are linearized around approximate values of the unknowns. The partial derivatives with respect to the observables $\delta F/\delta l$ are collected in matrix **A**. They are the coefficients of the corrections **v** (residuals) to the observables. The partial derivatives with respect to the unknowns $\delta F/\delta x$ are collected in matrix **B**. They are the coefficients of the corrections **x** to the approximate values of the unknowns. The structure of the matrices **A** and **B** are shown in Fig. 1 and 2 for the case of five images of a testfield with known geodetic coordinates of all 10 object space points. All images are taken with a camera that has eight fiducial marks. The adjustment illustrated in the Figures includes also interior and distortion parameters as unknowns.

The system of linerized observation equations is in matrix and vector notation

$$\mathbf{A}\mathbf{v} + \mathbf{B}\mathbf{x} = \mathbf{w}$$

where **v** are the residuals, **x** the corrections to the unknowns and **w** the discrepancies. The equations are given weights which are shown in the diagonal matrix **P**. The normal equations for the least squares solution have a structure as shown in Fig. 3. The least squares estimator of **x** is

$$\mathbf{x} = [\,\mathbf{B}^* \,(\mathbf{A}\,\mathbf{P}^{-1}\mathbf{A}^*)^{-1}\,\mathbf{B}\,]^{-1}\,\mathbf{B}^*\,(\mathbf{A}\,\mathbf{P}^{-1}\mathbf{A}^*)^{-1}\,\mathbf{w}$$

The variance - covariance of the estimates of the unknowns **x** is estimated by

$$\mathbf{C_{xx}} = [\,\mathbf{B}^*\,(\mathbf{A}\,\mathbf{P}^{-1}\mathbf{A}^*)^{-1}\,\mathbf{B}\,]^{-1}$$

If the images are placed in the comparator in such a way that the axes of the comparator (x,y) and the camera (x', y') (image coordinates) are parallel, then $a_2 = 1$, $a_3 = 0$, $b_2 = 0$ and $b_3 = 1$. After studying the partial derivatives with respect to the observables, we will find that the matrix $\mathbf{A} = \mathbf{I}$, and then $(\mathbf{A}\,\mathbf{P}^{-1}\,\mathbf{A}^*)^{-1} = \mathbf{P}$. The adjustment is thus in this case reduced to the common adjustment by elements.

$$\mathbf{B}\mathbf{x} = \mathbf{w} - \mathbf{v} \quad \text{with weight } \mathbf{P}.$$

The vector of unknowns **x** can be divided into four parts

$\mathbf{x} = (\mathbf{x_a}, \mathbf{x_e}, \mathbf{x_f}, \mathbf{x_i})^*$

$\mathbf{x_a}$ affine transformation parameters, 6 for each image

$\mathbf{x_e}$ exterior orientation, 6 for each image

$\mathbf{x_f}$ fiducial coordinates, 2 for each fiducial point

$\mathbf{x}_i$ interior orientation and distortion parameters, one set for the camera. The vector of discrepancies can be divided into four groups, too,
$\mathbf{w} = (\mathbf{w}_p, \mathbf{w}_r, \mathbf{w}_d, \mathbf{w}_c)^*$
$\mathbf{w}_p$ F1, F2: image observations on object points
$\mathbf{w}_r$ F3, F4: image observations on fiducial marks
$\mathbf{w}_d$ F5, F6: ficticious observations on approximate fiducial marks
$\mathbf{w}_c$ F7, F8, F9: ficticious observations on interior orientation and distortion parameters. The matrix **B** is composed by the corresponding submatices. The structure is shown in Fig. 2.

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_{pa} & \mathbf{B}_{pe} & \mathbf{B}_{pf} & \mathbf{B}_{pi} \\ \mathbf{B}_{ra} & \mathbf{B}_{re} & \mathbf{B}_{rf} & \mathbf{B}_{ri} \\ \mathbf{B}_{da} & \mathbf{B}_{de} & \mathbf{B}_{df} & \mathbf{B}_{di} \\ \mathbf{B}_{ca} & \mathbf{B}_{ce} & \mathbf{B}_{cf} & \mathbf{B}_{ci} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{pa} & \mathbf{B}_{pe} & \mathbf{0} & \mathbf{B}_{pi} \\ \mathbf{B}_{ra} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} \end{pmatrix}$$

SOME INTERESTING MATRIX ELEMENTS

Of particular interest to us are now the elements of the matrix **B** that are related to the estimation of fiducial marks coordinates. Of immediate concern are $\mathbf{B}_{pa}$, $\mathbf{B}_{pi}$, $\mathbf{B}_{ra}$, $\mathbf{B}_{rf}$, $\mathbf{B}_{df}$, $\mathbf{B}_{ci}$.

Matrix $\mathbf{B}_{pa}$ is composed of n hyperdiagonal submatrices, where n is the number of images, i. Each submatrix has six columns, and 2*m rows, where m is the number of object points, j, measured in that image. The columns are ordered as a_1 ,a_2 , a_3, b_1, b_2 , b_3 . Assuming radial and decentering distortion to be zero, the elements of a pair of rows for one point are

$$\begin{matrix} 1 & x_{ij} & y_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{ij} & y_{ij} \end{matrix}$$

Matrix $\mathbf{B}_{pe}$ is composed of n hyperdiagonal submatrices, where n is the number of images, i. Each submatrix has six columns, and 2*m rows, where m is the number of object points, j, measured in that image. The elements are the classical ones for single point resection in space.

Matrix $\mathbf{B}_{pi}$ is a full matrix with n submatrices, where n is the number of images, i. Each submatrix has 2*m rows, where m is the number of object points, j, measured in that image. The number of rows depend on the number of interior orientation and distortion parameters. The columns begin with elements of x'_0 , y'_0 , c. Then follows distortion parameter elements. The first three columns for a pair of rows are

$$\begin{matrix} -1 & 0 & U/W \\ 0 & -1 & V/W \end{matrix}$$

The matrix $\mathbf{B}_{ra}$ has the same structure as $\mathbf{B}_{pa}$, the only difference is that fiducial marks, r, are measured instead of object points, j. The number of rows of the submatrices are twice the number of fiducial marks measured. The elements of a pair of rows are

$$\begin{matrix} 1 & x_{ir} & y_{ir} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_{ir} & y_{ir} \end{matrix}$$

The matrix $\mathbf{B}_{rf}$ and $\mathbf{B}_{ci}$ are both diagonal matrices and $\mathbf{B}_{rf} = -\mathbf{I}$ and $\mathbf{B}_{ci} = -\mathbf{I}$.

SINGULAR CASES

The interesting question is now, whether there are any columns of **B** that are linear dependent. The linear combinations between photo stations and the principal point and principal distance, when 2-D testfields are used, is well known. That singularity can be avoided by multiple station convergent photography with different combinations of X_0, Y_0, Z_0, ϕ, ω, κ, or by using a 3-D testfield.

However, in our case of fiducial calibration, we have to be especially suspicious against combinations of coefficients of fiducial marks, principal point, and the constant term of the affine transformation. We find

$$\Sigma(\delta F1/\delta a_1)_j + \delta F1/\delta x_0' + \Sigma(\delta F3/\delta a_1)_j + \Sigma\Sigma(\delta F3/\delta x'_r)_j + \Sigma(\delta F5/\delta x'_r) + \delta F7/\delta x_0' = \Sigma(\delta F5/\delta x'_r) + \delta F7/\delta x_0'$$

$$\Sigma(\delta F2/\delta b_1)_j + \delta F2/\delta y_0' + \Sigma(\delta F4/\delta b_1)_j + \Sigma\Sigma(\delta F4/\delta y'_r)_j + \Sigma(\delta F6/\delta y'_r) + \delta F8/\delta y_0' = \Sigma(\delta F6/\delta y'_r) + \delta F8/\delta y_0'$$

The index j denotes an image and index r a fiducial mark. The summations are always over the letter index in question. It is evident that the equations F5, F7 and F6, F8 are necessary for a solution. Without them the above linear combinations would always be zero, and the system singular.

À PRIORI WEIGHTS

The weights of the linearized observation equations are by definition inversely proportional to the variance of the observables. We need à priori estimates of these variances. To find such estimates we should have some idea of a variance component model, a stochastic model. Such a model is presented by Torlegård 1989.

The observations in F1, F2 are x, y and their variance components comprise effects from testfield coordinates, target excentricity, not corrected and irregular athmospheric refraction, radial and tangential distortion, film flatness, film shrinkage, comparator and operator. This variance can vary from $(2\ \mu m)^2$ to $(10\ \mu m)^2$.

The observations in F3, F4 are x_r, y_r and their variance components comprise effects from film shrinkage, comparator and operator, and sometimes film flatness. This variance can vary from $(1\ \mu m)^2$ to $(9\ \mu m)^2$.

The observations in F5, F6 are the approximations to define the datum of the fiducial coordinate system. In this case the variance should reflect how good our approximations are, what differences do we expect between the approximation values and the final result. For a 9" aerial camera the absolute value of the fiducial coordinates would be 110 mm. As we do not like this value to influence the result to much, we should say that our approximation has a variance of e.g. (1 mm)^2 .

The observations in F7, F8, F9 are the approximations of the principal point and principal distance. In this case the variance has to describe how well we have guessed the values. For a single vertical photograph of a completely flat test field we can use any value, because of the full correlation between interior and exterior orientation elements. The à posteriori variance of the estimates of the unknowns will be the same as the à priori variance used for the weighting. There is no other information to separate the interior form the exterior orientation elements. Usually our guessing of principal distance and principal point would correspond to a variance of (1 - 3 mm)^2.

SELF-CALIBRATION

In self-calibration the coordinates of the object space points (X, Y, Z) are unknown. They can be estimated in the adjustment simultaneously with the calibration. When all object space points are unknown the datum has to be defined. One way of doing this is to select two planimetric and three elevation control points and let them define the datum. No discrepancies are introduce by this, and the calibration result is independent of the choice of control point combination. Another way to define the datum is to make a free net adjustment. A third way could be to introduce ficticious observations on all object point coordinates with a very low weight. This third possibility has the advantage that it can be used when the calibration images contain a mixture of both given and unknown points. The given ones are then given a higher weight corresponding to the variance of their precision, while the unknown points have lower weights which correspond to the difference between the approximate values and the final result.

CONCLUSION

This study shows that it is possible to include the estimation of coordinates of fiducial marks simultaneously with estimation of interior orientation and distortion. The adjustment has to include equations that determine the position of the origin and the direction of axes of the camera coordinate system, so as to neutralize and avoid the linear dependency between coefficients for principal point, for fiducial marks, and for translation parameters in the fiducial coordinate transformation. The expected precision of estimates of fiducial coordinates will be studied for various testfields, cameras and photogrammetric network design.

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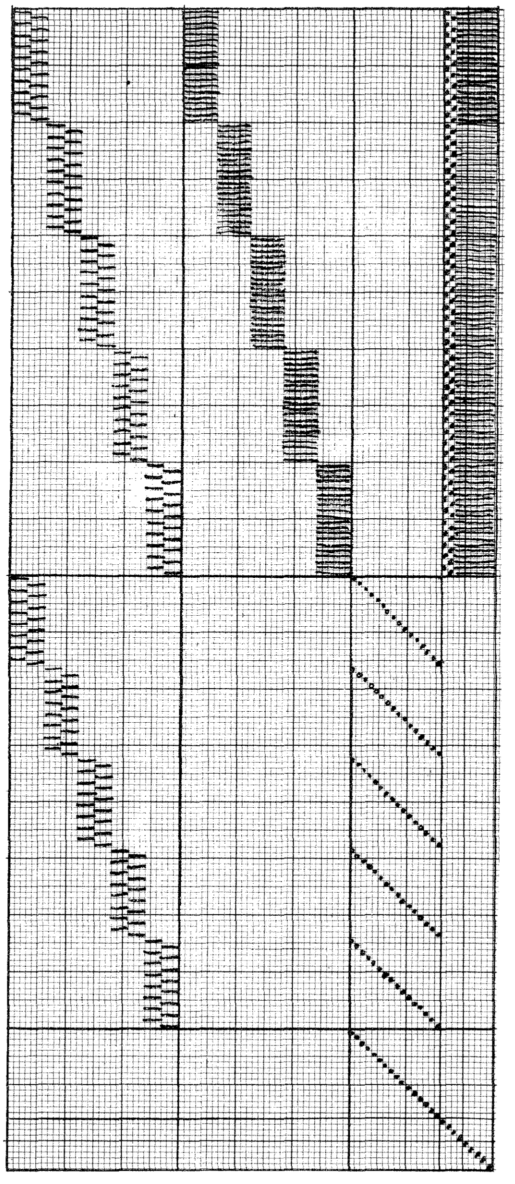

Fig. 2. Structure of matrix **B** with coefficients of the corrections, **x**, to the unknowns. From left to right: affine transformation parameters, exterior orientation, fiducial coordinates, interior orientation and distortion parameters.

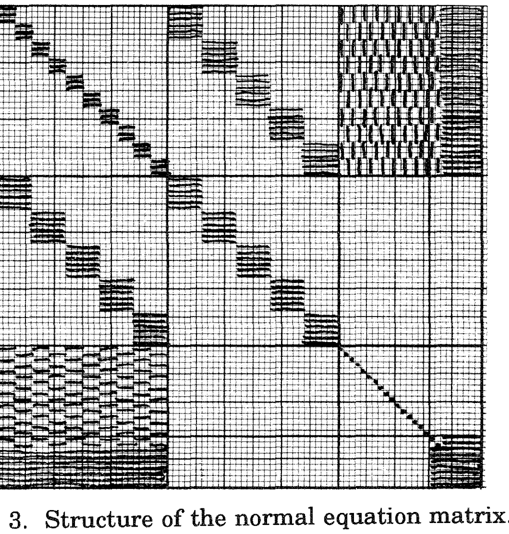

Fig. 3. Structure of the normal equation matrix.

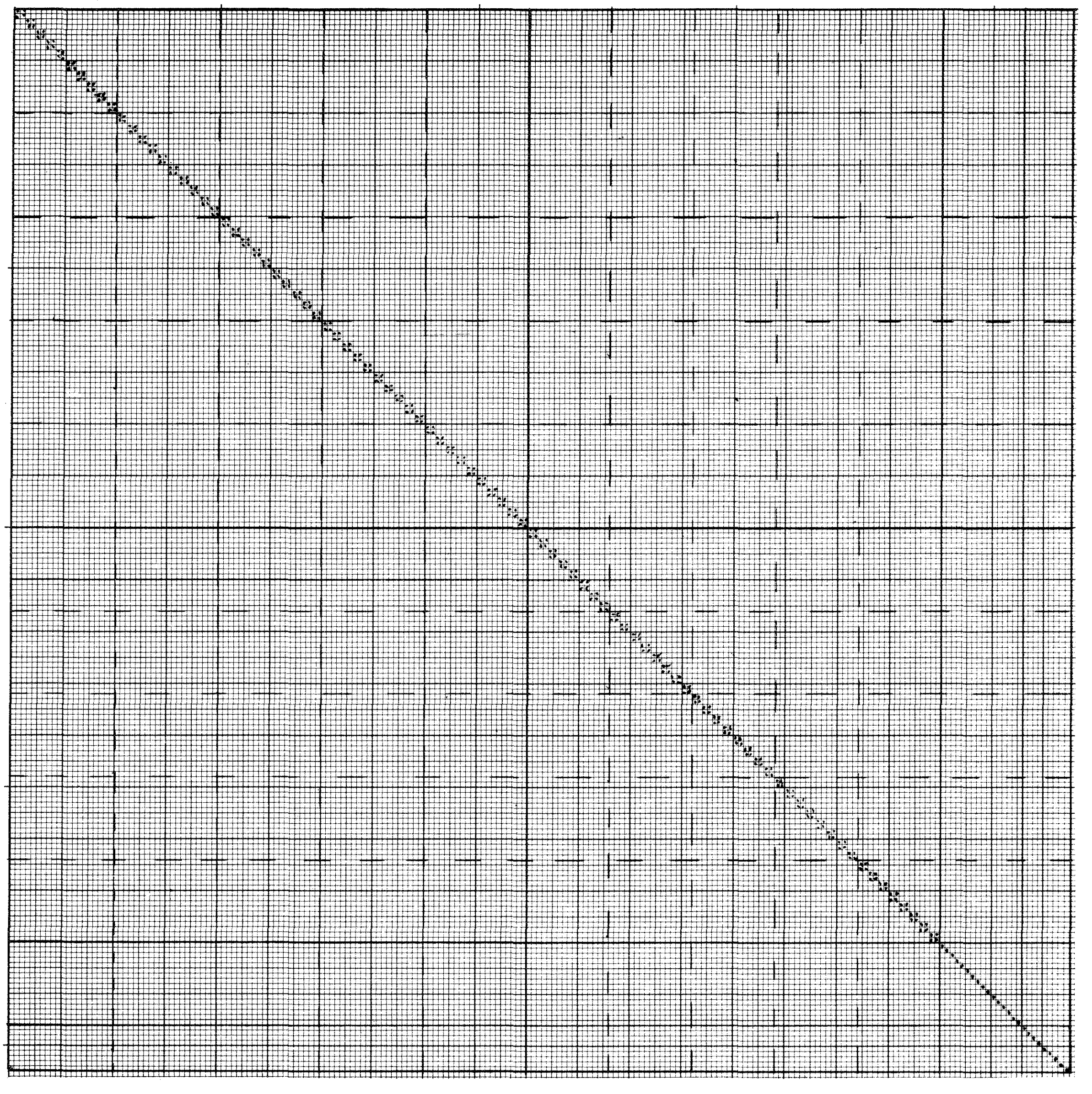

Fig. 1. Structure of matrix **A** with coefficients of the corrections, **v**, to the observables, **l**.