ATMOSPHERIC CORRECTION OF ANGULAR MEASUREMENTS ABOVE AN INHOMOGENEOUS AND NON-LAMBERTIAN SURFACE

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ABSTRACT

This paper discusses a method for solving the direct and reverse atmosphere optics problem for the atmosphere bounded by an inhomogeneous, non-Lambertian underlying surface. The method involves construction of the atmosphere optical transfer operator. For the Lambertian surface space-frequency response (SFR) is the kernel of the transfer operator, while for the non-Lambertian surface complex Green's function is the kernel of the transfer operator. A method to define surface reflectance using angular and spectral measurements of the upward radiation has been developed. For the known reflection indicatrices the surface albedo can be identified using measurements of the upward radiation in a single direction. With the absence of information about the surface reflectance a method of angular measurements to simultaneously identify reflection indicatrix and surface albedo is used.

Key words: inhomogeneous and non-Lambertian surface, optical transfer operator, reverse problem.

1. INTRODUCTION

Non-Lambertianity effects have to be taken into account to provide more accurate interpretation of the remote measurements from satellites. This problem has been addressed in many papers. In paper [1] non-Lambertianity problems are being addressed by means of solving an equation of radiation transfer in the atmosphere bounded by a non-Lambertian but homogeneous surface. In papers [2,3] the authors use two-dimensional Fourier transform to solve three-dimensional transfer equation for the atmosphere bounded by an inhomogeneous and non-Lambertian surface.

The method of solving reverse problem of atmosphere optics developed in this paper is based upon optical transfer operator theory. Paper (4) is devoted to construction of the optical transfer operator in the three-dimensional inhomogeneous atmosphere bounded by an inhomogeneous but Lambertian underlying surface. This involves the use of perturbation theory technique and Fourier transform in terms of horizontal coordinates. As a result instead of the original three-dimensional boundary problem of the transfer theory, one-dimensional problems for the SFR being the kernel of the optical transfer operator have to be solved. In papers [5,6] the results obtained in paper [4] are generalized to the equation of transfer in the atmosphere bounded by a non-Lambertian surface. On the assumption, however, that the reflection operator is factorized by angular and space coordinates. Solution of the transfer equation for arbitrary reflection is discussed in paper [7].

Method of the optical transfer operator enables to solve reverse problems of atmosphere optics since the operator connects upward radiation intensity and optical characteristics of the surface. An approach to solving reverse problems based upon measurements of the upward radiation intensity in many directions is discussed in papers [8,9]. Here atmosphere is assumed to be horizontally homogeneous and vertically inhomogeneous bounded by an inhomogeneous but Lambertian surface.

In this paper we solve a reverse problem for an inhomogeneous and non-Lambertian surface. However, we are supposed to have the information about atmosphere. The method is based upon the optical transfer operator theory for the atmosphere bounded by an inhomogeneous and non-Lambertian surface. Section 2 contains an outline of this theory.

2. OPTICAL TRANSFER OPERATOR FOR THE ATMOSPHERE BOUNDED BY AN INHOMOGENEOUS AND NON-LAMBERTIAN SURFACE

Consider plane-parallel vertically inhomogeneous anisotropically scattering atmosphere $-\infty \le x,y \le \infty$, $0 \le z \le h$, illuminated by a solar flux. Radiation intensity $I(r,s)$, depending on the space $r=\{x,y,z\}$ and angular $s=\{\mu,\varphi\}$ coordinates, satisfies the boundary problem

$$\begin{cases} \hat{D}I = \hat{S}I, \\ I|_{z=0},\ s\in\Omega^+ = \pi S_\lambda \delta(s-s_0), \\ I|_{z=h},\ s\in\Omega^- = \hat{R}I, \end{cases} \qquad (1)$$

where

$$\hat{D}I = \mu\frac{\partial I}{\partial z} + \sin\theta\cos\varphi\frac{\partial I}{\partial x} + \sin\theta\sin\varphi\frac{\partial I}{\partial y} + \sigma_t(z)I$$

is a transfer operator;

$\hat{S}I = \sigma_s(z)\int_\Omega I(r,s')\gamma(z,s,s')ds'$ is a collision integral,

$\gamma(z,s,s')$ is a scattering indicatrix; $ds' = d\mu' d\varphi'$;

$\hat{R}I = \frac{1}{\pi}\int_{\Omega^+} I(r,s')R(r_\perp,s,s')\mu' ds'$ is a reflection operator,

$R(r_\perp,s,s')$ is a reflection factor $(r_\perp=\{x,y\})$.

A set of all directions $s=\{\mu,\varphi\}$ constitutes a unit sphere $\Omega \equiv [-1,1]\times[0,2\pi]$ with $\mu\in[-1,1]$ and $\varphi\in[0,2\pi]$; $\Omega^+ \equiv [0,1]\times[0,2\pi]$ and $\Omega^- \equiv [-1,0]\times[0,2\pi]$ are semispheres for the directions of the downward and upward radiation propagation. Solar radiation propagates in the direction $s_0=\{\mu_0,\varphi_0\}$. The boundary problem involves coefficients of extinction $\sigma_t(z)$ and scattering $\sigma_s(z)$.

In problem (1) single out direct non-scattered solar radiation and atmosphere haze

$$I = I_0 + D + \Phi,$$

where $I_0(\tau,s) = \pi S_\lambda \delta(s-s_0)\exp[-\tau(z)/\mu_0]$,

$\tau(z) = \int_0^z \sigma_t(z')dz'$ is the optical thickness.

Atmosphere haze $D(\tau,s)$ satisfies the boundary problem

$$\begin{cases} \mu\frac{\partial D}{\partial \tau} + \omega(\tau)D = \hat{S}D + \hat{S}I_0, \\ D|_{\tau=0},\ s\in\Omega^+ = 0, \\ D|_{\tau=\tau_h},\ s\in\Omega^- = 0, \end{cases} \qquad (2)$$

$\omega(\tau) = \sigma_s(\tau)/\sigma_t(\tau)$ is a single scattering albedo, $\tau_h = \tau(h)$.

The inhomogeneous component of the radiation field is found from the problem

$$\begin{cases} \hat{D}\Phi = \hat{S}\Phi, \\ \Phi|_{z=0}, \ s\in\Omega^+ = 0, \\ \Phi|_{z=h}, \ s\in\Omega^- = \hat{R}\Phi + \hat{R}[D + I_0]. \end{cases} \tag{3}$$

Allow constraints. Flat surface $\{x,y\}$ can be broken up into the finite number of sub areas $Q_i \subset R^2$ ($\cup Q_i = R^2$, $Q_i \cap Q_j = 0$, $i\neq j$). Each of then satisfies condition [7]

$$R(r_\perp, s, s') = q_j(r_\perp)R_j(s,s'), \quad r_\perp \in Q_j,$$

i.e. we can single out surface sections using the same reflection indicatrix. Let's extend each function $q_j(r_\perp)$ beyond Q_j area: $q_j(r_\perp) \equiv 0$, $r_\perp \notin Q_j$.

Consider a set of boundary problems for functions $\Phi_j(r,s)$:

$$\begin{cases} \hat{D}\Phi_j = \hat{S}\Phi_j, \\ \Phi_j|_{z=0}, \ s\in\Omega^+ = 0, \\ \Phi_j|_{z=h}, \ s\in\Omega^- = q_j\hat{R}_j\Phi_j + q_jE_j(s), \end{cases} \tag{4}$$

where

$$\hat{R}_j\Phi_j = \frac{1}{\pi}\int_{\Omega^+} \Phi_j(r,s')R_j(s,s')\mu' ds',$$

$$E_j(s) = \hat{R}_j[D + I_0]. \tag{5}$$

Replacing function $\Phi(r,s)$ by the sum

$$\Phi(r,s) = \sum_{j=1}^{J} \Phi_j(r,s),$$

we neglect photons having being reflected from area Q_j, and subsequently reflected from area Q_i ($i\neq j$). The share of such photons is obviously small as compared with the total radiation. Thus, instead of solving initial boundary problem for the arbitrary reflection law we solve several problems for the reflection function being factorized by angular and space coordinates. By analogy with the Lambertian surface function $q_j(r_\perp)$ will be referred to as a surface albedo. The normalization condition for the reflection indicatrix $R_j(s,s')$ is written

$$\frac{1}{\pi}\int_{\Omega^-} R_j(s,s')\mu ds = 1.$$

Solution of problem (4) for the arbitrary reflection law is described in book [5]. The case for Lambertian surface is discussed in paper [4]. Henceforth index j corresponding to a certain sub area will be omitted in boundary problems. We shall make use of the perturbation theory technique [4].

We shall designate the surface albedo as

$$q(r_\perp) = \bar{q} + \varepsilon\tilde{q}(r_\perp) \le 1,$$

where $\bar{q}$ is the mean albedo, ε is the small parameter, $\tilde{q}(r_\perp)$ is albedo variations, and expand function Φ into a series in powers of ε

$$\Phi(r,s) = \sum_{k=1}^{\infty} \varepsilon^k \Phi_k(r,s).$$

Using Fourier transform

$$\overset{v}{f}(p) = F[f](p) = \int_{-\infty}^{\infty} f(r_\perp)e^{i(p,r_\perp)}dr_\perp,$$

$$(p,r_\perp) = p_x x + p_y y, \quad dr_\perp = dxdy,$$

the initial three-dimensional boundary problem is reduced to a parametric set of one-dimensional boundary problems for the SFR of a surface with non-Lambertian reflection [5]. SFR of a single scattering satisfies the boundary problem

$$\begin{cases} \mu\dfrac{\partial W_1}{\partial z} + [\sigma_t(z) - i(p,s_\perp)]W_1 = \hat{S}W_1, \\ W_1|_{z=0}, \ s\in\Omega^+ = 0, \\ W_1|_{z=h}, \ s\in\Omega^- = E(s), \quad \text{where} \end{cases} \tag{6}$$

$$p = \{p_x, p_y\}, \quad (p,s_\perp) = p_x\cos\varphi\sin\theta + p_y\sin\varphi\sin\theta.$$

SFR of order k is expressed recurrently in terms of SFR of order $k-1$

$$\begin{cases} \mu\dfrac{\partial W_k}{\partial z} + [\sigma_t(z) - i(p,s_\perp)]W_k = \hat{S}W_k, \\ W_k|_{z=0}, \ s\in\Omega^+ = 0, \\ W_k|_{z=h}, \ s\in\Omega^- = \hat{R}W_{k-1}. \end{cases}$$

In case of a Lambertian surface, SFR is

$$W(z,p,s) = W_1(z,p,s) = \ldots = W_k(z,p,s)\ldots$$

and satisfies the boundary problem [4]

$$\begin{cases} \mu\dfrac{\partial W}{\partial z} + [\sigma_t(z) - i(p,s_\perp)]W = \hat{S}W, \\ W|_{z=0}, \ s\in\Omega^+ = 0, \\ W|_{z=h}, \ s\in\Omega^- = 1. \end{cases} \tag{7}$$

SFR $W(z,p,s)$ does not depend on the properties of the underlying surface and presents an atmosphere transfer function.

SFR $W_1(z,p,s)$ satisfying the boundary problem (6) can be expressed in terms of Green's function $W_\delta(z,p,s,s_s)$ [5]

$$W_1(z,p,s) = \int_{\Omega^-} E(s_s)W_\delta(z,p,s,s_s)ds_s, \tag{8}$$

where $s_s = \{\mu_s, \varphi_s\}$, $ds_s = d\mu_s d\varphi_s$,

that satisfies boundary problem

$$\begin{cases} \mu\dfrac{\partial W_\delta}{\partial z} + [\sigma_t(z) - i(p,s_\perp)]W_\delta = \hat{S}W_\delta, \\ W_\delta|_{z=0}, \ s\in\Omega^+ = 0, \\ W_\delta|_{z=h}, \ s\in\Omega^- = \delta(s-s_s) \end{cases} \tag{9}$$

and does not depend on the area Q_j.

SFR of the atmosphere bounded by a Lambertian surface $W(z,p,s)$ is related to Green's function $W_\delta(z,p,s,s_s)$ as

$$W(z,p,s) = \int_{\Omega^-} W_\delta(z,p,s,s_s)ds_s. \tag{10}$$

Solution of problem (4) in approximation of a single reflection from the surface is written

$$\Phi_{1j}(r_\perp,s) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} W_{1j}(z,p,s)\overset{v}{q}_j(p)e^{-i(p,r_\perp)}dp, \tag{11}$$

$$dp = dp_x dp_y.$$

The final solution of the problem we found by the formula

$$\Phi_1(r_\perp,s) = \frac{1}{(2\pi)^2}\sum_{j=1}^{J}\iint_{-\infty}^{\infty} W_{1j}(z,p,s)\overset{v}{q}_j(p)e^{-i(p,r_\perp)}dp. \tag{12}$$

Thus, solution of the boundary problem (1) reduces to solution of the boundary problems (2) and (9) that do not depend on the surface reflectance. Then function $E_j(s)$ and SFR $W_{1j}(z,p,s)$ are evaluated for each reflection indicatrix

$R_j(s,s')$ by expression (8). Radiation intensity variations are evaluated for each sub area using (11), while the final solution is found as a superposition of the special cases.

Expression (12) presents the transfer operator of the atmosphere bounded by an inhomogeneous, non–Lambertian underlying surface that connects radiation intensity in approximation of a single reflection from the surface albedo.

If we present the upward radiation SFR $W_{1j}(z,p,s)$ as a sum of non–scattered and scattered components

$$W_{1j}=E_j(s)\exp\{-[\tau(h)-\tau(z)-i(h-z)(p,s_\perp)]/|\mu|\}+ +W_{dj}(z,p,s),$$

the transfer operator can be written as

$$\Phi_1(r,s)=\sum_{j=1}^{J}[E_j(s)e^{-[\tau(h)-\tau(z)]/|\mu|}\,q_j(r_\perp-\tilde{r})+ +\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}W_{dj}(z,p,s)\overset{\vee}{q}_j(p)e^{-i(p,r_\perp)}dp=$$

$$=\sum_{j=1}^{J}[E_j(s)e^{-[\tau(h)-\tau(z)]/|\mu|}\,q_j(r_\perp-\tilde{r})+ +\iint_{-\infty}^{\infty}\Theta_{dj}(z,r_\perp-r'_\perp,s)q(r'_\perp)dr'_\perp, \quad (13)$$

$$\tilde{r}=(h-z)\,\mathrm{tg}\,\theta\{\cos\varphi,\sin\varphi\}.$$

Here $\Theta_{dj}(z,r_\perp,s)$ is the scattering function

$$\Theta_{dj}=F^{-1}[W_{dj}]=\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}W_{dj}(z,p,s)e^{-i(p,r_\perp)}dp \quad (14)$$

Expression (13) is another form of writing the atmosphere transfer operator, that henceforth will be used as a basis of the algorithm for the reverse problem solution.

3. IDENTIFICATION OF A SURFACE ALBEDO FOR THE KNOWN REFLECTION INDICATRICES

3.1. Background

Assume that the atmosphere model is familiar i.e. height dependence of the scattering $\sigma_s(z)$ and extinction $\sigma_t(z)$ coefficients and scattering indicatrix $\gamma(z,s,s')$ are specified. Also assume that reflection indicatrices of various surface areas are familiar and that albedo does not change within one area, i.e. $q_j(r_\perp)\equiv q_j \quad (r_\perp\in Q_j)$.

The problem consists in evaluating the surface albedo $q_j \quad (j=1,\ldots,J)$ using the known values of the upward radiation intensity $I(r,s)$ at altitude z and in direction $s=\{\mu,\varphi\}$.

The intensity of the upward radiation in approximation of a single reflection from the surface under the above assumptions is defined by

$$I=D(z,s)+\sum_{j=1}^{J}q_j[E_j(s)e^{-[\tau(h)-\tau(z)]/|\mu|}H_j(r_\perp-\tilde{r})+ +\iint_{-\infty}^{\infty}\Theta_{dj}(r_\perp-r'_\perp)H_j(r'_\perp)dr'_\perp], \quad (15)$$

where H_j is the Heaviside step function of the area Q_j

$$H_j(r_\perp)=\begin{cases}1, & r_\perp\in Q_j,\\ 0, & r_\perp\notin Q_j.\end{cases}$$

Allow a symbol

$$P_j(r_\perp)=E_j(s)e^{-[\tau(h)-\tau(z)]/|\mu|}H_j(r_\perp-\tilde{r})+ +\iint_{-\infty}^{\infty}\Theta_{dj}(r_\perp-r'_\perp)H_j(r'_\perp)dr'_\perp, \quad (16)$$

(we fix values of z and s, hence these arguments of the functions Q_{dj} and P_j can be omitted).

Instead of expression (15) we obtain

$$\Phi_1(r,s)=I(r,s)-D(z,s)=\sum_{j=1}^{J}q_jP_j(r_\perp). \quad (17)$$

Thus albedos q_i are coefficients of the function $\Phi_1(r_\perp)$ expansion into series in functions $P_j(r_\perp)$.

Reverse problem algorithm involves

1). Solution of the boundary problem (2) for an atmosphere haze and evaluation of function $\Phi_1(r,s)=I(r,s)-D(z,s)$.

2). Calculation of function $E_j(s)$ by the formula (5).

3). Solution of the boundary problem (9) for Green's function $W_\delta(z,p,s,s_s)$.

4). Evaluation of SFR $W_1(z,p,s)$ by the expression (8).

5). Calculation of the scattering function $\Theta_{dj}(r,s)$ using Fourier transform (14).

6). Calculation of functions $P_j(r_\perp)$ using convolution (16).

7). Evaluation of coefficients of the function $\Phi_1(r_\perp)$ best approximation by functions $P_j(r_\perp)$ (under fixed z and s values).

3.2. Numerical realization of the algorithm

Boundary problem (2) for an atmosphere haze is solved using the method of iterations in multiplicity of scattering [5]. We calculate integral (5) for the function $E_j(s)$ using trapezoid formula when integrating with respect to azimuth, and Gauss' quadrature when integrating with respect to zenith angle.

Calculation of Green's function $W_\delta(z,p,s,s_s)$ is the most tedious part of the algorithm. Boundary problem (9) is solved using iteration in multiplicity of scattering. Collision integral is evaluated for each iteration using the efficient algorithm developed in [7]. The algorithm is based upon two properties of the collision integral of the non–polarized radiation, namely:

– scattering indicatrix (being the kernel of the integral operator) at a particular point z is a function only of the total scattering angle,

$$\gamma(z,s,s')\equiv\gamma(z,\cos\chi), \quad \text{where}$$

$$\cos\chi=\mu\mu'+\sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\varphi-\varphi').$$

– collision integral is the operator of the circular convolution over azimuth.

In order to more efficiently use the second point a uniform grid over azimuth has to be used. In this case point $|\varphi_i-\varphi_{i'}|$ is also a node of the grid under any values of i and i′. Here we can evaluate matrix $\gamma(\mu_j,\mu_{j'},\varphi_i)$. We shall break up the interval of the scattering angle change into L sub intervals $\mathbb{R}_l$ so that relative change of the scattering indicatrix within one interval does not exceed the preset value ε. We shall break up a set of the grid nodes $\mu_j\times\mu_{j'}\times\varphi_i$ into S_l sub sets:

$$(\mu_j,\mu_{j'},\varphi_i)\in S_l, \quad \text{if}$$

$$\mu_j\mu'_{j'}+\sqrt{1-\mu_j^2}\sqrt{1-\mu'^2_{j'}}\cos\varphi_i\in\mathbb{R}_1.$$

In addition to that we shall replace a three-dimensional matrix of the scattering indicatrix values by the one-dimensional array γ_l, $l=1,\ldots,L$ of the scattering indicatrix mean values for each interval. To calculate collision integral we take the summation of the integrand values over each of the sub sets S_l and subsequently multiply the sum by γ_l. Thus we substitute IxJ multiplications with L multiplications such that value L does not depend on the grid dimensions. Efficiency of this algorithm considerably increases with the grid dimensions growth since a number of multiplications increases in proportion to the grid dimensions rather than to dimensions square. The algorithm relative error does not exceed value ε.

In contrast to problem (7) Green's function $W_\delta(z,p,s,s_s)$ (9) does not have cylindrical symmetry. Due to this fact function $W_\delta(z,p,s,s_s)$ depends on the four parameters: p_x, p_y, μ_s, φ_s. However, calculations may be done for $\varphi_s=0$ only, since

$$W_\delta(z,p,\mu,\varphi,\mu_s,\varphi_s)=W_\delta(z,p,\mu,\varphi+\varphi_s,\mu_s,0).$$

Besides, the symmetry about plane $\varphi=0$:

$$W_\delta(z,p,\mu,\varphi,\mu_s,0)=W_\delta(z,p,\mu,2\pi-\varphi,\mu_s,0),$$

allows to reduce amount of calculations two times.

For the large frequency values ($p_x\geq 5$ or $p_y\geq 5$) evaluation of the collision integral is a difficult task which is primarily caused by oscillations of the real and imaginary components of the function W_δ. Calculations, however, show that with $p\geq 1$ it is sufficient to take into account only unscattered and singly-scattered components of the Green's function. Here the error is compatible with one associated with the failure to take into account multiple reflection from the surface.

An unscattered component of the Green's function is defined by

$$W_{\delta 0}=\begin{cases}0, & s\in\Omega^+,\\ \delta(s-s_s)\exp\{-[\tau(h)-\tau(z)-i(h-z)(p,s_{s\perp})]/\\ \quad/|\mu_s|\}, & s\in\Omega^-,\end{cases}$$

$$(p,s_{s\perp})=p_x\cos\varphi_s\sin\theta_s+p_x\sin\varphi_s\sin\theta_s,\quad \theta_s=\arccos\mu_s.$$

A singly-scattered component of the Green's function is calculated analytically provided an atmosphere has a laminated structure [7], i.e. coefficients $\sigma_t(z)$, $\sigma_s(z)$ and a scattering indicatrix are piecewise constant functions in z. Since the formulae are too clumsy we omit them.

Accuracy of the boundary problem (9) solution for the Green's function W_δ is evaluated using the relationship (10). Boundary problem (7) solution accuracy for SFR $W(z,p,s)$ was discussed in book [5].

The next step of the algorithm is calculation of SFR W_1 by (8). According the paper [7] $\lim_{|\mu s|\to 0} W_{\delta 1}=0$, where $W_{\delta 1}=W_\delta-W_{\delta 0}$ is a scattered component of the Green's function. In addition to that calculations show that $W_{\delta 1}$ for $|\mu_s|\to 0$ decreases very rapidly (see below) therefore, there is no need to evaluate function W_δ close by point $\mu_s=0$. Integral (8) is evaluated using quadrature formulae.

The scattering function is calculated by (14) using the Fast Fourier Transform algorithm. Here it is sufficient to evaluate function W_δ for the frequencies $p_x,p_y\leq 20$.

To calculate functions $P_j(r_\perp)$ by means of convolution (16) we use the trapezoid formula with respect to x and y.

3.3. Identification of the surface albedo

Approximation of the function $\Phi_1(r_\perp)$ by the functions $P_j(r_\perp)$ is a correct problem despite the fact that $P_j(r_\perp)$ are not orthogonal functions. This is primarily results from the fact that essentially each $P_j(r_\perp)$ function can be considered as the finite function approximating the function Φ_1 for the area Q_j. Functions P_i and P_j overlapping area depends on the Green's function W_δ influence area. Thus the problem of approximation may be solved using classical technique.

Make use of the least-squares method. For this purpose we shall construct a square functional

$$T(q_1,\ldots,q_J)=\int_{-\infty}^{\infty}\int[\Phi_1(r_\perp)-\sum_{j=1}^{J}q_jP_j(r_\perp)]^2dr_\perp \qquad (18)$$

and find the minimum of this functional under $q_j\in[0,1]$. The problem of functional minimization reduces to a set of equations

$$\frac{\partial T}{\partial q_i}=0,\quad i=1,\ldots,J,$$

that can be transformed to

$$\sum_{j=1}^{J}a_{ij}q_j=b_i,\quad i=1,\ldots,J, \qquad (19)$$

where

$$a_{ij}=\int_{-\infty}^{\infty}\int P_i(r_\perp)P_j(r_\perp)dr_\perp,\quad b_i=\int_{-\infty}^{\infty}\int\Phi_1(r_\perp)P_i(r_\perp)dr_\perp.$$

The maximum coefficients of the matrix $\{a_{ij}\}$ are on the main diagonal and rapidly decrease with the increase of $|i-j|$ difference. Therefore, the matrix $\{a_{ij}\}$ is well conditioned and solution of the set (19) does not present difficulties.

4. IDENTIFICATION OF THE SURFACE ALBEDO AND THE REFLECTION INDICATRICES

Like in the previous Section let's assume that the atmosphere model is familiar. Make use of the approximation of the reflection indicatrices by orthogonal polynomials, Legendre polynomials, for example, like in [2]

$$P_j(s,s')=\sum_{k=0}^{K}d_{jk}B_k(s,s'), \qquad (20)$$

where $B_k(s,s')=B_k(\cos\chi)$ are orthogonal polynomials, χ is a reflection angle. Let's consider that approximation of each reflection indicatrix P_j requires no more than K polynomials. In view of function P_j normalization condition we obtain normalization relation for the coefficients d_{jk}

$$\sum_{k=0}^{K}d_{jk}=1\quad\text{for each j.} \qquad (21)$$

In view of expression (20) we obtain:

$$E_j(s)=\hat{R}_j[D+I_0]=\sum_{k=0}^{K}d_{jk}\bar{B}_k,$$

where $\bar{B}_k(s)=\int_{\Omega^+}B_k(s,s')[D(h,s')+I_0(h,s')]\mu'ds'$. (22)

Instead of expression (8) we obtain

$$W_{1j}(z,p,s)=\sum_{k=0}^{K} d_{jk}[\bar{B}_k \exp\{-[\tau(h)-\tau(z)- -i(h-z)(p,s_\perp)]/|\mu|\}+\bar{W}_k],$$

where $$\bar{W}_k(z,p,s)=\int_{\Omega^+}\bar{B}_k(s_s)W_{\delta 1}(z,p,s,s_s)ds_s. \quad (23)$$

Instead of expression (17) for the fixed z and s values we obtain

$$\Phi_1(r_\perp)=\sum_{j=1}^{J}\sum_{k=0}^{K}\bar{d}_{jk}\bar{P}_{jk}(r_\perp), \quad (24)$$

where $\bar{d}_{jk}=q_j d_{jk}$,

$$\bar{P}_{jk}(r_\perp)=\bar{B}_k(s)e^{-[\tau(h)-\tau(z)]/|\mu|}H_j(r_\perp-\tilde{r})+ +\iint_{-\infty}^{\infty}\bar{\Theta}_k(r_\perp-r'_\perp)H_j(r'_\perp)dr'_\perp, \quad (25)$$

$$\bar{\Theta}_k(r_\perp)=F^{-1}[\bar{W}_k].$$

Thus, the problem reduces to approximation of function $\Phi_1(r_\perp)$ by functions $\bar{P}_{jk}(r_\perp)$. As compared with problem (17) evaluation of the approximation coefficients in this case is not a correct problem, since functions $\bar{P}_{jk}$ ($k = 1,\ldots,K$; j is a fixed value) insufficiently differ, i.e. we have a non–orthogonal approximation by the similar functions. In order to make the problem of approximation correct we can involve measurements of the upward radiation intensity in several directions.

Suppose we have measurements of radiation intensity $I(r,s_m)$ in directions $s_1,..,s_M$. Evaluate functions $\Phi_1^m(r_\perp)=I(r,s_m)-D(z,s_m)$ (z is a fixed value) and instead of (24) we obtain

$$\Phi_1^m(r_\perp)=\sum_{j=1}^{J}\sum_{k=0}^{K}\bar{d}_{jk}P_{jk}^m(r_\perp), \quad (26)$$

where functions $P_{jk}^m(r_\perp)$ are evaluated using a normal pattern for the direction s_m.

The problem of approximation (26) is solved as follows. Like in the previous Section for each sighting direction, using the least–squares method we obtain a set of $J\times K$ linear equations in $\bar{d}_{jk}$, since $\bar{d}_{jk}$ do not depend on the sighting direction

$$\sum_{j=1}^{J}\sum_{k=0}^{K}\bar{d}_{jk}a_{i1\,jk}^m=b_{i1}^m, \quad \begin{matrix} i=1,\ldots,J,\\ l=1,\ldots,K\\ m=1,\ldots,M\end{matrix} \quad (27)$$

where

$$a_{i1\,jk}^m=\iint_{-\infty}^{\infty}P_{i1}^m(r_\perp)P_{jk}^m(r_\perp)dr_\perp,$$

$$b_{i1}^m=\iint_{-\infty}^{\infty}P_{i1}^m(r_\perp)\Phi_1^m(r_\perp)dr_\perp.$$

Thus we obtain a set of $J\cdot K\cdot M$ equations with $J\cdot K$ unknowns. Eq. (27) can be rewritten in a matrix fashion

$$Ad=b, \quad (28)$$

where $A=\{a_{i1\,jk}^m\}$ is a matrix of the set of equations (27), $d=\{\bar{d}_{jk}\}$ is a vector of unknowns, $b=\{b_{i1}^m\}$ is a vector of the Eq. (27) right side. The set of equations (28) can be solved using the least–squares method [10]

$$d=A^{\neq}b,$$

where

$$A^{\neq}=(A^TA)^{-1}A^T \quad (29)$$

is a pseudo inverse matrix. Matrix (29) is well conditioned when $M\geq K$. Hence, to evaluate K harmonics in the reflection indicatrix we must measure the upward radiation intensity at least in K directions.

On evaluating coefficients $\bar{d}_{jk}$ coefficients q_j and d_{jk} can be easily found in view of normalization condition (21)

$$q_j=\sum_{k=0}^{K}\bar{d}_{jk}, \qquad d_{jk}=\bar{d}_{jk}/q_j.$$

Consider utilization of this method to determine a reflection indicatrix for a homogeneous but non–Lambertian surface. Here J=1 and index j as well as summation over j in all expressions can be omitted. Formula can be rewritten as

$$\Phi_1(r_\perp)=\sum_{k=0}^{K}\bar{d}_k\bar{P}_k(r_\perp), \quad (29)$$

where

$$\bar{P}_k(r_\perp)=\bar{B}_k(s)e^{-[\tau(h)-\tau(z)]/|\mu|}+\iint_{-\infty}^{\infty}\bar{\Theta}_k(r_\perp-r'_\perp)dr'_\perp=$$

$$=\bar{B}_k(s)e^{-[\tau(h)-\tau(z)]/|\mu|}+W_k(z,p=0,s).$$

With the fixed z and s values $P_k\equiv \text{const}$ and Eq. (29) have an infinite number of solutions.

Using the angular method we obtain a set of M equations

$$\Phi_1^m=\sum_{k=0}^{K}\bar{d}_kP_{km}, \quad (30)$$

where $\Phi_1^m=\Phi_1(z,s_m)$, $P_{km}=P_k(z,s_m)$ with the fixed z values. With $M<K$ the set of equations (30) has an infinite number of solutions. With $M\geq K$ the set is unambiguously solvable using the least–squares method, provided matrix $\{P_{km}\}$ columns are linear independent. Linear independence of the matrix $\{P_{km}\}$ columns is easily derived from polynomials $B_k(s,s')$ independence.

5. CONDITIONS AND RESULTS OF CALCULATION PERFORMANCE

To solve the key problems of radiation transfer in the atmosphere bounded by an inhomogeneous and non–Lambertian underlying surface i.e. to evaluate SFR $W(z,p,s)$ and Green's function $W_\delta(z,p,s,s_s)$ SFC2 computer code was developed. The estimated values of the SFR and Green's function are entered into the archives of solutions. The RADIAT code package allows to solve direct and reverse problems using the archives of the atmosphere transfer functions.

The atmosphere model for $\lambda = 0.6943$ μm [8] was used in the calculations. Fig. 1–3 show azimuth dependencies of the upward radiation Green's function $W_\delta(z,p,s)$ components for the upper atmosphere boundary ($z = 0$) for $p_x = 0$ (only W_δ^{Re}, since $W_\delta^{Im}=0$) and for $p_x = 2$. The calculations confirm the fact that W_δ rapidly decreases with $\mu_s\to 0$. Here the speed of the decrease increases with the p growth. Hence, there is no need to have a large number of nodes μ_s about 0.

For modeling anisotropic surface reflectance the described in paper [3] BDRF (bidirectional reflectance function) was used. The function was modified in view of normalization condition

$$P(\mu,\mu',\varphi,\varphi') = \frac{1+g(\mu')[\,|\mu|\mu' + \sqrt{1-\mu^2}\,\sqrt{1-\mu'^2}\cos(\varphi-\varphi')]}{1+\frac{2}{3}\mu' g(\mu')},$$

where $g(\mu')$ is an anisotropy factor which varies from −1 to 1. For a Lambertian surface $g \equiv 0$. For the model results, the BDRF was taken to be forward scattering and we chose $g = 0.5$ for normal incidence and $g = 1$ for grazing incidence i.e.

$$g(\mu') = 1 - \mu'/2.$$

Fig. 4 shows dependence of function $P(\mu,\mu_0,\varphi,0)$ on a zenith angle for the azimuth values of $\varphi = 0$, $\varphi = 180^\circ$ for the three sun zenith angles $\theta_0 = \arccos \mu_0 = 30^\circ; 45^\circ; 60^\circ$. Fig. 5 shows dependence of function $E(s)$ on the zenith angle cosine for the same three sun zenith angles. Maximum $E(s)$ (as well as reflection indicatrix) falls at the area of the solar glare. Fig. 6 shows comparison of the amplitude–frequency response values obtained by the formula (8) for Lambertian and non Lambertian surfaces. Fig. 7 illustrates distribution of the upward radiation intensity for the various sighting angles for the "step" albedo

$$q(x) = \begin{cases} 0.2 & x<0, \\ 0.4 & x\geq 0. \end{cases}$$

Calculation results for the Lambertian and non–Lambertian reflection from the right side of the surface $(x \geq 0)$ are compared (the left side $(x < 0)$ is a Lambertian surface). Fig. 8 shows similar results, but for the "belt" albedo

$$q(x) = \begin{cases} 0.2 & x < -2\,\text{km}, \\ 0.6 & -2\text{km} \leq x \leq 2\text{km}, \\ 0.4 & x > 2\text{km}. \end{cases}$$

It should be noted that the graphs show total intensity in approximation of a single scattering from the surface in view of atmosphere haze. The calculations were done in the relative units with $S_\lambda = 1$.

Based on the results of the direct problem using the algorithm described in Section 3, identification of the surface albedo for the known reflection indicatrices was done. The error of albedo, both "step" and "belt", identification is some 3–4% for the various combinations of reflection indicatrices.

The error of the reflection indicatrices identification on the basis of the method described in Section 4 did no exceed 15% for the 3–angular measurements ($\mu = -0.997$; -0.755; -0.461) and 5% for the 9–angular measurements ($\mu = -0.997$, -0.755, -0.461; $\varphi = 0^\circ$, 90°, 180°). The error of albedo identification did not exceed 10% in the first case and 5% in the second case.

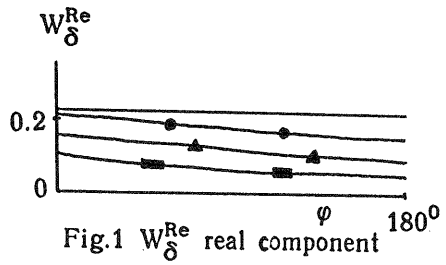

Fig.1 W_δ^{Re} real component
$p_x = 0$ $\mu = -0.997$

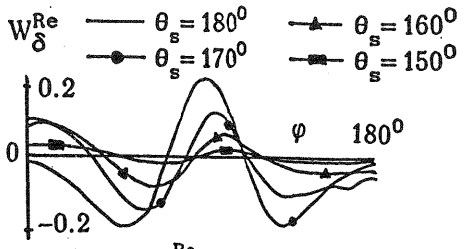

Fig. 2 W_δ^{Re} real component
$p_x = 2$ $\mu = -0.997$

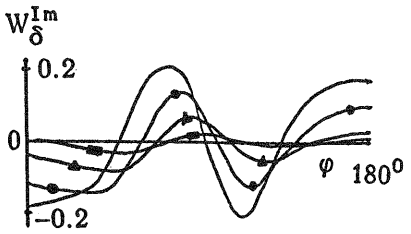

Fig. 3 W_δ^{Im} imaginary component
$p_x = 2$ $\mu = -0.997$

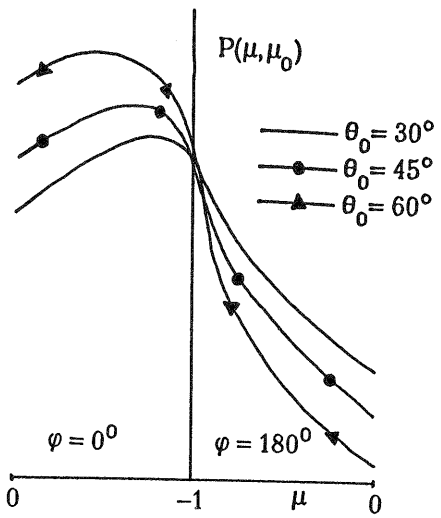

Fig. 4. Dependence of BDRF $P(\mu,\mu_0)$ from μ

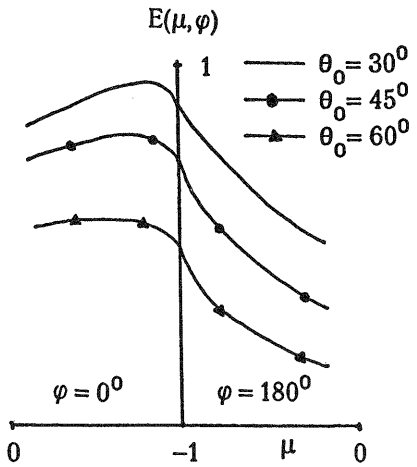

Fig.5 Dependence of E(s) from μ

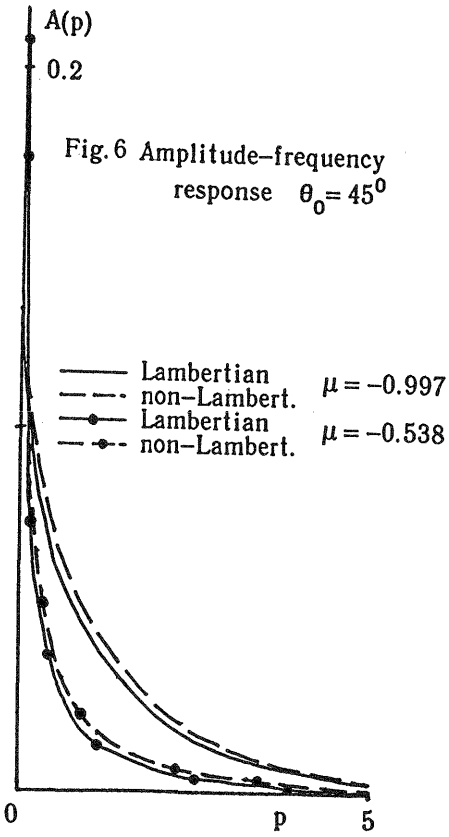

Fig. 6 Amplitude-frequency response $\theta_0 = 45^0$

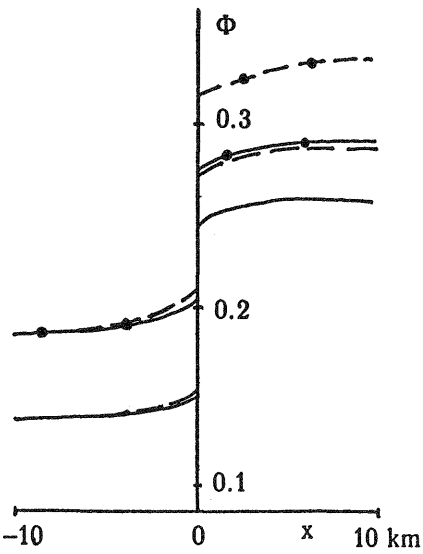

Fig. 7. Upward radiation. "Step" albedo. $\theta_0 = 45^0$

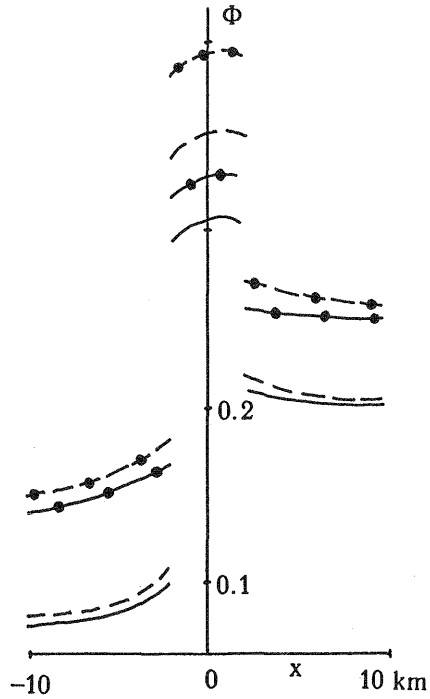

Fig. 8. Upward radiation. "Belt" albedo. $\theta_0 = 45^0$

6. CONCLUSIONS

The results presented in this paper demonstrate, that for the known atmosphere model and surface reflection indicatrices surface albedo can be rather accurately identified using atmosphere optical transfer operator. In case the reflection indicatrices are unknown angular measurements of the upward radiation intensity can be involved to identify indicatrices and albedo. Here the number of the reflection indicatrix harmonics to be identified does not exceed the number of angular measurements.

It should be noted that independent definition of the optical characteristics of the underlying surface at various wave-lengths allows to obtain a spectral set of the optical characteristics.

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