A COMPARATIVE STUDY ON THE METHODS FOR ESTIMATION OF MIXING RATIO WITHIN A PIXEL

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ABSTRACT

A comparative study has been made with the following three methods for estimation of mixing ratio within a pixel. The first one is the well known least square method by means of Generalized Inverse Matrix. The second one is Maximum LikeliHood method and last one is the Least SQuare method minimizing the square error of estimated Mixing ratio. It was found that the estimation accuracy of the Maximum LikeliHood method is superior to the other methods in the case for the simulated data with S/N ratio of up to about 28.

KEY WORDS: Mixed Pixel, Estimation of Mixing Ratio, Inverse Problem Solving, Cloud Coverage.

1. INTRODUCTION

There are many classification methods of remortly sensed imagery data. Many of them put label in a pixel basis. Even for the pixels consist of plural categories, ordinary classification method give one category to the pixels. Then, it was considered that information of mixing ratio within a pixel was taken out without abandoning that information.
In remortly sensed image, estimation of partial class mixing ratio within a pixel have the significant role for land coverage. For example, cloud coverage estimation based on category decomposition is useful for making products of sea surface temperature.
Category decomposition give us the way get that proportion from a mixed pixel(MIXEL).
In this paper, we picked up the three methods for estimation of category proportion within a pixel which are proposed. Comparing these estimation accuracies with simulation data consists of the mixels added nomal distributed random number.

2. THE METHODS OF CLASS MIXING RATIO ESTIMATION

2.1 Least square method by means of Generalized Inverse Matrix

Let an observed vector be I with the dimensionality M, mixing ratio or proportion vector be B with the number of classes N and the matrix representing the spectral response of all classes be A.

$$\mathbf{I} = \mathbf{A}\,\mathbf{B} \tag{1}$$

$$\mathbf{I} = (I_1, I_2, \ldots, I_M)^t \tag{2}$$

$$\mathbf{A} = \begin{vmatrix} A_{11}, A_{12}, \ldots, A_{1N} \\ A_{21}, \ldots\ldots, A_{2N} \\ \ldots\ldots\ldots\ldots \\ A_{M1}, \ldots\ldots, A_{MN} \end{vmatrix} \tag{3}$$

$$\mathbf{B} = (B_1, B_2, \ldots\ldots, B_N)^t \tag{4}$$

Since I is given vector, if matrix A is assumed, then vector B is determined under constraint that minimizing norm of estimation error E_i,

$$\left| \sum_{i=1}^{M} E_i^2 \right| \text{--->min}, \quad \mathbf{E} = \mathbf{I} - \mathbf{A}\,\mathbf{B} \tag{5}$$

$$\mathbf{B} = (\mathbf{A}^t\ \mathbf{A})^{-1}\ \mathbf{A}^t\ \mathbf{I} \tag{6}$$

Then, under constraint that the sum of mixing ratio has 1.0, equation(6) is solved the following analyticaly.

$$\mathbf{B} = \mathbf{A}^+\ \mathbf{I} + \frac{1 - \mathbf{u}^t\ \mathbf{A}^+\ \mathbf{I}}{\mathbf{u}^t\ (\mathbf{A}^t\ \mathbf{A})^{-1}\ \mathbf{I}}\ (\mathbf{A}^t\ \mathbf{A})^{-1}\ \mathbf{I} \tag{7}$$

At this point, **u** is a vector has factors with all of 1.0.

2.2 Maximam LikeliHood method

If independent variables $X_i\,(i=1,2,\ldots,N)$ follows normal distribution as a function of $N(\mu_i, \sigma_i^2)$, the following equation is shown X and N.

$$\mathbf{X} = \sum_{i=1}^{N} a_i\ X_i,\quad N\left(\sum_{i=1}^{N} a_i \mu_i,\ \sum_{i=1}^{N} a_i \sigma_i^2 \right) \tag{8}$$

So that when the response of spectral reflectance is independent of each other in the classes, the following equation is conducted on the observed vector I and N.

$$\mathbf{I} = \sum_{i=1}^{N} \mathbf{A}_i\ \mathbf{B}_i,\quad N(\ \mathbf{A}_i{}^t\ \mathbf{B},\ \mathbf{B}^t\ \mathbf{Z}_i\ \mathbf{B}\ ) \tag{9}$$

$$\mathbf{A}_i = (\ A_{i1},\ A_{i2},\ \ldots\ldots,\ A_{iN}\ ) \tag{10}$$

$$\mathbf{Z}_i = \text{diag.}(\ \sigma_{i1}^2,\ \sigma_{i2}^2, \ldots\ldots,\ \sigma_{iN}^2\ ) \tag{11}$$

The probability P_i is shown equation(12) when I_i is observed in the i band and proportion vector has B.

$$P_i = \frac{\exp\left\{ -\dfrac{1}{2}\ \dfrac{(\ I_i - \mathbf{A}_i{}^t\ \mathbf{B}\ )^2}{(\ \mathbf{B}^t\ \mathbf{Z}_i\ \mathbf{B}\ )} \right\}}{\{\ 2\pi\ (\ \mathbf{B}^t\ \mathbf{Z}_i\ \mathbf{B}\ )\ \}^{1/2}} \tag{12}$$

Then probability P that the observed vector is I and the proportion vector is B is shown the next equation(13),

$$P = \prod_{i=1}^{M} P_i \qquad (13)$$

and a logarithm of the equation(13) is

$$Q = -\ln P \qquad (14)$$

These problem result in nonlinear optimum problem minimizing the equation(14), but the general solution do not exist. Then we used the mesh search method stated the fllowing. Firstly, the N dimensional closed domain is divided in the meshes which has 1/128 length of the N sides. In the second place, Q of the equation(14) is caluculated at the each meshes, and the proportion vector **B** is got when Q has minimum value. Moreover, the vector **B** has constraint of the equation(15).

$$\sum_{j=1}^{N} B_j = 1, \ B_j > 0 \ (j=1, 2, \ldots\ldots, N) \qquad (15)$$

2.3 The Least SQuare method minimizing the square error of estimated Mixing ratio

In this method, scattering reflection property of observed objects, property change due to observational condition, error of measurement and so on, is considered. Method2-1 is an approach under constraint that minimizing the remainder between observation value and estimation value, but this one is the method which has the terms of constraint to class mixing ratio. This uses the least square algorithm directly to the mixing ratio vector. These equation is shown the following.

$$| \mathbf{B} - \mathbf{A}^{+} \mathbf{I} | \longrightarrow \min, \ \mathbf{u}^{t} \mathbf{B} = 1 \qquad (16)$$

$$\mathbf{B} = \mathbf{A}^{+} \mathbf{I} + \frac{(1 - \mathbf{u}^{t} \mathbf{A}^{+} \mathbf{I})\,\mathbf{u}}{|\mathbf{u}|^{2}} \qquad (17)$$

At this point, U is a vector has factors with all of 1.0.

3. EXPERIMENTS

3.1 Data used

Data used in this paper was got in the sea near Japan at 25 Apr 1989 by NOAA - 11/ AVHRR. As trainning sample data, two categories are picked up 100 points each. One category is cloud and the other is sea surface. Table 1 shows average andvariance of CCT counts selected 100 points fromcloud and sea surface respectively.

3.2 Simulation data maked

The simulation data ware maked by the above trainning data, which are divided value of pure pixel average in 10 equal parts on the each two classes of cloud and sea surface. Then mixed pixels ware done every 10 percent from 0% to 100% on cloud coverage. we picked up 4 classes of MIXEL data of 20%, 40%, 60%, 80%, in addition to that random noises of normal distribution (σ=1.0, 3.0, 5.0, 7.0, 9.0) ware added to that data. The number of simulation data for estimation is 128 each class. Fig.1 shows simulation data set used.

3.3 Estimation using the simulation data

Each accuracies of the following three methods on cloud coverage estimation within a pixel added random noise ware compared by means of the simulation data maked.

1.) Least square method by means of Generalized Inverse Matrix

2.) Maximum LikeliHood method

3.) Least SQuare method minimizing the square error of estimated Mixing ratio

Table 1 Average and Variance of trainning data

Category	sea surface	cloud
Average		
BAND 1	53.030	254.30
2	42.920	241.84
3	115.62	229.45
4	73.050	2.8600
Variance		
BAND 1	11.049	7.8700
2	8.7535	165.77
3	474.77	464.30
4	17.427	28.300

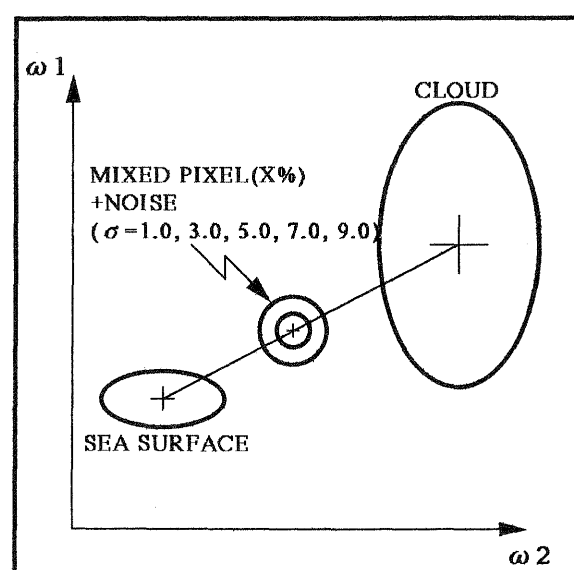

Fig.1 Used simulation data set

4. RESULTS

From the experimental results which are shown in Figure 2, 3, 4, 5, there are a relationship between σ of random noises added and root mean square error depended on each three methods. X axis has the random noise σ and Y axis has RMS error. In this paper, GIM stands for Least square method by means of Generalized Inverse Matrix, MLH stands for Maximum LikeliHood method, and LSQM is Least SQuare method minimizing the square error of estimated Mixing ratio.

4.1 Result depended on GIM

Result depended on GIM method is shown in Fig.2, 3. Fig.2 shows estimation accuracy without constraint which sum of mixing ratio is 1.0, what cloud coverage normalized that sum of these is equal to 1.0 is conducted after equation(6) is caluculated. That vary every MIXEL maked. Moreover, Fig.3 shows RMS error by equation(7), the second terms of that equation include the constraint that sum of cloud coverge is 1.0. Table 2, 3 shows value of the first term of equation(7) and the second one. As a result of three methods, this accuracy is third.

4.2 Result depended on MLH

Result depended on MLH method is shown in Fig.4. According as the observed noise σ increase, estimation error is larger too, and estimation accuracies vary every mixed pixel. The RMS error has a tendency to increase with the variance of anchor point data, but as a whole this method is superior to the other methods.

4.3 Result depended on LSQM

Result depended on LSQM method is shown in Fig.5. This method include the constrains that minimizing square of error of estimated mixing ratio, which and what sum of mixing ratio is 1.0 is represented the second term of equation(17). The second term exert influence on accuracy of proportion estimation. Table 4,5 shows value of first and second terms of equation(17) on each random noise added.

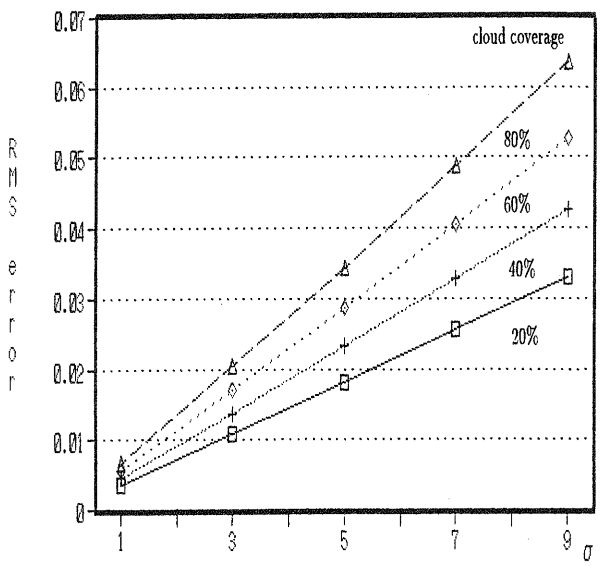

Fig.2 Estimation accuracy in terms of RMS error of cloud coverage within a pixel for GIM that normalized after equation(6) is calculated.

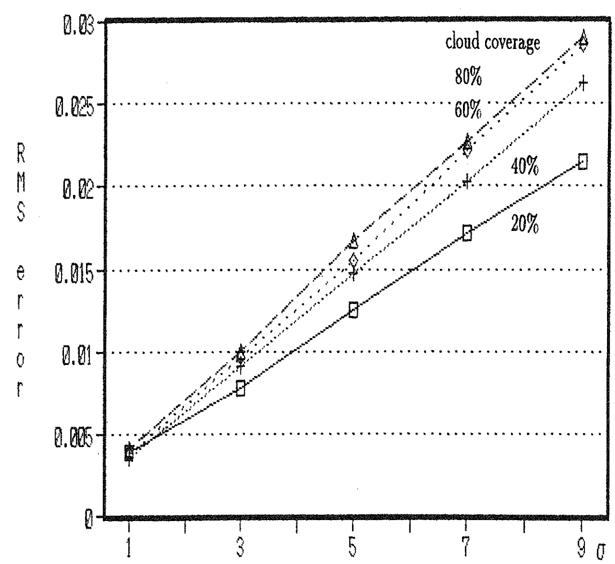

Fig.4 Estimation accuracy in terms of RMS error of cloud coverage within a pixel for MLH.

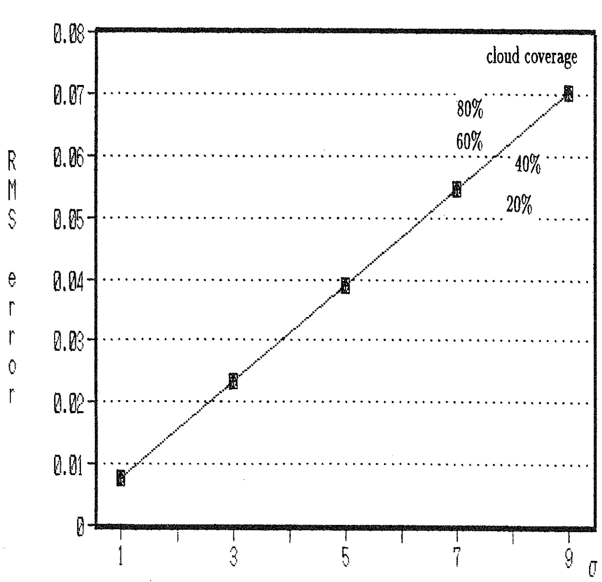

Fig.3 Estimation accuracy in terms of RMS error of cloud coverage within a pixel for GIM.

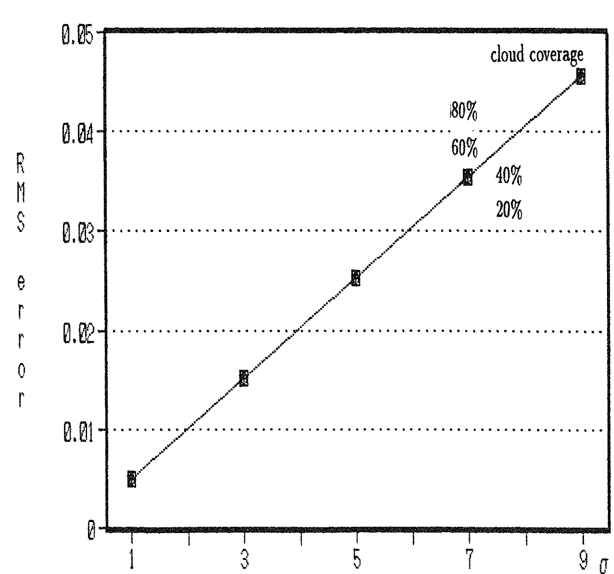

Fig.5 Estimation accuracy in terms of RMS error of cloud coverage within a pixel for LSQM.

Table 2 The first and second terms of equation(7) for GIM. (CLOUD : SEA SURFACE = 0.4 : 0.6)

σ \ Terms	The 1st term	The 2nd term
0.1 CLOUD SEA	3.9927e-01	-8.0001e-04 -5.3994e-08
0.3 CLOUD SEA	3.9783e-01	-2.4002e-03 -1.6246e-07
0.5 CLOUD SEA	3.9639e-01	-3.9997e-03 -2.7152e-07
0.7 CLOUD SEA	3.9494e-01	-5.6001e-03 -3.8126e-07
0.9 CLOUD SEA	3.9350e-01	-7.1942e-03 -4.9120e-07

Table 3 The first and second terms of equation(7) for GIM. (CLOUD : SEA SURFACE = 0.8 : 0.2)

σ \ Terms	The 1st term	The 2nd term
0.1 CLOUD SEA	8.0149e-01	3.8787e-03 3.3660e-07
0.3 CLOUD SEA	8.0447e-01	1.1629e-02 1.1015e-06
0.5 CLOUD SEA	8.0745e-01	1.9386e-02 1.7022e-06
0.7 CLOUD SEA	8.1043e-01	2.7143e-02 2.3972e-06
0.9 CLOUD SEA	8.1341e-01	3.4894e-02 3.0997e-06

Table 4 The first and second terms of equation(17) for LSQM. (CLOUD : SEA SURFACE = 0.4 : 0.6)

σ \ Terms	The 1st term	The 2nd term
0.1 CLOUD SEA	3.9927e-01	-4.0003e-04 -4.0003e-04
0.3 CLOUD SEA	3.9783e-01	-1.2002e-03 -1.2002e-03
0.5 CLOUD SEA	3.9639e-01	-1.9999e-03 -1.9999e-03
0.7 CLOUD SEA	3.9494e-01	-2.8002e-03 -2.8002e-03
0.9 CLOUD SEA	3.9350e-01	-3.5973e-03 -3.5973e-03

Table 5 The first and second terms of equation(17) for LSQM. (CLOUD : SEA SURFACE = 0.8 : 0.2)

σ \ Terms	The 1st term	The 2nd term
0.1 CLOUD SEA	8.0149e-01	1.9395e-03 1.9395e-03
0.3 CLOUD SEA	8.0447e-01	5.8153e-03 5.8153e-03
0.5 CLOUD SEA	8.0745e-01	9.6942e-03 9.6942e-03
0.7 CLOUD SEA	8.1043e-01	1.3573e-02 1.3573e-02
0.9 CLOUD SEA	8.1341e-01	1.7448e-02 1.7448e-02

5. CONCLUSION

In consequence, whichever of three techniques of estimation have better accuracy below 0.8 RMS error in terms of cloud coverage within a pixel under the random noise S/N=28. Among three methods, Maximum LikeliHood method show the best estimtion accuracy. The reason for that is that MLH takes into account a variance of spectral characteristics of the classes of interest. The least square method minimizing the square error of estimation mixing ratio (LSQM) is superior accuracy without considering variance of sample training data of spectral characteristics. The estimation accuracy is enough to apply image classification. Next step is to consider the variance of data, the other constraint and determination of anchor point data. It is necessary that the methods of mixing ratio estimation are improved.

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