

INVESTIGATIONS OF THE ACCURACY OF THE DIGITAL PHOTOGRAMMETRY SYSTEM DPS, A RIGOROUS THREE DIMENSIONAL COMPILATION PROCESS FOR PUSH BROOM IMAGERY

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Abstract:

MBB developed a new concept for a digital stereo-scanner with three line sensor arrays working on the push broom principle and a suitable analytical compilation method.

It delivers the orientation data of the camera along the flight trajectory of aircraft, spacecraft, or missiles, the three-dimensional coordinates of the digital elevation model (DEM), ortho- and stereo-orthophotos and geometrically rectified multispectral images. The procedure involves a digital correlation process and does not need, beside the scanner data, any additional informations with the exception of a few control points for the absolute orientation. Special stabilizations or measurements of flight data are not required. By computer simulated models the accuracy of the process was tested.

1. Introduction

In the past years the opto-electronic scan technology known under the name "push-broom principle" has made remarkable progress and is applied and involved in present and future programs, for example in the German space camera MOMS (Modular Opto-electronic Multispectral Scanner) and the French SPOT-program.

The outstanding features of this technology are known, they are founded by the immediate conversion of radiation into electric signals and yield among others important advantages of instantaneous and unlimited data transmission, direct digital image processing and precise measurement of radiation in a wide spectral band.

These advantages have to be traded off against a severe disadvantage: the loss of geometrically exact stereocompilation, as a consequence of the loss of the central-perspective geometry. The push-broom principle forms the image strip line by line and each image cycle has its own set of orientation parameters influenced by the movements of the camera carrier. Therefore the reproduced image strip is distorted and up to now no really practicable process for rectification was known. We have solved this problem by a purely analytical photogrammetric process. Solely image data are used, external measurements or aids are not necessary. The whole hard- and software equipment to be developed is called "Digital Photogrammetry System, DPS".

2. The Working Principle of the DPS

The DPS-Stereo-Camera contains in the focal plane of an objective three linear semiconductor sensor arrays which are oriented perpendicular to the direction of flight. Their mutual separations determine the stereo base. In an equivalent arrangement three individual objectives can be used whose optical axes are convergent. In each focal plane one linear sensor array is arranged in the optical axis and perpendicular to the direction of flight. The latter arrangement is advantageous with long focal lengths which are

required for imagery from outer space. The convergent angle  $\gamma/2$  determines the accuracy of the terrain elevation and can be selected as an optimum.

During the flight these arrays A, B, C, scan the terrain according to the push-broom principle simultaneously by the synchronised image cycle N thereby producing three overlapping image strips  $A_S, B_S, C_S$  of the same terrain but with different perspectives (Fig. 1).

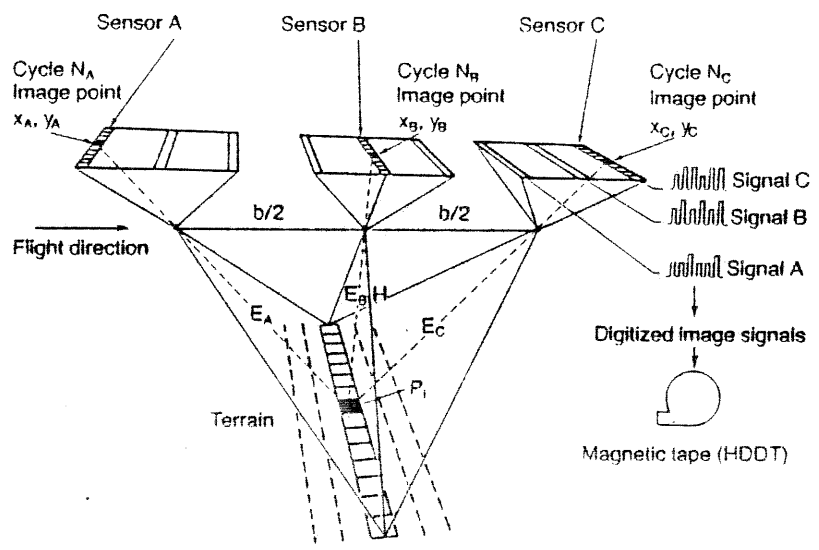


Fig. 1 DPS data acquisition principle

On account of the movement of the carrier, a different set of orientation parameters ( $X_N, Y_N, Z_N, \omega_N, \psi_N, \alpha_N$ ) applies to each image cycle. On the other hand each terrain point  $P_i (X_i, Y_i, Z_i)$  is imaged in three different image cycles  $N_A, N_B, N_C$  and corresponding orientation positions on the sensors A, B, C in the pixel numbers  $m_A, m_B, m_C$ . As the positions of the linear arrays within the image plane and the pixel intervals are known and calibrated, the image coordinates  $x_A$  and  $y_A, x_B$  and  $y_B, x_C$  and  $y_C$  of any imaged and identified terrain point  $P_i$  can be determined (Fig. 2).

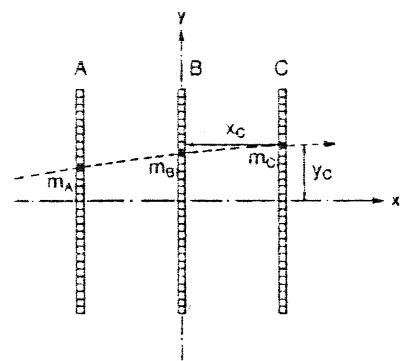


Fig.2 Trace of a terrain point  $P_i$  in the image plane of the DPS-camera

This imaging process can be explained as the travelling of an imaged terrain point across the image plane crossing the line-sensor arrays A, B, C in the pixels  $m_A$ ,  $m_B$ ,  $m_C$  and during the image cycles  $N_A$ ,  $N_B$ ,  $N_C$ .

The compilation process for the reconstruction of the model now consists of two main procedures:

- a) Determination and identification of the three image points belonging to the same terrain point  $P_i$  (DEM-point) and the corresponding image cycles by area correlation.
- b) Computation of the orientation parameters of orientation points  $P_j$  along the flight trajectory and the coordinates of the DEM-points  $P_i$  in a closed analytical process of bundle adjustment. The density of the orientation points  $P_j$  and the terrain points  $P_i$  depend mainly on the smoothness of the flight, the required accuracy of reconstruction, the map scale and the structure of the terrain.

### 3. Computation of the DPS Model

In order to reconstruct the whole model the so-called coplanarity condition or the collinearity condition can be used. We have decided to adopt the latter method as it allows a straightforward least-squares adjustment for the whole model, including the parameters of orientation and the coordinates of the DEM points.

The fundamental geometric condition imposed is the requirement that the three rays  $E_A$ ,  $E_B$ ,  $E_C$  (Fig. 1) through the three conjugate image points and the corresponding perspective centres intersect in the DEM point. This requirement is implemented by setting up the pairs of collinearity equations belonging to the same DEM point. Normally three pairs of collinearity equations can be set up; at the beginning and the end of the strip only two pairs.

They are of the following form:

$$\bar{x}_{i,N} = c \frac{a_{11} (X_i - X_N) + a_{21} (Y_i - Y_N) + a_{31} (Z_i - Z_N)}{a_{13} (X_i - X_N) + a_{23} (Y_i - Y_N) + a_{33} (Z_i - Z_N)} = F_x (p, k) \quad (1)$$

$$\bar{y}_{i,N} = c \frac{a_{12} (X_i - X_N) + a_{22} (Y_i - Y_N) + a_{32} (Z_i - Z_N)}{a_{13} (X_i - X_N) + a_{23} (Y_i - Y_N) + a_{33} (Z_i - Z_N)} = F_y (p, k) \quad (2)$$

Here  $c$  is the focal length of the camera,  $X_i$ ,  $Y_i$ ,  $Z_i$  are the coordinates of the DEM point to be determined.  $X_N$ ,  $Y_N$ ,  $Z_N$  are the coordinates of the instantaneous scan position at the image cycle  $N$  ( $N$  stands for  $N_A$ ,  $N_B$ ,  $N_C$ ) and  $a_{11} \dots a_{33}$  are the coefficients of the matrix of rotation of the camera which contains the roll  $\omega$ , the pitch  $\varphi$  and the swing  $\alpha$ .

As we do not determine the orientation parameters of all image cycles  $N$  but only those of the orientation points  $P_j$  (Fig. 3), the individual values of the orientation parameters are represented by interpolation between the orientation parameters of the orientation points  $P_j$  to be determined.

It is convenient to represent all parameters of orientation by the vector  $p$  and all terrain coordinates of the DEM points  $P_i$  by the vector  $k$ .

The unknowns  $p$  and  $k$  in the equations (1) and (2) are not yet known but they are approximately determined by the approximately known parameters of flight, altitude, speed and direction of the carrier at the image cycles  $N_A$ ,  $N_B$  and  $N_C$ . Roll, tilt and swing are set equal to zero.

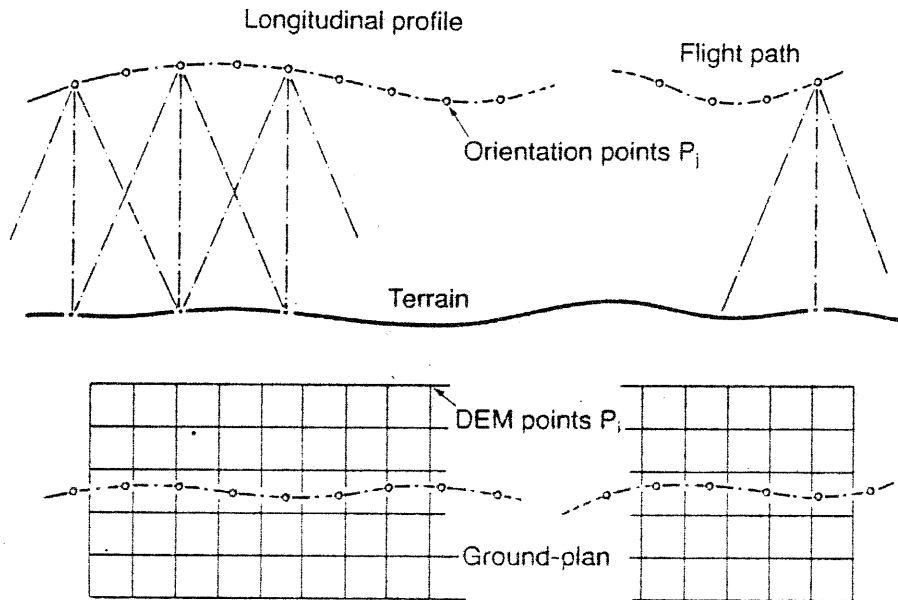


Fig. 3 DPS-model

The terrain points  $P_i$  are determined approximately by intersection with the correlated image coordinates and the approximate orientation parameters.

In consequence of these approximate values the computed image coordinates  $\bar{x}$  and  $\bar{y}$  of the collinearity equations (1) and (2) differ from the correlated (measured) image coordinates  $x$  and  $y$  and we set up the error equations,

$$v_{x,i,N} = \bar{x}_{i,N} - x_{i,N} = F_x(P_N, k_i) - x_{i,N} \quad (3)$$

$$v_{y,i,N} = \bar{y}_{i,N} - y_{i,N} = F_y(P_N, k_i) - y_{i,N} \quad (4)$$

three (two) pairs for each DEM point.

For the absolute orientation simple error equations with the known control points can be added. By least-squares adjustment the residual errors  $v$  can be minimized and the most probable values of  $p$  and  $k$  are computed.

As  $F_x$  and  $F_y$  are not linear functions they have to be linearized by truncation of a Taylor series. In brief form the error equations can be written:

$$v = A p + B k - H = M x - H \quad (5)$$

$A$  and  $B$  are the coefficient matrices of the unknowns  $p$  and  $k$ , which can be represented by the matrix  $M$  and the vector  $x$ . The absolute values are

represented by the vector  $H$ .

The normal equations

$$(M^T G M) x = M^T G H \quad (6)$$

are set up and can be solved by a Gauss's algorithm. In equation (6)  $G$  is the weight matrix which represents the accuracies.

There are two groups of unknowns, the orientation parameters  $p$  at orientation points  $P_j$  and the coordinates  $k$  of the DEM-point  $P_j$ . In the normal equations the unknowns  $k$  can be eliminated and the reduced normal equations possess band structure. They are solved by a direct method. Reinsertion of the computed orientation parameters  $p$  into the normal equations yields the unknowns  $k$  ( $X_j, Y_j, Z_j$ ) of the other group. Because of the non-linearity of the collinearity equations several iteration steps are necessary.

#### 4. Accuracy and Stability of the DEM Model

It must be emphasized that the DEM strip is form-invariant and stable even without any control points. Without them it could be computed in any scale and orientation within a local system. The insertion of measured orientation parameters and/or control points establishes the absolute orientation and tends to stabilize and possibly improve the accuracy of the strip.

The accuracy of the results can be expressed by the equation

$$\sigma_x = \sigma_0 \frac{h}{c} \sqrt{Q_{xx}} \quad (7)$$

The least-squares adjustment yields the standard error  $\sigma_0$ . It depends on the accuracy of the calibration of the camera, the image correlation and the errors of interpolation between the orientation points  $P_j$ . Flight altitude  $h$  over focal length  $c$  determines the image scale factor.

The factor  $Q_{xx}$  is the appropriate diagonal element of the cofactor matrix (the inverse of the normal equations). This cofactor is influenced by the geometry of the strip, its length and width, distribution and density of orientation points  $P_j$ , DEM-points  $P_j$  and control points, and the camera parameters.

A simple consideration reveals that the compilation of the strip is highly overdetermined:

Each DEM point leads to a total of six  $x$  and  $y$  image coordinates (observations). Thus in principle two DEM points suffice to compute the six orientation parameters at one orientation point  $P_j$ . As a rule, however, many more than two DEM points per orientation interval are available, producing a large redundancy. Therefore, many correlations can fail before the computation of the strip becomes impossible.

To prove the feasibility and the accuracy of the DPS procedure it was checked by a computer-generated synthetic model. This consists of a strip of terrain with a length of 3 km, a width of 800 m with 114 DEM points at intervals of 160 m. In a computer simulation this terrain was "overflowed" with a three-line DPS camera, focal length 52 mm, altitude 1000 m (Fig. 3). All orientation parameters along the flight path were disturbed by low-frequency sinusoidal oscillations and then the image coordinates of the DEM points

crossing the sensor lines A, B and C were computed. The half angle of convergence between the sensor line B and A respectively C was assumed to be  $\gamma/2 = 22^\circ$ .

The DEM points and orientation parameters in intervals of  $\Delta j = 200$  m were then computed with the help of the image coordinates and four control points at the corners of the strip. The results revealed the good coincidence between the given and the reconstructed model and its stability. According to equations (7) the theoretical errors of all orientation and DEM-points were computed. The errors of the DEM points  $P_i$  in X, Y and Z are separated and respectively represented vertically in perspective figures 4,5,6.

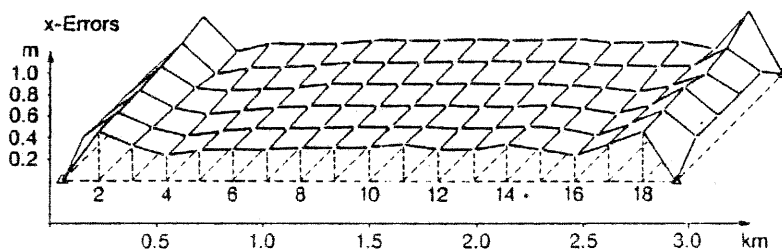


Fig. 4 X-errors with computer simulated data

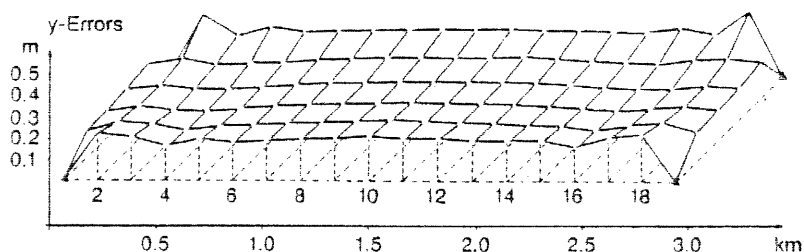


Fig. 5 Y-errors with computer simulated data

The residual errors between the true and the computed values are the following:

Standard error of the image points in the focal plane:

$$\sigma_0 = \pm 4.6 \text{ } \mu\text{m}$$

Digital elevation model:

$$m_x = \pm 0.157 \text{ m}$$

$$m_y = \pm 0.202 \text{ m}$$

$$m_z = \pm 0.466 \text{ m}$$

The standard error  $\sigma_0$  in the image plane is caused in this computer simulated test model<sup>0</sup> only by the deviations between the real values of the orientation parameters and their linear interpolated values between the

orientation points  $P_j$ . Correlation errors are not inserted. In operational cases these errors of interpolation can be reduced by the following provisions and conditions:

- smooth undisturbed flight of the carrier (for example of a high altitude jet aircraft, ideal preconditions are given for spacecraft) and/or damped suspension of the camera;
- small intervals between the orientation points  $P_j$ ;
- non-linear interpolation between the orientation points, for example by spline functions.

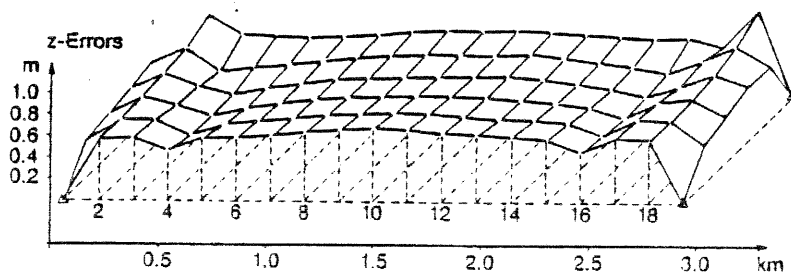


Fig. 5 Z-errors with computer simulated data

As a consequence for operational cases, the standard error  $\sigma_0$  in the image plane is mainly influenced by errors of interpolation and correlation. The correlation error will be of the order of  $\pm 2 \mu\text{m}$ .

The most remarkable result of this simulated test is the flatness of the error curves of the DEM-points in all three coordinates X, Y and Z, they are nearly equal over the whole length of the strip. In the four corner points the errors are zero because of the placement of the points of absolute orientation at these points. At the beginning and the end of the strip the errors are greater. This result is caused by the fact that in these parts of the strip the DEM points are determined only by two intersections of image rays. It would be possible to eliminate these parts and arrange the points of absolute orientation within those parts of the strips where threefold overlapping exists.

Of great influence for error propagation is the number of base lines within the whole strip. In this case the length of the base is about 800 m, and therefore the whole strip contains about for base lengths.

Of course, other and more operational models can be simulated and tested and we do this presently. But for error estimations this "normalized" model and its results can be extrapolated to any other dimensions. The model itself can be enlarged (or reduced) by the factor flight altitude  $h/\text{m}$  / 1000  $[\text{m}]$ , and the DEM-errors can be extrapolated by applying formula (7). The DEM errors of the figures 4,5,6 have to be multiplied by the factor

$$\frac{\sigma_0 [\mu\text{m}] \cdot h/\text{m} \cdot 52 [\text{mm}]}{4.6 [\mu\text{m}] \cdot 1000 [\text{m}] \cdot c [\text{mm}]} \quad (8)$$

where  $\sigma_0$  (standard image error),  $h$  (flight altitude),  $c$  (focal length) are the new selected parameters of the model.

#### References

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- /2/ Hofmann, O., 1982. Digitale Aufnahmetechnik, Bildmessung und Luftbildwesen, 50, (1982), Heft 1, S. 16-32.
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