SEQUENTIAL DATA PROCESSING FOR PHOTOGRAMMETRIC ACQUISITION OF DIGITAL ELEVATION MODELS

Anders Östman
Department of Photogrammetry
Royal Institute of Technology
Sweden
Commission IV

ABSTRACT

The selection of measured points is very important when sampling elevations for Digital Elevation Models (DEM). In many cases human operators are needed to perform this selection. To guide the operator, a system can be used, where the DEM is continously updated and contours are generated and displayed on a graphic screen.

When updating the DEM, sequential algorithms can be used. The paper describes a system for real-time monitoring of contours during the photogrammetric data acquisition. Sequential algorithms corresponding to different interpolation methods are analysed.

1 INTRODUCTION

One central problem in photogrammetric data acquisition of Digital Elevation Models (DEM) is the selection of the points to be measured. This selection can be controlled by the operator as in the case of selective sampling, (manual selection of points), by a computer as in the case of progressive sampling or by predefined patterns as in the case of sampling in a regular grid.

To obtain a high quality DEM, the measured points must be selected with respect to the topography. Such a selection can be performed by using for example selective sampling or progressive sampling. In production, different sampling modes are often combined. Selective sampling of breaklines can for example be combined with profiling or progressive sampling. This paper treats problems concerned with selective sampling.

Manual selection of densification points are performed by an operator. The decision of where to densify is based on the topography of the terrain as judged by the operator. Such a selective sampling is usually performed in off-line mode, separated from the calculation of the DEM. The operator simply feeds a "black box" with co-ordinates, without receiving information whether the selected points are relevant or not. In some areas the points measured are redundant and in other areas they are not dense enough. The only response the operator gets is requests of remeasurements two to three days later, because of the off-line computation for checking of the data. Thus the experience is gradually built up, based on requests of remeasurements. The problems described can be summarized as follows

i) The operator will not see in which areas he/she have been measuring redundant information

ii) In area where the measurements are not dense enough the quality of the DEM will be decreased. If this is not discovered when checking the data, or when calculating the DEM, the usefulness of the DEM will be decreased

iii) If remeasurements are necessary, they are expensive to perform at a later stage

iv) It is hard for the operator to gain experience of the necessary sampling density when the result of the work is obtained days later, and perhaps another project has been started.

These problems are related to communication and feed-back of information. The operator is not receiving the necessary information while measuring the DEM. To solve this problem a two-way communication channel can be set up between the operator and the computer. This channel might consist of an interactive graphic system, where the operator is feeding the computer with co-ordinates, the computer analyses the data and displays for example contours on a graphic screen.

A comparison between the terrain in the stereomodel and the contours on the graphic screen gives information whether a densification is necessary. In addition, the operator can also evaluate the effect of a new registration by comparing the contours on the screen before and after the registration. In this way the experience of the operator is continously widened.

2 ALGORITHMS FOR SEQUENTIAL DATA PROCESSING

The contours are not recorded directly in the stereoinstrument, they have to be generated from a DEM. Since observations are continously recorded, the DEM has to be updated when an observation is entered or deleted from the system. From this updated DEM, new contour lines are generated, showing the present status of the DEM. The procedure to update the DEM is a sequential data processing procedure.

There are a number of interpolation methods available for DEM purposes. In a review by Shut, 1976, six different categories were presented. In this chapter four of these will be analyzed with respect to

i) The computing effort (the number of multiplication), ω.

It can be assumed that the CPU-time is proportional to the number of multiplications

ii) The number of variables to be updated when entering a new measurement, μ.

The performance (speed) of the system is depending on the number of disc accesses needed to update the DEM when a new measurement is entered. The number of disc accesses is depending on the buffer sizes, the number of variables to be updated and the method for accessing the information. The buffer sizes are partly depending on the primary memory space and will not be analysed here. The access methods are of course important, but they will only be shortly commented.

The analysis will be performed with respect to single point measurement only. This is due to the fact that algorithms and data structures for handling breaklines are not available for all methods.

2.1 The Method of Moving Surfaces

Assume that we have a number of measured points and a number of secondary points, where the elevations are to be estimated. The secondary points can be formed as a regular grid. From this regular grid, contours can be generated and displayed on a graphic screen.

The method of moving surfaces means that for each secondary point a surface is fitted to the surrounding measured points in a least squares adjustment. The surface can be a level plane, a tilted plane or a polynomial of higher degree. For each measured point observation equations are formed and weighted with respect to the distances to the secondary points. If the surface to be adjusted is a level plane, the method is also called the weighted mean method.

For an updating of the elevations of the secondary points, the elevations and the equation systems for each secondary point have to be stored. If a tilted plane surface is used, nine variables per secondary point are to be updated. If a level plane surface is used, only two variables per secondary point are to be updated.

The computing effort (ω) and the number of variables to be updated (μ) for the weighted mean method are

$$\omega_{wm} = 4 \cdot s\,(3 + y) \tag{2.1}$$

$$\mu_{wm} = 8 \cdot s \tag{2.2}$$

where

$$s = \left(\frac{d_{max}}{\delta}\right)^2$$

d_{max} is the limiting distance for the weight function

δ is the spacing between the secondary points

y is the number of multiplications necessary for the calculation of the weight of an observation.

For derivation of the formulas, see Östman, 1984.

A special case of weighted mean interpolation is linear interpolation. In this case, $y = 5$ and $s = 4$, giving

$$\omega_{LI} = 128 \tag{2.3}$$

$$\mu_{LI} = 32 \tag{2.4}$$

2.2 The Method of Adding Surfaces

- Interpolation methods belonging to this group are often called linear prediction least squares interpolation, collocation, etc. The term "adding surfaces" is based on a geometric interpretation of the methods.

The elevations between two arbitrary points, minus trend-surface, are supposed to be correlated according to function $K(d)$, which is a function of the distance d between the two points.

The elevation z_p in point p is then determined by

$$z_p = \bar{q}^T C^{-1} \bar{z} \tag{2.5}$$

where

$\bar{q}$ = vector with correlations between the point p and the measured point

C = autocorrelation matrix of the measured points

$\bar{z}$ = vector with measured elevations minus trendsurface.

To satisfy the underlying statistical properties a trendfunction must be subtracted from all measured elevations before the calculations can start. A common trendfunction is a polynomial of second order or higher, but other trendfunctions can be used. When the elevations have been computed according to formula 2.5, the trendfunction has to be added to the result.

The dimension of the autocorrelation matrix C is here equal to the number of measured points. When adding an observation the dimension of the matrix C increases.

Let C_{n-1} be the autocorrelation matrix when n-1 points are measured. When the n:th point is measured the inverse of the C_n^{-1} matric can be updated as follows

$$C_n^{-1} = \begin{bmatrix} C_{n-1} & \bar{c}_n \\ \bar{c}_n^T & q \end{bmatrix}^{-1} = \begin{bmatrix} F_n & G_n \\ G_n^T & H_n \end{bmatrix} \tag{2.6}$$

According to Mikhail, 1976, the matrices F_n, G_n and H_n can be computed as

$$H_n = (q - \bar{c}_n^T C_{n-1}^{-1} \bar{c}_n)^{-1}$$

$$F_n = C_{n-1}^{-1} + C_{n-1}^{-1} \bar{c}_n H_n \bar{c}_n^T C_{n-1}^{-1}$$

$$G_n = - F_n \bar{c}_n q^{-1}$$

If we count the number of multiplications and the number of variables needed to perform this updating, we obtain

$$\omega_{as} = 7 n^2 + n((c + 1) \cdot v - 10) + 5 - v \tag{2.7}$$

$$\mu_{as} = n(3.5 + 0.5 n) \tag{2.8}$$

where

n = number of measured elevations

m = number of secondary points

v = number of multiplications needed to calculate one correlation coefficient

and

$$c = \frac{m}{n}$$

When comparing equations 2.7 - 2.8 with 2.3 - 2.4 one finds that

$\omega_{as} < \omega_{LI}$ when $n < 3$, $c = v = 4$

$\mu_{as} < \mu_{LI}$ when $n < 6$

The methods demand about equal computer resources only when the number of measurements is very small. Since this usually not is the case, the method of moving surfaces (linear interpolation) is to be preferred compared to the method of adding surfaces.

2.3 Simultaneous Adjustment of Local Polynomials

The most well-known method of this type is Height Interpolation by Finite Elements (HIFI) as described by Ebner et al, 1980. The area is here divided into a grid, where the elevations at the grid nodes are unknown in a large equation system. The grid cells can be described by bilinear or bicubic polynomials, but in this paper only bilinear local polynomials will be discussed.

According to Mikhail, 1976 a sequential algorithm for the updating of the inverse of the normal equations and the solution vector is performed by

$$N_i^{-1} = N_{i-1}^{-1}(E - a_i^T(p_i^{-1} + a_i N_{i-1}^{-1} a_i^T)^{-1} a_i N_{i-1}^{-1}) \qquad (2.9)$$

$$X_i = X_{i-1} + N_{i-1}^{-1} a_i^T (p_i^{-1} + a_i N_{i-1}^{-1} a_i^T)^{-1} (l_i - a_i x_{i-1}) \qquad (2.10)$$

The calculation efforts and the number of variables to be updated are

$$\omega_{lp} = r^2 + 6r + 8 \qquad (2.11)$$

$$\mu_{lp} = \frac{r}{2}(r + 3) \qquad (2.12)$$

where r is the number of unknowns in the equation system. It can be noted that the calculation efforts are not depending on the bandwidth as in batch-oriented solutions.

When comparing equations 2.11 - 2.12 with the corresponding equations for linear interpolation (2.3 - 2.4) one finds that

$\omega_{LP} < \omega_{LI}$ when $r < 9$

$\mu_{LP} < \mu_{LI}$ when $r < 7$

The methods demand about equal computer resources only when the number of unknowns is very small. Since this usually is not the case, the method of simultaneous adjustment of local polynomial is of less interest for updating of the DEM.

2.4 Interpolation in a Net of Triangles

This interpolation method is used when deriving contours directly from the measured points, without using an intermediate regular grid. The method consists of two steps

i) Formation of triangles

   The formation is performed with respect to some geometric condition, where "goodlooking" triangles are created and stored in tables

ii) Interpolation in a triangle

   When interpolating in a triangle a local surface can be adjusted to the triangle nodes. The surface can be a plane, corresponding to linear interpolation.

The analysis is here performed only for the algorithms for the formation of triangles.

It is desirable that the resulting triangulation should be independent of the starting point or the order of measurements. This is not always the case (McCullagh and Ross, 1980). If a unique solution is to be preferred, the number of triangles influenced by an additional measured point will vary depending on the geometry. Figures 1 - 2 show two examples where a different number of triangles are updated.

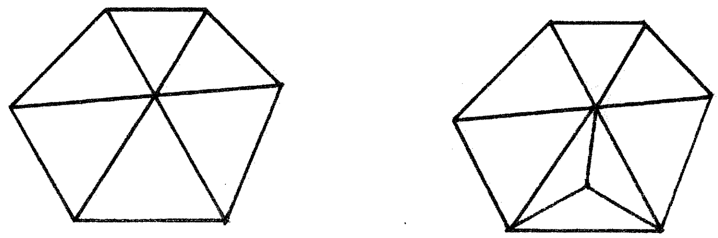

Figure 1 Updating of triangle formation.
One triangle is split into three triangles

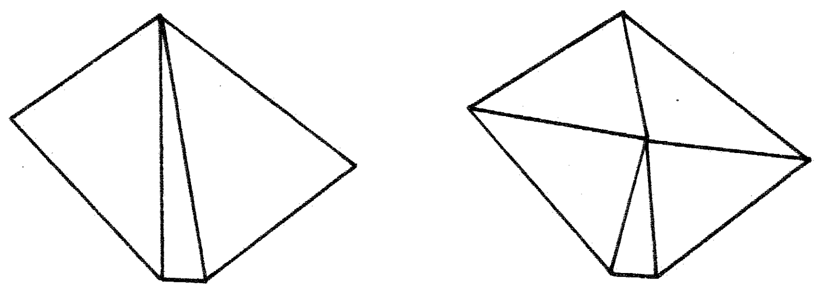

Figure 2 Updating of triangle formation.
Three triangles are split into five triangles

The number of parameters to be updated and the computing effort can be estimated as

$$\mu_{NT} = 9 \cdot k + 12 \qquad (2.13)$$

$$\omega_{NT} \approx 20 \cdot (k + 1)^2 \qquad (2.14)$$

where k is the number of triangles affected by each registration, $k > 1$.

For derivation, see Östman, 1984.

When comparing the method with the moving surface method (figures 3 - 4) one finds that the method of moving surfaces is slightly better than interpolation in a net of triangles. In addition, when the moving surface method is used, the necessary information can be retrieved by a direct access method. This is not the case when interpolating in a triangular network, because of the random distribution of measured points.

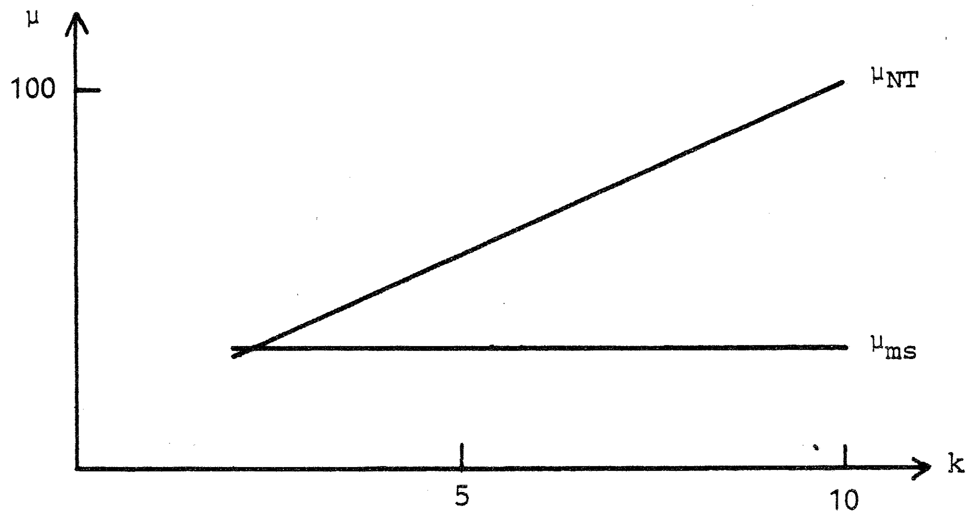

Figure 3 Number of variables to be updated when interpolating in a net of triangles and the method of moving surfaces, s = 4

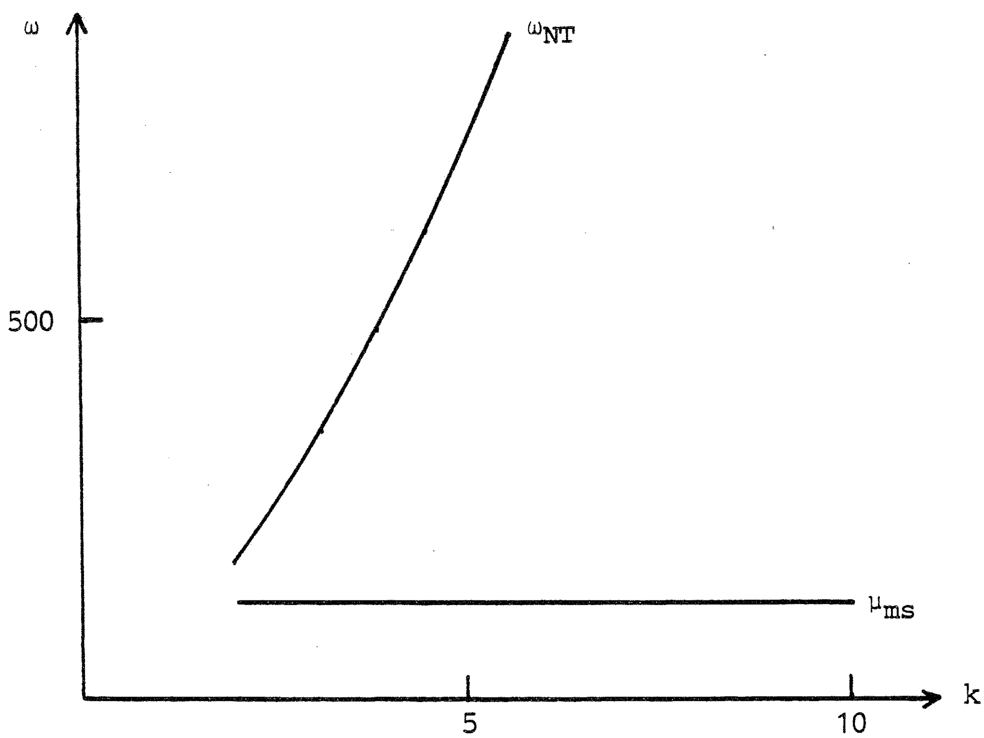

Figure 4 Computing efforts for interpolation in a net of triangles and the moving surface method s = 4, y = 5

3 ALGORITHMS AND DATA STRUCTURES

The present hardware used at the Department of Photogrammetry, Royal Institute of Technology, Stockholm, consists of

i) Wild stereoautographs A7 and A8 with co-ordinate recording devices

ii) A general-purpose minicomputer with 42 kbytes of core-memory available for the user

iii) An intelligent graphic screen, Tektronix 4112.

The system uses the moving surface method when updating the DEM. The reasons are mainly the following

i) The method demands less computer resources compared with other interpolation methods

ii) Direct access methods can be used when retrieving the information.

The elevation in a grid node (i, j) is calculated as

$$\tilde{z}_{i,j}(n) = \frac{\sum_1^n p_k z_k}{\sum_1^n p_k}$$

where

$\tilde{z}_{i,j}$ = calculated elevation in node (i, j)

z_k = measured elevation in point k

p_k = weight, depending on the distance between point k and node (i, j).

When adding a new single point observation, the updated elevation in grid node (i, j) is calculated as

$$\tilde{z}_{i,j}(n+1) = \frac{\sum_1^n p_k z_k + p_{n+1} z_{n+1}}{\sum_1^n p_k + p_{n+1}}$$

or

$$\tilde{z}_{i,j}(n+1) = \frac{\tilde{z}_{i,j}(n) \cdot \sum_1^n p_k + p_{n+1} z_{n+1}}{\sum_1^n p_k + p_{n+1}}$$

The information needed in this procedure is

i) The observed elevation of point (n+1) and the corresponding weight, (z_{n+1} and p_{n+1})

ii) The elevation $\tilde{z}_{i,j}(n)$

iii) The accumulated weight $\sum_1^n p_k$

The data structure of the system, is described in figure 5.

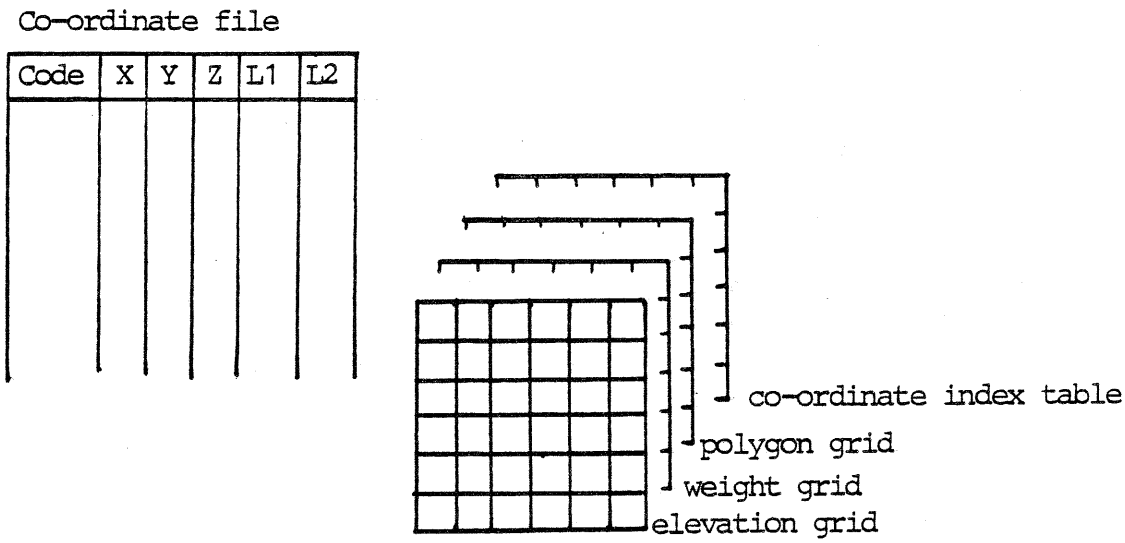

Figure 5 Data structure

The data are stored in four grids and one sequential file. To enable up-dating, all files should be of random access. When a single point is entered, the elevation grid is updated by using the weight and polygon grids. The polygon grid contains information concerning breaklines and other line-features. The co-ordinates of the measured points are stored in the sequential co-ordinate file. For accessing this file, the co-ordinate index table is used, which is also formed as a grid.

When a polygon (breakline) is entered, it will modify the weight functions of the surrounding observations. Then the co-ordinate file must be searched and the response time will increase. But since the index table is gridded and sparse, this procedure will also be fast, but not as fast as in the case of single point measurement.

For the display of contours only the elevation grid is used. Linear inter-polation is performed along the borders of the grid cells and polygons are constructed from intersections between the contours and the border of the grid cells.

4 DISCUSSION

The system here described is rather limited. Examples of possible extensions are the abilities of measuring ridges and displaying the measured line-features such as breaklines as dotted lines. The structure of the software admits such extensions.

The output of the system consists of the displayed contours, the edited observations and the elevations at the grid nodes. The edited observations can be used as input to another system which, for some reason, is better for the present application. In such cases, the interactive system described, acts as an on-line graphical editor of the measured elevations. In other cases the elevations at the grid nodes can be used, for example when gene-rating contour maps.

Recent work by Förstner, 1983, shows that the morphological quality of a DEM increases when additional form measurements are used. When introducing such measurements in a production environment, the operator has to gain experience. In such a case an interactive graphic system will be valuable.

There seems to be a need for more sophisticated sampling methods than the "blind registration". One such a sophisticated method is the method of progressive sampling. A limitation in this method is the fact that the computer is analysing the data and conducting the sampling process using only a limited amount of information concerning the terrain elevations. The interactive approach described in this paper implies that the computer is analysing the data and the operator is conducting the sampling procedure. Since the operator can see the terrain surface in the stereomodel and compare it with the result of the computer analysis, all information is available for the conductor of the sampling process. On the other hand, the progressive sampling is much faster than an operator-conducted sampling. Depending on the terrain type and the requirements on the DEM, the optimal solution will differ. There might also be procedures where progressive sampling is combined with interactive graphics.

The system is not yet in production but future research and practical experience will show whether the approach is successful or not.

LITERATURE

Ebner H, Hofman-Wellenhof B, Reiss P, Steidler F. HIFI - A Minicomputer Program Package for Height Interpolation by Finite Elements. International Archives of Photogrammetry, Vol XXIII, Part B4, pp 202-215, 1980

Elfick M H. Contouring by Use of a Triangular Mesh. The Cartographic Journal, Vol 16, No 1, pp 24-29, 1979

Förstner W. On the Morphological Quality of Digital Elevation Models. Proc of International Colloquium on Mathematical Aspects of Digital Elevation Models. Department of Photogrammetry, Royal Institute of Technology, Stockholm, 1983

Grün A. An Optimum Algorithm for On-Line Triangulation. Presented paper to the symposium of Commission III of the ISPRS, Helsinki, 1982

Kevany M J. Interactive Graphics as the Mechanism for Communication between Information and User. 8th European Urban Data Management Symposium, Oslo, June 2-6, 1981

Knuth D E. The Art of Computer Programming - Volume 3. Addison-Wesley Publishing Company, Reading, Massachusetts, 1973

Leupin M. Interaktive Geländeerfassung mit analytischen Plottern. Vorträge der 39. Photogrammetrische Woche. Institut für Photogrammetrie, Stuttgart, 1983

Makarovic B. From progressive to composite sampling for digital terrain models. Geo-processing, Vol 1, No 2, pp 145-166, 1979

McCullagh M J, Ross C G. Delauney Triangulation of a Random Data Set for Isarithmic Mapping. The Cartographic Journal, Vol 17, No 2, pp 93-99, 1980

Mikhail E M. Observations and Least Squares. IEP, New York, 1976

Shut G H. Review of Interpolation Methods for Digital Terrain Models. International Archives of Photogrammetry, Vol XXI, Part 3, Helsinki, 1976

Östman A. Design of Interactive Graphics for DEM Sampling. International Archives of Photogrammetry, Vol 24, Part 3, Helsinki, 1982

Östman A. Interactive Graphical Control of Photogrammetric Measurements for Digital Elevation Models. Department of Photogrammetry, Royal Institute of Technology, Stockholm, 1984.