A High-Efficient Algorithm for Geometric Rectification of remote Sensing Digital Imagery

Chang Benyi
Research Institute for Survey and Mapping
People's Republic of China

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Abstract: The geometric rectification of LANDSAT MSS digital imagery is generally performed by use of polynomials. In this peper an efficient increment operation algorithm based on the features of digital image processing is developed. It is shown, that in this algorithm all multiplications necessary in geome - tric rectification of digital imagery can be replaced by the addition operations, thus a considerable reduction in number of operations is achieved, and a relatively high-efficient algo- rithm is obtained. According to the developed algorithm, a FORTRAN computer program is written and tested on a small com- puter with satisfactory results.

The geometric rectification of LANDSAT MSS digital imagery is generally performed using polynomials because the algorithms ba sed on polynomials are simple and high-efficient, showing litt- le differences in computing accuracy, as compared to others [1]. Nevertheless, in the case of digital processing of remote sensing imagery the simple polynomial interporation needs much time, because the information to be processed is bulky. For example, there are 7.5 Mp (megapixels) in a single band of LANDSAT MSS imagery with the pixel dimension on ground of 79 x 56 m. By reducing the pixel dimension to 50 x 50 m, the total number of pixels is increased to 13 M. Obviously, it takes very long computer time even if the simple 3rd order polyno- mials are used. According to Yang Kai and Wen Wogeng [1], 10.5 minutes of CPU time are needed to process 512 x 512 pixels on VAX 780 with 3rd order polynomials, using the nearest neighbor- hood assignment method. It means, processing of 13 M pixels re quires more than 9 hours of computer time. It is concluded, therefore, that any geometric rectification method relating to LANDSAT MSS imagery processing would win practicability only when a convenient and efficient algorithm is available.

In digital image processing, application of an increment opera- tion method based on that the digital image is an orderly arranged two-dimensional data array often leads to improvement in operating rate.

Let x, y be coordinates of a pixel in the original image and X, Y the rectified coordinates of the same pixel with relation- ship

$$x = f_x(X,Y), \quad y = f_y(X,Y). \tag{1}$$

In the case of two-dimensional array, the coordinates of neigh- bouring pixels are related by

$$X_{i+1,j} = X_{i,j} + dX, \quad Y_{i+1,j} = Y_{i,j}, \tag{2}$$

and

$$X_{i,j+1} = X_{i,j},$$
$$Y_{i,j+1} = Y_{i,j} + dY. \quad (3)$$

For the convenience of discussion the recurrence formula for function f_x in (1) will be derived only. Besides, it begins with relation (2) with i being variable and j being fixed, therefore the subscript j will be omitted.

By expanding function f_x at X_i into Taylor series we obtain

$$f_x(X_{i+1}, Y) = f_x(X_i, Y) + f_x'(X_i, Y)\, dX + \frac{f_x''(X_i, Y)}{2!} dX^2 + \frac{f_x'''(X_i, Y)}{3!} dX^3 + \ldots\ldots \quad (4)$$

Assuming function f_x to be a 3rd order polynomial as

$$x = a_0 + a_1X + a_2Y + a_3XY + a_4X^2 + a_5Y^2 + a_6X^2Y + a_7XY^2 + a_8X^3 + a_9Y^3, \quad (5)$$

it is possible to determine the coefficient by each term in / series (4):

$$f_x'(X_i, Y) = a_1 + a_3Y + a_7Y^2 + (2a_4 + 2a_6Y + 3a_8X_i)\, X_i,$$
$$f_x''(X_i, Y) = 2a_4 + 2a_6Y + 6a_8X_i, \quad (6)$$
$$f_x'''(X_i, Y) = 6a_8.$$

Substituting (6) into (4) and ordering by merging of similar terms we obtain following recurrence formula:

$$x_{i+1} = x_i + \Delta x_1 + \Delta x_{2i} + \Delta x_{3i}, \quad (7)$$

where

$$\Delta x_1 = (a_1 + a_3Y + a_7Y^2 + (a_4 + a_6Y)\, dX + a_8dX^2)\, dX,$$
$$\Delta x_{2i} = (2\,(a_4 + a_6Y) + 3a_8dx)\, dX \cdot X_i,$$
$$\Delta x_{3i} = 3a_8dX \cdot X_i^2.$$

Formula (7) shows the capability to provide the same results as expression (5), but is considerably simplified. In this formula Δx_1 is a constant, independent on X_i, while Δx_{2i} varies with X_i and Δx_{3i} varies with X_i^2. Moreover, similarly (7) it is / also possible to derive corresponding recurrence formulas for Δx_{i2} and Δx_{i3} thus converting all the computations in (7) into addition operations only.

Now we return to derive the expression for computing (7). In practice, it is convenient to assume $X_1 = 0$, $dX = 1$ (i.e. distance between pixels is normalized), then the expression is obtained as following:

$$x_{i+1} = x_i + \Delta x_1 + \Delta x_{2i} + \Delta x_{3i}, \qquad (8)$$

where

$$\Delta x_1 = C_1,$$

$$\Delta x_{2i} = \Delta x_{2i-1} + C_2,$$

$$\Delta x_{3i} = \Delta x_{3i-1} + d\Delta x_{i-1},$$

$$d\Delta x_{i-1} = d\Delta x_{i-2} + C_3.$$

In expression (8), the initial values (i.e. for i=1) are

$$x_1 = a_0 + (a_2 + a_5 Y + a_9 Y^2)\,Y,$$

$$\Delta x_1 = \Delta x_{21} = \Delta x_{31} = 0,$$

$$d\Delta x_1 = C_4;$$

and the constants are

$$C_1 = a_1 + a_4 + a_8 + (a_3 + a_6 + a_7 Y)\,Y,$$

$$C_2 = 2a_4 + 3a_8 + 2a_6 Y,$$

$$C_3 = 6a_8,$$

$$C_4 = 3a_8.$$

It is seen from (8), that 6 addition operations are enough to compute x_{i+1} with x_i as given and C_1, C_2, C_3 and C_4 as known. In this case, 3 addition operations are needed to evaluate the function itself, 2 operations are required to compute the cor - rection for the function value, and the last one is performed to obtain the secondary correction. The another 6 addition operations are needed to evaluate y_{i+1} with y_i as known. Therefore, a total of 12 addition operations are enough for each pixel to be processed, provided the described in this paper algorithm is used in geometric rectification of degital imagery.

Now we'll condider the case in (3) with i being fixed and j being variable. It is convenient to initiate our discussion from expression (8) by assuming C_1 and C_2 to be varied with Y. In analogy to the derivation of recurrence formula for function / f_x, it is assumed that $Y_1 = 0$, $dY = 1$, and the expression for practical computation can be obtained then.

The recurrence formula for x_j will be similar to (8), because / x_j is a 3rd order polynomial too.

Thus

$$x_{1j+1} = x_{1j} + \Delta x_{11} + \Delta x_{12j} + \Delta x_{13j}, \qquad (9)$$

where

$$\Delta x_{11} = D_1,$$

$$\Delta x_{12j} = \Delta x_{12j-1} + D_2,$$

$$\Delta x_{13j} = \Delta x_{13j-1} + d\Delta x_{1j-1},$$
$$d\Delta x_{1j-1} = d\Delta x_{1j-2} + D_3.$$

In (9), the initial values are

$$x_{11} = a_0,$$
$$\Delta x_{11\,1} = \Delta x_{12\,1} = \Delta x_{13\,1} = 0,$$
$$d\Delta x_{1\,1} = D_4;$$

and the constants are

$$D_1 = a_2 + a_5 + a_9,$$
$$D_2 = 2a_5 + 3a_9,$$
$$D_3 = 6a_9,$$
$$D_4 = 3a_9.$$

C_1 is a 2nd order polynomial and its recurrence formula is

$$C_{1j+1} = C_{1j} + \Delta C_{11} + \Delta C_{12j}, \qquad (10)$$

where

$$\Delta C_{1\,1} = E_1,$$
$$\Delta C_{12j} = \Delta C_{12j-1} + E_2.$$

In formula (10), the initial values are

$$C_{11} = a_1 + a_4 + a_8,$$
$$\Delta C_{11\,1} = \Delta C_{12\,1} = 0;$$

and the constants are

$$E_1 = a_3 + a_6 + a_7,$$
$$E_2 = 2a_7.$$

C_2 is function linearly varying with Y and its expression is

$$C_{2j+1} = C_{2j} + \Delta C_{21}, \qquad (11)$$

where

$$C_{21} = 2a_4 + 3a_8,$$
$$\Delta C_{21\,1} = 0 \quad (\text{for } j=1),$$
$$\Delta C_{21} = 2a_6 \quad (\text{for } j \neq 1).$$

Expressions (9), (10) and (11) show, that for processing a new image line following j-th line, all initial values varying with Y, as well as all constants can be estimated by means of the / above derived recurrence formulae and performed with unconsiderable number of addition operations. The conclusion is, therefore, that the addition operation is the only one, which is necessary to perform the geometric rectification of LANDSAT MSS /

digital imagery by polynomials, using the algorithm developed in this paper.

The efficiency of this algorithm has been well demonstrated in practical computations, as shown in table 1. It is seen from the data summarized in table 1, that processing speed of our algorithm is much faster than that reported in[1].

Table 1

computer type	capacity of internal memory	number of pixels	system time	external storage
LSI-2	32K	7000 x 70	9^{m}	disc
LSI-2	32K	2300 x 1900	$3^{h}24^{m}$	disc
HP-~~MX~~ 21 MS	128K	2400 x 2000	$1^{h}02^{m}$	tape
HP-~~MX~~ 21 MS	128K	3200 x 3000	$1^{h}45^{m}$	tape

A system for geometric rectification of LANDSAT MSS digital imagery featured mainly by the above described algorithm has been developed. In addition to relatively high economic efficiency, the system is able to provide satisfactory accuracy of processing. The results of first batch of LANDSAT MSS imagery processed by this system with use of 1:100,000 topographic map as control base have shown, that the RMS error in position is about 90 m, which satisfies in general the accuracy requirements for 1:250,000 scale topographic mapping.

References

[1] Yang Kai and Wen Wogeng, The Geometric Rectification of Digital Multispectral Scanner (MSS) Imagery in Remote Sensing, Journal of the Wuhan College of Geodesy, Photogrammetry and Cartography, 1982, No. 1, 77 - 90.