

COMBINED ADJUSTMENT OF AIRBORNE NAVIGATION DATA AND PHOTOGRAMMETRIC BLOCKS

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1. Introduction

1.1 Combined adjustment of different kinds of observation data, also referred to as hybrid observations, is a most powerful tool of modern data processing. It has become practicable by the appearance of new measuring techniques, by computer processing, and by the development of statistical adjustment theory. In this paper, our considerations are restricted to aerial triangulation which is a prime example of the expediency of combined adjustment.

It is a particularly interesting case of hybrid observations if a main set of standard observations in a system is complemented and supported by another preferably small set of additional observations of different kind, by which certain deficiencies or weaknesses of the main system can be efficiently and economically remedied, in the sense of good systems engineering. With aerial triangulation in mind we can define the following list of tasks for which combined adjustment of additional observations may be expedient or necessary:

- (1) restore rank deficiencies concerning datum
- (2) restore rank deficiencies concerning configuration
- (3) control systematic errors (model deficiencies)
- (4) control propagation of random errors (precision)
- (5) ensure reliability

It must be kept in mind, however, that additional observations of different kind usually imply an extended mathematical model, especially with regard to tying them into the system. The consequences are additional datum problems and transformation relations with additional unknown parameters, and additional systematic errors and reliability problems as well.

1.2 In aerial triangulation image coordinates (or model coordinates) constitute the standard type of observations. The resulting network of points has deficiencies and weaknesses which, in classical aerial triangulation, are almost entirely remedied by ground control. Ground control coordinates are now regularly treated as weighted observations. Therefore, conventional block adjustment with ground control can be seen as an example of combined adjustment.

However, the concept of combined adjustment has hitherto been used in aerial triangulation in a more specific way, concerning introduction of observed camera orientation data, in particular statorscope- or APR-data, into combined block adjustment, (1), (2). In those days already, when proper combined adjustment was not yet practicable, the prime purpose of such "auxiliary" observations was reduction of (vertical) ground control with regard to items (3) and (4) of the above list. The example of the statorscope also points to the problem, that additional data imply extension of the mathematical model and require certain datum transformations in order to be linked into the system. Combined adjustment of photogrammetric blocks with statorscope- or APR-data has proven to be particularly effective and economic, thus demonstrating the success of well chosen auxiliary observations. It is regrettable that practical application in small-scale and medium-scale aerial triangulation has not been as widespread as it would have deserved. One might even say that, from the point of view of systems engineering, aerial triangulation in that range of application has fallen back behind the performance level of 30 years ago. Has it happened in spite of or because of the development of modern block adjustment?

It may be mentioned that in close-range-photogrammetry the principle of combined and in various ways extended adjustment is considerably further developed than in aerial survey applications.

1.3 This paper is concerned with a new class of observations, relating to airborne navigation systems, and their utilisation for aerial triangulation by combined adjustment. Such navigation systems are, (3), (4):

- INS inertial navigation system
- CPNS computer controlled photo navigation system, by in-flight trilateration to ground-transponders
- GPS Navstar Global Positioning System.

The primary task of such systems is positioning in real time for navigation purposes. However, by post-processing of recorded data high precision positioning can be obtained off-line, for subsequent use. In this paper we are solely concerned with post-processed positioning data, relating ultimately to the position of the air survey camera in the aircraft, at the times of exposure. All 3 navigation systems are capable of delivering position data (CPNS only x , y , unless supported by barometric data; INS may give, in addition, attitude data). We shall deal, in the following, only with xyz position data, and concentrate on the GPS system, as it is a most precise and certainly the most economic system. As far as the method of combined adjustment is concerned, CPNS and INS data are also implied, (5).

Camera positioning data as derived from navigation systems resemble very closely the well known case of barometric statorscope data. Hence, a similar approach to the combined adjust-

ment is taken in the following. And similar effectiveness is expected.

2. GPS kinematic positioning

2.1 Navstar GPS (global positioning system) is a universal satellite positioning system for real time navigation, (6). It is, at present, in an incomplete experimental stage, and can locally be used only within certain time windows which change in time and geographic position. When fully operational (in 1990?) practically anywhere on the earth and at any time at least 4 satellites will be visible, allowing permanent signal receiving for real time navigation or for post-processing of data. We consider here only post-processing of data for the purpose of high precision assessment of the trajectory of a moving air survey aircraft. It is known as kinematic positioning.

Each GPS satellite transmits permanently 2 electromagnetic carrier waves: L1 at 1575,42 MHz ($\lambda_1 = 19$ cm), L2 at 1227,60 MHz ($\lambda_2 = 24$ cm). The L1 carrier wave is phase-modulated by 2 PN (pseudo-noise) codes known as P (precision)- and C/A (coarse acquisition)-code, with code frequencies of 10 MHz and 1 MHz, respectively. The wave L2 carries only the P-code. Also, additional messages are transmitted in low frequency, carrying the identification of the satellite, the ultra-precise time signal, orbit data etc.

GPS positioning is based on simultaneous distance measurements from a stationary or moving receiver to at least 4 satellites. There are 2 types of distance observations possible. The first method assesses essentially the travelling time of the signal, giving a distance measure which is known as "pseudo-range". It relies on the C/A-code or on the P-code. As the clock error of the receiver constitutes a major unknown parameter simultaneous pseudo-range measurements to at least 4 satellites are required for positioning. Their inherent precision (resolution), amounting to 1% of the wavelength, is 0.3 m and 3 m for P-code and C/A-code, respectively. The second kind of measurements is essentially code-free, operating as phase measurement directly on the carrier waves L1 or L2. It is particularly precise, the resolution of 1% of the wavelengths of the L1 and the L2 carrier waves amounting to 19 mm and 24 mm, respectively. However, the main part of the range, constituted by the number N of integer wavelengths, remains unknown. N is called the unknown ambiguity parameter, associated with each phase measurement. It has to be solved in the data processing. Therefore phase observations are only used for positioning by post-processing.

The intended application of GPS kinematic positioning of camera air stations for combined aerial triangulation adjustment will essentially be based on phase observations, which are generally accessible.

There are various GPS receivers available which give pseudo-range and phase observations at a high rate, as it is required for kinematic positioning. It is suggested that on board the aircraft a receiver is used with preferably 5 (or more) channels for simultaneous and continuous lock on 5 satellites (or more), if visible. Such receivers seem not to suffer from cycle slips, other than by obstruction. Observations should be pseudo-ranges and phases. It is sufficient for our purpose if the observations refer to L1 C/A-code and L1 carrier phase. Most kinematic receivers allow observations at rates of about 1 sec (0.6 sec). In order to transfer the inherent observation accuracy without noticeable loss to the exact time of camera exposure by interpolation observation rates of 0.2 sec or less would be desirable. There are receivers which can observe at the exact moment of camera exposure, triggered by the time signal.

It is preferable, although not absolutely necessary, to use a stationary GPS receiver on the ground, in addition to the on-board receiver. The stationary receiver is placed, if possible, on a known ground control point, preferably within the block area or close to it. The stationary observations are taken at the same time rate, in order to allow differencing.

2.2 There are many concepts of processing GPS observations, distinguished especially by the various ways of modelling and eliminating systematic errors. With regard to the kinematic mode the possibilities are restricted. Further elaboration is still a matter of research. We shall assume, in the following, the simplest and most straight forward mode of data processing which, as we shall see, will sufficiently serve our purpose. It means that the phase observations of either receiver, relating to the respective observation times t_1 , are directly and independently processed into position coordinates, by using the conventional GPS positioning formulae, based on simultaneous phase measurements to 4 or more satellites. The main point is, that the observations are used directly, only corrected for by a priori known corrections as submitted by the satellite message signals. In particular, no attempt is assumed for modelling or eliminating systematic errors. Consequently, the computed positions, which refer to the phase centre of the antenna of a receiver, will be badly affected by the non-compensated systematic errors. They are in particular: Satellite position and orbit errors, satellite clock error, unknown ionospheric and topospheric refraction. The ambiguity parameter N of the phase measurements is supposed to be approximately determined, for instance from the pseudo-ranges observed in the C/A- or P-code. Other calibration of N (by going over a fixed point at the airport) is feasible. Once a channel of a receiver is locked onto a satellite it can be assumed that the changes of N are monitored. The unknown rest in the preliminary determination of N acts as additional systematic error in the GPS position. As it is against the philosophy of stationary geodetic GPS positioning it is emphasized once more that no

particular effort is made to compensate systematic errors at that stage.

Computed GPS positions refer to the WGS 84 coordinate system, which is a cartesian coordinate system centered at the gravity centre of the earth. All subsequent computations, including the aerial triangulation block adjustment, may be done in that coordinate system, especially if all ground control points are also determined by GPS. It is expedient, however, to apply a preliminary transformation into a local horizon system, or even into a given map projection and a local vertical datum.

The ultimate purpose of the kinematic GPS positioning of the aircraft are the GPS-coordinates of the camera stations of the air survey camera at the moments of exposure. More precisely it is the perspective centre of each photographic exposure which is to be determined. Hence, a time signal is needed, precise to about 1 msec (preferably 0.1 msec, as we shall see), indicating the central moment of exposure. For that moment the GPS position (of the receiver antenna) is to be determined, by simple interpolation or by sophisticated filtering and prediction from the surrounding positions, unless measured directly. It would, therefore, be sufficient to process only some GPS recordings which are close (in time) to the camera exposure. However, for safety and reliability reasons it is suggested to calculate positions, as described, for all GPS recordings. Any discontinuities or disturbances would be spotted in this way.

Camera perspective centre and GPS antenna cannot physically coincide in the aeroplane. There is an off-set between the 2 points. The magnitude of the off-set is constant, whilst the coordinate components are not, being influenced by the attitude parameters during the flight. It is suggested to measure as good as possible the off-set in the aircraft, and also the coordinate components with regard to an aircraft-fixed coordinate system. If possible the GPS antenna may be placed directly vertical above the camera, thus the direction of flight (κ) has a negligible effect only. It is not suggested to reduce the GPS antenna coordinates directly for the off-set onto the camera position (nor only approximately by constant terms), as it is not necessary in our approach. It should be mentioned, however, that correction for the off-set between GPS antenna and sensor position can be a tricky problem in other cases.

2.3 The utilization of kinematic GPS positions in the combined adjustment with aerial triangulation depends vastly on the essential error properties, about which new experimental results have become available very recently. They demonstrate strikingly the outstanding accuracy of kinematic GPS positioning and are the basis and justification for the approach to combined adjustment which will be suggested in chapter 3. Therefore, the relevant empirical results are briefly summarized. They refer to the GPS-test "Flevoland" carried out by the Rijkswaterstaat authority in the Netherlands, joined by Stuttgart University. The investigation, as far as completed, concerns 7 large-scale photo strips (image scale 1 : 3 800, 130

photographs), covering an area of about 2,5 x 4 km² containing 47 ground control points, see (7), (8). GPS data (L1 C/A-code and phase observations) were recorded, at a rate of 0.6 sec, with 2 Sercel receivers: type TR5SB in the aircraft, type NR52 on one of the ground control points in the test area. Both receivers have 5 parallel channels for continuous lock on upto 5 satellites. Actually signals from the satellites no. 6, 8, 9, 11, 12 were received. During one part of strip 2 the aircraft receiver lost lock on satellite 11, leaving the kinematic positioning for that part (strip 2.1) based on the remaining 4 satellites. In the latter case the geometric strength of the solution deteriorates considerably, the DOP-factor (dilution of precision) jumping from values of around 3.5 to 40. The data were analyzed per strip, giving data sets of only 60 sec duration, because of the short strip lengths of only 4 km. The stationary receiver recorded continuously all 5 satellites over periods of 1 h 33 min and 1 h 18 min, during the flight mission on 10 and 12 June 1987.

After independent and straight forward computation of all GPS positions the original WGS 84 coordinates were transformed into a local cartesian horizon coordinate system and centered on the stationary receiver. Thus, the aircraft positioning corresponds to a differencing method, by which some systematic errors are supposed to be considerably reduced.

In this way preliminary coordinates of the GPS antenna in the moving aircraft were obtained in dense sequence, from which the GPS positions at the moments of camera exposure were interpolated. The latter were compared with the "true" camera air stations as obtained by independent high precision aerial triangulation, relating to the ground control coordinate system which is also cartesian in this case. The precision of the photogrammetric coordinates of the air stations was assessed, by simulation, to 4.3 cm, 4.1 cm, 1.9 cm in x,y,z, respectively.

stages	ΔX		ΔY		ΔZ		ΔKoo		vector Δs	
	a	b	a	b	a	b	a	b	a	b
	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm
(1) strips exc. strip 2.1		3 m 33 m	-	32 m 356 m						
(2) strips exc. strip 2.1	7.3 197	7.8 318	8.9 582	12.9 940	5.6 117	14.3 196	7.4 361	12.0 584	8.0 624	12.0 1011
(3) all strips strip 2.1	5.5 4.5	5.5 5.4	6.1 7.3	7.6 13.5	3.9 3.3	4.6 4.0	5.2 5.3	6.0 8.7	3.4 3.7	3.9 6.3
(4) all strips	3.4	3.4	4.5	6.4	3.4	4.2	3.8	4.8		

Table 1. Test Flevoland; R.m.s. differences between GPS- and "true" camera positions.
a: with stationary receiver, b: without stationary receiver
(1) brute GPS coordinates, (2) after shift corrections,
(3) after linear corrections,
(4) after subtraction of photogrammetric errors of camera station coordinates

The empirical accuracy investigations considers 3 stages of reduction of systematic errors. Also 2 cases of kinematic positioning are distinguished: (a) making use and (b) making no use of the stationary receiver. The results are summarized in table 1. They give rise to the following comments:

- (1) The brute coordinates of the GPS antenna in the moving aircraft are off by tens and hundreds (strip 2.1) of meters. They are not useful for photogrammetric positioning.
- (2) The large differences between the GPS positions and the true camera positions can be drastically reduced by applying shifts in x,y,z per strip. The remaining coordinate differences amount to only 7.4 cm and 12.0 cm, with and without stationary receiver, respectively. For strip 2.1 (4 satellites), however, the respective r.m.s. differences are 3.6 m and 5.8 m. Thus, although GPS positioning may be, at that stage, more than precise enough for photogrammetric application one cannot really rely on it, depending on the satellite constellation used. The differences may exceed magnitudes of 10 m, especially with longer strips.
- (3) A closer look at the results of stage (2) shows that there remain linear drift effects in all cases. The drift rates range from -4 to +6 mm/sec (-12 to +9 mm/sec without stationary receiver), however they jump up to magnitudes of 0.7 m/sec (and 1.1 m/sec, respectively) in strip 2.1 (4 satellites). After removing the linear systematic errors by linear regression the r.m.s. coordinate differences between GPS coordinates and camera positions are reduced to 5.2 cm and 6.0 cm, with and without stationary receiver, respectively. It is remarkable that now strip 2.1 also reaches the same level of precision.
- (4) The "true" camera stations in the above comparison still contain the errors of their photogrammetric determination. They may be subtracted, giving as final result 3.8 cm and 4.8 cm as the r.m.s. coordinate - accuracy of kinematic GPS positioning, after stripwise removal of linear systematic errors. Most remarkable is the value of 4.8 cm which is obtained with the on-board receiver alone.

The experimental results of table 1 still contain 2 error effects which have not been removed:

- The effects of attitude variations on the coordinate differences between camera and GPS antenna in the aircraft. (They are visible in the vector as compared to the coordinate components.)
- Linear interpolation of GPS coordinates onto the actual time of camera exposure, within the 0.6 sec time interval during which the aircraft moved on for about 40 m.

It is certainly a surprise to most photogrammetrists that under practical operational conditions of kinematic GPS positioning coordinate accuracies of a few cm are reached in a rather straight forward way. There were no particular difficulties encountered, also no cycle slips occurred. The accuracy results are not far off the theoretical expectation, see (8), (9).

Also, the results were anticipated as some previous tests had reached similar figures, (10) - (12).

Δt	60 sec	20 sec	300 sec	600 sec
r.m.s. ΔX	0.6 cm	1.2 cm	2.8 cm	6.1 cm
r.m.s. ΔY	1.3 cm	3.0 cm	8.0 cm	21.4 cm
r.m.s. ΔZ	1.8 cm	3.3 cm	7.6 cm	16.5 cm
r.m.s. ΔK_{00}	1.3 cm	2.7 cm	6.6 cm	16.0 cm

Table 2. Test Flevoland, stationary receiver, recording period 77 min. R.m.s. coordinate residuals after successive linear regressions over various time intervals Δt .

It is essential to realize that the high accuracy of kinematic GPS positioning is only obtained if systematic errors are effectively compensated. The Flevoland results show that linear corrections are highly effective and sufficient, if applied independently per strip. However, the strips were very short (4 km, 60 sec). Therefore the question remains over which time intervals and distances the GPS drifts stay reasonably linear, especially as photogrammetric flight strips in small scale applications can take 10 min or more. A preliminary empirical answer can be given by looking at the GPS positions derived from the continuous recordings of the stationary receiver. Over a time period of 1 h 18 min the computed stationary GPS positions drift off in a highly non-linear and partly irregular way, by about 10 m in plan and about 50 m vertically. However, if the total drift is replaced by discrete linear regressions of certain duration the r.m.s. residuals stay, for instance, below 10 cm magnitude for time intervals of 5 min, see table 2. Thus, application of linear corrections seems vastly sufficient for standard photogrammetric requirements.

3. Combined Adjustment

3.1 We consider here the combined adjustment of photogrammetric block data and airborne GPS data. The GPS data are supposed to be given as brute GPS coordinates, derived essentially from phase observations and relating to the antenna on board the aircraft, for the moments of camera exposure. An additional stationary receiver may or may not have been used. The original GPS coordinates (WGS 84 coordinates) are assumed to be transformed into a local cartesian horizon system (or properly into a given geodetic system). Those coordinates are treated, for the time being, as uncorrelated observations

(autocorrelation subsides after about 2 sec; the 3 x 3 covariance matrices may be considered), weighted with regard to the variance factor σ_0^2 as defined by the photogrammetric observations.

The GPS coordinates are related to the block coordinate system by additional linear transformation terms, the parameters of which are considered unknown to be solved for in the combined adjustment. The terms are set up independently for X,Y,Z coordinates and per strip. GPS approximations should be close enough to allow such simplified approach. Thus we can formulate the following observation equations for the GPS coordinates, relating to camera station i in photo strip k:

$$\begin{aligned} V_{X_{ik}}^{\text{GPS}} &= X_{ik} - (a_{0k} + a_{1k} \bar{x}_{ik}) - X_{ik}^{\text{GPS}} \\ V_{Y_{ik}}^{\text{GPS}} &= Y_{ik} - (b_{0k} + b_{1k} \bar{x}_{ik}) - Y_{ik}^{\text{GPS}} \\ V_{Z_{ik}}^{\text{GPS}} &= Z_{ik} - (c_{0k} + c_{1k} \bar{x}_{ik}) - Z_{ik}^{\text{GPS}} \end{aligned} \quad (1)$$

Here the coordinates $(X,Y,Z)_{ik}$ refer to the camera projection centres. They tie the GPS observation equations (1) into the photogrammetric block adjustment. The parameters $a_{0k} \dots c_{1k}$ represent the linearized datum transformations (and corrections for systematic errors) via which each sequence of GPS positions is linked with the photo camera stations of the respective strip. There included is the off-set between camera and GPS antenna in the aircraft. The coordinates x_{ik} refer to an auxiliary local axis fitted to the flight axis of strip k. A time parameter t_{ik} may be used instead.

With regard to the photogrammetric part of combined adjustment there are no special requirements, as the standard cases are dealt with. The photogrammetric observations can be either image coordinates (for bundle block adjustment) or model coordinates (for independent model block adjustment). Either case can be combined with GPS data equally well. Hereafter only the bundle adjustment is considered. The blocks are assumed to be complete in the conventional way, with standard overlap and standard tie connections. In particular the photogrammetric block must be geometrically determined in itself. It means that the following minimum ground control configuration is asked: 4 horizontal ground control points more or less located in the 4 corners of a block; 2 chains of vertical control points at the front ends of the block. It is assumed, for compatibility reasons, that the ground control points are also determined by GPS and refer to a local GPS coordinate system

in which the adjustment takes place. Otherwise appropriate corrections for earth curvature and geodetic map projection would have to be applied, also for the airborne GPS data.

3.2 With the above approach the combined (bundle-) block adjustment deals here with 3 groups of observation data, which are all treated as uncorrelated, for the time being:

- xy image coordinates (vector \underline{x}), weights 1
- (XYZ)^c ground control points (vector \underline{X}^c), weights p^c
- (XYZ)^g GPS coordinates (vector \underline{X}^g), weights p^g .

The mathematical model of the combined bundle adjustment contains 4 groups of unknowns:

- the camera orientation parameters \underline{t} , including the coordinates of the camera stations
- the coordinates \underline{k} all of terrain points of the block, including ground control points
- additional parameters \underline{c} for selfcalibration
- drift parameters \underline{d} for the GPS observations.

Thus, the complete set of linearized observation equations for the combined bundle adjustment is:

$$\begin{aligned}
 \underline{v}_x &= \underline{A}t + \underline{B}x + \underline{C}c && - f_x && \text{weights 1} \\
 \underline{v}^c &= && \overline{\underline{B}}x && - f_c && p^c \quad (2) \\
 \underline{v}^g &= \underline{A}t && + \underline{D}d && - f_g && p^g
 \end{aligned}$$

The additional coefficient matrices $\overline{\underline{A}}$ and \underline{D} are most simply generated. $\overline{\underline{A}}$ and $\overline{\underline{B}}$ contain only the elements 1 in the respective columns, and only 1 non-zero element per equation. The matrix \underline{D} , according to (1), is composed of 2-columns submatrices, with the elements 1 and \overline{x}_{ik} in the columns, separately per strip and independent for X, Y, and Z.

The linearized observation equations (2) give normal equations, the coefficient matrix \underline{N} of which may conveniently be partitioned according to the 4 groups of unknowns:

$$\underline{N} = \begin{bmatrix} \overline{N}_{tt} & N_{tk} & N_{tc} & N_{td} \\ N_{kt} & N_{kk} & N_{kc} & N_{kd} \\ N_{ct} & N_{ck} & N_{cc} & N_{cd} \\ N_{dt} & N_{dk} & N_{dc} & N_{dd} \end{bmatrix} \quad \begin{aligned} N_{tt} &= \underline{A}'\underline{A} + \overline{\underline{A}}'p^g\overline{\underline{A}} & N_{tk} &= \underline{A}'\underline{B} \\ N_{tc} &= \underline{A}'\underline{C} & N_{td} &= \overline{\underline{A}}'p^g\underline{D} \\ N_{kk} &= \underline{B}'\underline{B} + \overline{\underline{B}}'p^c\overline{\underline{B}} & N_{kc} &= \underline{B}'\underline{C} \\ N_{kd} &= 0 & N_{cc} &= \underline{C}'\underline{C} \\ N_{cd} &= 0 & N_{dd} &= \underline{D}'p^g\underline{D} \end{aligned} \quad (3)$$

We recognize that the introduction of GPS data into the combined adjustment leads in first instance only to a slight extension of the conventional bundle adjustment normal equations by the unknowns \underline{d} (6n parameters for n strips). The additional submatrix N_{dd} disintegrates into 2 x 2 blockdiagonal submatrices, because of the simple structure of the \underline{D} matrix. Also the other additional non-zero submatrix N_{td} has a simple structure composed of D-type submatrices. Within the main body of the original normal equations it is only the submatrix N_{tt} (compo-

sed of 6 x 6 blockdiagonal submatrices) which is slightly altered: Because of the special structure of \bar{A} the weights p^c of the GPS coordinates need simply be added onto the respective diagonal elements of the unknown camera station coordinates. Thus, apart from a narrow additional border, the structure of the normal equations of conventional block adjustment is not altered by the additional GPS observations. Therefore conventional solution techniques may be applied. In particular also the well known partial reduction of the normal equations of the unknown terrain coordinates k , which leads to an efficient band-border structure, remains applicable.

It was observed earlier that the drift parameters d take implicitly care of the off-set between camera and GPS antenna. This is true, however, only for the constant parts. As the coordinate off-sets depend on the aircraft attitude an iterative correction procedure is suggested, as soon as approximate camera orientation parameters for azimuth and tilt are available. Initial off-set components must be given, and the camera crab must not be altered within a strip.

3.3 The approach to combined adjustment, as suggested here, has not yet been confirmed empirically. Within a short time results are awaited. It is not expected that the solution would meet special difficulties, as this approach to combined adjustment is geometrically stable, with no rank deficiencies. However, there are a number of open questions which warrant further investigation and experimentation. A major point will be further reduction of vertical control, which is certainly possible with GPS cross strips. Also non-linear drift corrections may be studied, with the necessary caution concerning rank deficiencies and hence additional control. Variance component estimation may be applied, and self-calibration and reliability questions should be studied. From a methodical point of view special investigations are warranted concerning the possibilities of effectively reducing or eliminating systematic errors in kinematic GPS positioning, by differencing the original observations.

4. Evaluation

4.1 Some time ago simulation studies were published (13), concerning the accuracy of combined block adjustment in relation to the accuracy of GPS positioning. At that time very little information was available about the accuracy to be expected for kinematic GPS positioning. The main result was, however, that rather poor GPS accuracy would be sufficient to give the combined block adjustment the accuracies required for photogrammetric mapping, as is shown in table 3, taken from (14). Equally important was the finding, that GPS carries the absolute accuracy performance, replacing ground control in that function, except for the datum- and configuration-deficiencies which arise from the unknown linear transformation parameters and which still have to be remedied by some ground control points, for the time being.

Table 3 Required precision of navigation data for combined block adjustment with minimum ground control

(photo) map scale	photo scale	required accuracy of AT		contour interval	required precision of na- vigation data (position)	
		$\mu_{x,y}$	μ_z		$\sigma_{x,y}$	$\sigma_z^{3)}$
1:100 000	1:100 000	5 m	4 m	20 m	30 m	16 m
1: 50 000	1: 70 000	2.5	2	10	15	8
1: 25 000	1: 50 000	1.2	1.2	5	5	4
1: 10 000	1: 30 000	0.5	0.4	2	1.6	0.7
1: 5 000	1: 15 000	0.25	0.2	1	0.8	0.35
1: 1 000	1: 8 000	5 cm	10 cm	0.5	0.4 ¹⁾	0.15
numerical point de- termination	1: 4 000	1-2	6		0.15 ²⁾	0.15

Assumption: $\sigma_0 = 15 \mu\text{m}$, ¹⁾ $\sigma_0 = 6 \mu\text{m}$, ²⁾ $\sigma_0 = 3 \mu\text{m}$, ³⁾ $\sigma_{0z} = 15 \mu\text{m}$

4.2 Comparing the requirements of table 3 with the actual GPS results as reviewed in section 2.3 it is evident that GPS supersedes by far the accuracies required for mapping at any scale, even as large as 1:1 000. 5 cm GPS accuracy would even be sufficient for high accuracy aerial triangulation, as applied to cadastral photogrammetry or network densification. It means that the camera air stations, as determined by airborne GPS, can be considered as control points of superior precision, except for their orientation by free parameters.

This gives a rather convenient situation from the practical point of view, at least for the pending phase of tentative application and further experimentation. It means that successful practical application does not, at this stage, critically depend on the theoretically correct treatment of the error properties of GPS data, nor on the linear treatment of GPS drifts.

4.3 The combined adjustment of photogrammetric blocks and airborne positioning data, especially GPS data, represents certainly a great step in improving the accuracy, efficiency, and economy of aerial triangulation. This is true although the combined adjustment still relies on a complete block triangulation, as is assumed and required throughout this paper, in order to have a safe basis. It is suggested that the experimental application of the method in near future operates on that basis. However, there is a vast accuracy surplus by GPS. Thus further research could study the possibilities of reducing the aerial triangulation part to a simplified, in itself incomplete system. The ultimate goal might be to measure all camera orientation parameters directly in-flight to a degree of absolute accuracy that all essential functions of conventional aerial triangulation would eventually be substituted.

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