PATTERN-ORIENTED ROBUST FILTERING OF IMAGES

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Abstract

The problem dealed with is the robust filtering and detection of the 2D signals. After re-defining the robust filtering for these signals, specific 2D criteria are introduced and discussed, with reference to image enhancement by digital methods. The enhancement problem is re-analysed from the point of view of the criteria used for optimality, and the pattern-oriented filtering of the 2D signals is defined and analysed. Finally, fuzzy robustness and fuzzy filtering are briefly discussed. These new methods could be fruitful as a link between pre-processing methods and artificial intelligence-based algorithms in data analysis.

Key words: filtering; detection; image enhancement; robust signal processing; fuzzy methods

Introduction

Optimal (Wiener) filtering of signals refers to *one* signal, *one* signal disturbance and *one* optimality criterion. The same is valid for optimal detection in the classical sense. Robustness is a global, generalized version of optimality, as it refers to a *class* of signals, a *class* of noise and a specific criterion of optimality on these classes. Usually, optimality is defined as based on Gaussian, MSE criterion, i.e. the output of the optimal filter has the minimum square error (MSE) with respect to the original signal. On the other hand, the usual optimal detector is matched to the probability of the original signal.

In this paper we consider the original signal carries a high level information, i.e. they represent patterns. In this case, classes of signals are involved, thus signal processing systems have to be robust, eventually adaptive. On the other hand, the use of the MSE criterion or the probability of the original signals is no more justified if they do not allow the best pattern recognition or detection. Following, specific criteria are to be defined.

Pattern-oriented optimal filtering

In this paragraph we define the pattern-oriented filter (pof) and contrast it with the usual one. Note $\underset{\cdot}{s}$ the original signal, F(s) the transfer function of the filter, $\hat{s}$ = F(s), where s is the input (noisy) signal. Consider the filtered signals are to be used for pattern recognition, the recognition process consisting in a mapping

$$r: S \rightarrow P$$

where S is the class of signals and P the class of patterns (usually a discrete class). Consider a norm // // is defined on the space of patterns. Than, the natural optimality criterion is:

$$(1) \qquad \min_F // r(F(s)) - r(\underset{\cdot}{s})// = \min//r(\hat{s})-r(\underset{\cdot}{s})//$$

Contrast (1) to the usual optimality criterion:

$$(2) \qquad \min_F / F(s) - \underset{\cdot}{s}/ = \min /\hat{s} - \underset{\cdot}{s}/$$

where / / represents the norm on the space of signals, eventually defined in the Gaussian manner. Note that (2) does not generally implies (1), i.e. the crresponding optimal filters differ. In many practical cases, signals represent patterns (at least the original signals). Thus, the use of a pattern-oriented criterion as defined by (1) is largely justified.

Pattern-oriented robust filtering

If the problem of filtering involves a class of noisy versions of the same signal $\underset{\cdot}{s}$, the class being denoted by S, than criterion (1) can lead to various robustness criteria, e.g.:

$$(3) \qquad \min_F \; {}_S // r(\hat{s}) - r(\underset{\cdot}{s})//$$

or:

$$(4) \qquad \min_F \max_S // r(\hat{s}) - r(\underset{\cdot}{s})//$$

Conditions (3) and (4) are straightforward extended to the case of a class of original signals $\underset{\cdot}{S}$ corrupted by a class of noise signals, by replacing in (3) and (4) the class S by the class Z of all original signals as corrupted by all possible noise signals. Note that (3) requires a global optimum, while (4) implies the minimization of recognition error for the worst case.

It is obvious that in general, the po robust filter and the usual (MSE) robust filter are different, for the same reasons the po optimal and the optimal filters are different.

Comparison of the po optimal- and optimal filters

The following remarks help in contrasting the two types of filters and in choosing the usual optimal filters for recognition purposes.

Remark 1. If //r(ŝ) - r(ş)// is a monotonous function in /ŝ - ş/, than the po filter and the optimal filter (2) are the same.

Remark 2. If r(ŝ) = r(ş) for any ŝ s.t. /ŝ - ş/ is less than q, and if the optimal filter yields an error less than q, it is the optimal po filter.

Note 1. In Remark 1, the norm // // can be interpreted as a measure of recognition, e.g. the probability of recognition. The function involved, in variable /ŝ-ş/, must in this case be an increasing one. On the other hand, the domain of monotony can be restricted as follows. Let q be the value of the signal such as the function discussed has a minimum in q, and q is the value which is the ş-closer with this property. Let be u the value of the function corresponding to q and v the ş-closer value of ŝ such as the value of the function in v be u. If the function is monotonous in the interval 0...v, and if the optimal filter error is less than /v-ş/, than it is po-optimal. Otherwise, there is a filter which is po-optimal and which differs from the (usual) optimal filter.

Note 2. If // // is a function of ///ŝ - ş///, where /// /// is a norm different than / /, in general the po-optimal filter and the optimal filter are different.

Above, it was considered the case of a single pattern, i.e. one pattern and the 'chaos', or: "this is A (the pattern), or it is not" case. Note that this problem is equivalent to the detection one. Of course, the above remarks can be extended with more than a pattern.

Analysis of optimality for 2D signals processing

It is worth mentioning that in common inference, "image" means a collection of patterns. Without any pattern, a 2D signal is n o t an image, but merely a luminance process, requiering no filtering or detection. Thus, in what follows we restrict the discussion only to true images.

Probably the most important parameter of an image is its resolution, i.e. its capability to carry dense information. It is known that this property is directly connected to the contours of the domains in the image, that is to the higher frequency band in the spectrum. Following, the information content carried by various frequency bands in a image signal is not the same, as usually happens for 1D signals. Thus, the optimal filters for 2D signals must preserve the information in these HF bands in the first place. At least, some weights for various frequency bands must be used to accuratelly describe the optimality in 2D signals processing.

We consider two ways to include in the optimality and robustness criteria the specificity required by image signals. The first is to weight the signals in eq.(2) by a weighting function corresponding to a high pass or band pass filter. The second way is to use the po processing of the signal, as described by eq. (1). Of course, the first way is just a first step towards the po-filtering. It has however the advantage of using well established methods, already available.

Furtheron, robust processing of images is an extension of the above described optimal methods to classes of signals and noise, as described by eqs. (3) and (4).

Fuzzy optimal and robust processing of images

There are many cases when the classes of signals and noise, and possibly the systems used in signal processing can not be described in a crisp or probabilistic manner. For example, this can happens when only a partial statistic is available. In such circumstances, it is suitable to consider the classes as fuzzy sets, and the signals to be considered as belonging to these classes as by a membership function. Then, one can introduce specific optimality and robustness criteria, based on the membership functions, as in /1/, eventually also using fuzzy inference rules.

In this case, there is more than one possible criterion. Instead of that presented in /1/, one can use for example:

(5) $\min_F \max_S x$, where $x = \max /\hat{s}-s/$ s.t. $M_{/\hat{s}-s/}$ is max

More explicitelly, consider $/\hat{s}-s/$ is a fuzzy number with the membership function $M_{/\hat{s}-s/}(y)$. Choose from those y satisfying $M_{/\hat{s}-s/} = 1$ the greater y, noted x. For all signals,choose max of x. The filter is robust if it minimizes this value. The above definition does not necessary implies that the membership functions are triangular.

Pattern-oriented fuzzy filters are introduced in a similar manner. One can consider in this last case the optimality (or robustness) is reached when the membership function to the correct pattern reaches the maximum value.

Conclusions

The methods of filtering introduced in this paper fil a gap between the classical filtering and the artificial intelligence based methods. It was proved that image processing asks specific methods, mainly the po-processing methods. If the information about the patterns is available, than the results obtained by po-processing can be much better than those produced by classical methods.

Concluding, we have introduced a new powerful tool in signal processing, allowing the enhancement of information in images.

/1/ H.N. Teodorescu - Robustness in terms of fuzziness; Preprints 2nd IFSA Congress, Tokyo, 1987, p. 733.