COMPILER POSITIONING SYSTEM:
AN ARRAY ALGEBRA FORMULATION of DIGITAL PHOTOGRAMMETRY

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ABSTRACT

The evolution of array algebra and its applications in the components of Compiler Positioning System (CPS) will be reviewed, with emphasis on the 1984 - 88 development period. CPS removes the restriction of 2 ray stereo compilation using the array algebra techniques of -

- ***multi-ray Global Least Squares Correlation** (GLSC), which allows transfer of all pixels as internal tie points, and*
- *self-calibrating **grid triangulation** of the dense tie points, automatically forming Digital Image Maps (DIM).*

The resulting DIM simplifies and accelerates automatic feature compilation by simple monoscopic pointing. The high accuracy, reliability and speed of CPS makes advanced digital photogrammetry practical and economical.

1.0 INTRODUCTION

Photogrammetry today is facing fundamental changes due to the advances of digital computer technology. Similar changes started taking place few decades ago through the birth of analytical techniques in photogrammetric triangulation and stereo compilation. The input of these processes consisted of analog images. Their measured image co-ordinates were digitized for analytic computations in a computer.

Today, the emphasis of photogrammetric research is shifting toward digital photogrammetry that has the potential of automating most of the image measurements by new correlation techniques of digital or digitized multi-ray stereo models. The introduction of multiple rays for stereo compilation has the potential of solving the bottlenecks of analog and analytical photogrammetric mapping of 2-ray models. However, the rigorous formulation of the combined triangulation and compilation problem of digital photogrammetry has been computationally prohibitive.

Array algebra is a powerful computer and math technology of two decades development that already has solved several computational problems of digital and analytical photogrammetry. Array algebra has grown from the ideas of the photogrammetric and geodetic M.S., Ph.D. and Techn.D. thesis of Rauhala (1968, '72, '74) into a broad technology of modern computer and math sciences, (Rauhala, 1975-87). In a summary, the array algebra has extended and unified the foundations of :

- o Vector, matrix and tensor algebras
- o Theory of general matrix inverses
- o Estimation theory of mathematical statistics
- o Multi-linear numerical analysis
- o Digital signal processing and fast transform technology
- o Analytical and digital photogrammetry

- o Geodetic sciences and potentially all fields involving numerical solutions of large systems of linear and non-linear equations such as meteorology, structural analysis, etc.

The practical computational power of array algebra is based on its general fast transform technique and signal processing which is made applicable for the solution of general systems of linear and non-linear equations by a generalized estimation theory and numerical analysis. This paper shall give a brief overview of array algebra and its applications in the components of the compiler positioning system of digital photogrammetry from Rauhala (1986, '87). The practical proofs of the computational feasibility of on-line DTM validation and progressive sampling of Rauhala, Davis, and Baker (1988) are extended to the array algebra correlation techniques of Global Least Squares Correlation. The philosophy and early applications of array algebra grid triangulations are reiterated.

2. FOUNDATIONS AND LITERATURE REVIEW OF ARRAY ALGEBRA

Sections 2.1 - 2.3 briefly review the literature of array algebra and its connection to numerical analysis, mathematical statistics and fast transform techniques. A discussion of some early array algebra applications in the user problems of medical and industrial close-range photogrammetry follows in Section 2.4.

2.1 MULTI-LINEAR NUMERICAL ANALYSIS

The early development stages of array algebra are reviewed in the Helsinki ISPRS paper of Rauhala (1976). The paper included practical examples of computational solutions from the application of the array algebra function theory in a linear regression analysis. The coefficients of a math model for Hardy's multiquadrics, least squares interpolation and geodetic boundary value problem were computed at a five times higher speed than those of equally large 2-D Fast Fourier Transforms (FFT). The operations count is only linearly dependent on the number of parameters. The resulting speed of 1,000 nodes in 1.4 CPU seconds can be reached today already in the general purpose micro computers and low-end workstations, (Rauhala, Davis and Baker, 1988).

2.2 LOOP INVERSE ESTIMATION

A common sense interpolation math model for photogrammetric self-calibration and reseau corrections of the Hasselbad close-range moon camera resulted in the foundations of array calculus in Rauhala (1972). The same technique offered a common sense approach to the problem of a general matrix inverse by a new theory of loop inverses, (Rauhala, 1974, '75, '81, '82).

Loop inverse estimation converts a singular system into the classical full-rank case by replacing the original modelling parameters of an ill-poised problem with an estimable set of functions:

$$\underset{[p,1]}{L_o} = \underset{[p,n]}{A_o} \underset{[n,1]}{X} \quad , p \leq \text{rank } (A)$$

from the always estimable space of observables of the Gauss-Markov model:

$$\underset{[m,1]}{E(L)} = \underset{[m,n]}{A} \underset{[n,1]}{X} \quad .$$

The conventional least squares estimate:

$$\hat{L}_o = H\ L = A_o\ G\ L$$

provides the estimate:

$$\hat{X} = G\ L = A_o^m\ H\ L$$

by a back substitution of the parameter transformation:

$$\hat{X} = A_0^m \hat{L}_0 , \quad A_0^m = A_0^T (A_0 A_0^T)^{-1} .$$

The loop inverse operator $A_0^m H$ turned out to be more general than the pseudo-inverse A^+ satisfying all but the condition $A G A = A$ of a general inverse. The unique filter operator:

$$H = (K^T K)^{-1} K^T , \quad K = A A_0^m ,$$

provided an elegant link between the classical condition and element adjustment techniques (Rauhala, 1974, p. 71). Their expansion to singular systems was straight forward. Yet, the resulting solutions of the singular adjustment problems only involve computations of non-singular square matrices.

Some studies of statistical estimation in Rauhala (1976) further limited the usefulness of the g-inverse. The fundamental definition of estimability of functions

$$L_0 = A_0 X \quad \text{by} \quad A_0 = A_0 G A$$

was extended beyond the restriction of $A G A = A$. These highly theoretical studies resulted in the key philosophy of the Compiler Positioning System for modelling the parameters of Global Least Squares Correlation and automated feature compilation in the always unbiasedly estimable image space of photogrammetry. Summaries of these estimation studies can be found in Rauhala (1981, '82, '86, '87).

2.3 GENERAL FAST TRANSFORM and SIGNAL PROCESSING TECHNOLOGY OF ARRAY ALGEBRA

I.J. Good (1958), Cooley and Tukey (1965) applied a special tensor or Kronecker product technique in the derivation of FFT. The transformations of these special tensor products of square matrices were generalized in the array calculus of Rauhala (1972) beyond the Einstein summation convention of tensor calculus. This allowed the expression of overdetermined grid observations as multi-linear functions of separable modelling parameters. Their general solutions by loop inverse estimation in (Rauhala, 1972, '74) provided the tools of covering the unexplored and fertile fields between signal processing, numerical analysis and mathematical statistics of the adjustment calculus.

The FFT and other fast transforms are very restricted and specialized. Yet, they are successfully serving several sciences, technologies and industries. The studies of these special operators helped the array algebra reasearch in finding the philosophy of Array Relaxation (AR) through fast special convolution algorithms. AR solution of the array algebra finite element and fractal technique of Rauhala (1980a) was applied in the DTM, GLSC and grid triangulation problems of the compiler positioning system in Rauhala (1986, '87) with some further experiments shown in Rauhala, Davis and Baker (1988).

2.4 ARRAY ALGEBRA USER APPLICATION OF PHOTOGRAMMETRY

Rauhala (1972 - 76) reported the early applications of the efficient integration rules of separable math modelling in some odontological and orthopaedic problems of medical photogrammetry. A combined analytical triangulation and multi-ray compilation of few pre-targeted discrete object points provided their accurate object space co-ordinates. The array algebra regression technique was used for an efficient least squares fitting of a mathematical model into the measured co-ordinates. The resulting model was used to derive volumetric, relative bone movement and other data of the user interest, as shown in Lippert (1973), Bergstrom (1974) and Rauhala (1976).

The development of the STARS close-range system of industrial photogrammetry and the use of its predecessor for accurate volumetric and other calibrations of LNG transport and storage tanks occupied most of the 1975-83 professional implementations of array algebra (Brown, 1984), (Rauhala, 1976-87). This photogrammetric tank calibration procedure became the accepted standard of the industry after the National Bureau of Standards had certified the technique through independent tests, (Siegwarth, La Brecque and Carroll, 1984).

The analytical system of STARS is a specialized form of the digital CPS concept as discussed in Rauhala (1987). The new automated image mensuration system of Global Least Squares Correlation in future digital or digitized systems of industrial photogrammetry could eliminate the need for manual preselection and retrotargeting of the object points to be measured. An on-line AR feedback loop of finite element progressive surface modelling could automatically determine the optimal number and location of the points to be measured until a prespecified modelling accuracy of the quantities of the user interest is achieved.

Examples of the on-line AR modelling algorithms are the experimental 3-D (time dependent) STARS reseau reduction technique of array algebra and the automated DTM validation technique. The technique of entity LSC of the object surface will reduce or eliminate the need of discrete point targeting in several applications. Examples include the calibration of the actual object shape against its design shape, and the removal of the cumbersome monumentation and pretargeting requirements in the CPS and hypertriangulation technology of national and continental surveys down to the scale of cadastral surveying, (Rauhala, 1968, '86, '87). The beauty of GLSC and the array algebra reseau correction technique is that one high quality reseau or retro-target measurement of a discrete point can be replaced by several lower quality and very fast sub point measurements of GLSC. The final product quality is controlled on-line such that the overall production of the user application is accelerated by orders of magnitude. For more literature, see Masry (1981), Lugnani and Souza (1984), Rosenholm and Torlegard (1987).

3. Compiler Positioning System (CPS)

The concept of CPS in digital photogrammetry integrates an automated multi-ray stereo mensuration of image-to-image pixel transfer functions with a rigorous self-calibrating array algebra grid triangulation of their accurate object space coordinates. The main philosophical and practical problems of this new digital formulation of photogrammetry are discussed in Rauhala, (1986, '87) with some practical examples of its key algorithms summarized in Rauhala, Davis and Baker (1988). In a very brief summary, these main problems are:

- o <u>Data acquisition, storage, transfers, reformating and processing of digital stereo images:</u> The image processing techniques typically have handled only small non-metric images so even a brute-force conversion of 2-ray analytical stereo compilation into the digital domain appears cumbersome. The 2-ray digital stereo compiler would still share the main shortcomings of the old analog and analytical photogrammetry leaving the main potential advantages of digital photogrammetry unexplored.

- o <u>New systems components:</u> The systems and operations concepts tailored for 2-ray compilation in analog and analytical photogrammetry have to be replaced by new ones before the full advantage can be taken from digital photogrammetry. The computational problems of the new systems components of multi-stereo CPS have prompted several inventions and practical developments of the array algebra technology that make the introduction of CPS practically feasible in the foreseeable future.

Sections 3.1 - 3.3 will reiterate the philosophy of the three main systems components, namely:

1) automated mensuration of the pixel mapping functions of Global Least Squares Correlation (GLSC),
2) grid triangulation and,
3) automated information extraction from a Digital Image Map (DIM).

Section 3.4 will summarize some experimental work in the implementation of these new techniques and concepts.

3.1 Pixel Transfer Technique of GLSC

The GLSC process replaces the selective, usually manual, single-point image coordinate measurements of a few discrete multi-ray tie and 2-ray compilation points of the analytical photogrammetry by a rigorous simultaneous multi-ray image-to-image transfer solution of <u>all</u> pixels. This involves rigorous least squares solutions of the finite element modelling parameters of the shifts and the radiometric biases. The minimum number of the modelling parameters depends on the behavior of these shift and bias functions. Therefore, the feedback loop of progressive sampling and on-line validation of the modelling quality is integrated into GLSC.

For many practical applications, we are not only interested in the minimum number of the parameters for a sufficient modelling accuracy but in their evaluations of the shift parameters at a dense regular grid. This grid simplifies the evaluation of the shifts at the pixel level by a simple local interpolation. Thus, once this grid of the finite element technique is established, we have implicitly transferred the geometric location (and radiometric distortion) of every single pixel of the reference image to the slave images.

In order to automate the compilation problem with a sufficient accuracy the grid has to be quite dense. Ideally we would like to capture the image space shift behavior of the object's micro-surfaces such as the resolvable terrain canopy. This typically requires the use of such high node density and small correlation windows that the single-point correlation techniques would breakdown in reliability and production speed. The philosophy of the simultaneous global solution prevents this breakdown as will be shown in some examples of practical tests in Section 3.4.

One would think that the computational solution of the global formulation is unfeasible as its special case of solving the unconstrained diagonal system of the single-point approach of the conventional formulation would be too slow. However, here the power of the GLSC philosophy and array algebra computer technology can be utilized such that Rauhala (1977-87) is envisioning the global solution and concurrent sample speeds of over millions of parameters per second. The feasibility of these future advanced systems of taylored hardware is shown by the slower software experiments of Section 3.4.3.

3.2 Grid Triangulation

The dense point transfer grid of GLSC in the image space provides the input of grid triangulation. The self-calibration math model is extended to image variant self-calibration to allow for shear free and high density object space coordinates to the dense image transfer grid. Each image has local image-to-object and object-to-image grid transforms so they can be considered as Digital Image Maps (DIM), (Rauhala, 1986-87).

3.3 Use of Digital Image Map

DIM makes the use of orthophotos and compilation of line maps obsolete from the point of view of the map user. A monoscopic DIM pointing yields the object surface coordinates in real-time. Rauhala (1986-87) discusses the future possibility of digital multi-ray stereo instrument for a "hologrammetric" type of mensuration of multi-layer features and the final edit of DIM.

3.4 Experimental Results

3.4.1 Grid Triangulation

Rauhala (1972, '74, '75) reported some 1970-71 grid triangulation results of a 5x9 strip test field constructed on a wall of a camera calibration laboratory in Stockholm. Both image variant and invariant "discrete" self-calibration parameters were applied with and without simultaneous geodetic observations. Full variance - covariances among all control points could be incorporated because the reduced normals were expressed in terms of the coordinate and image invariant self-calibration parameters. These experiments introduced multiple exposures and discrete image and point variant reseau corrections for self-calibration of the only discretely unbiasedly estimable systematic errors of the "tie grid". This concept grew into "hyper-triangulation" and "hyper-compilation" of digital systems of enormous accuracy potential. The fundamental realization that the unbiased self-calibration is possible only at the discrete image grid locations of the regular tie grid is not yet commonly understood in photogrammetry. It explains a critical flaw of the control transfer in analytical photogrammetry for arbitrarily distributed control points (Rauhala, 1980, '86, '87). This and several other critical flaws of analytical triangulation are removed by the concept of hyper-triangulation.

A new solution of the photogrammetric bundle adjustment was experimented in the early '70's using the loop inverse modifications of orthogonalization and hyper Cholesky techniques (LSQCHOL) of singular and non-singular matrices (Rauhala, 1975, '87). This algorithm is computationally superior to the traditional elimination, banded-border solution and error propagation techniques of analytical photogrammetry as discussed in more detail in Rauhala (1987). The achieved accuracy of 0.1 - 0.2 micrometers in the image scale of the 1970-71 "tie grid" triangulation is not yet common in photogrammetric literature. The feasibility of the expansion of the sparse 5x5 "image tie grid" triangulation of the early experiments to the dense GLSC image transfer grid was established computationally in Rauhala (1982, '84). The applications of the self-calibrating net adjustments of the North American Datum and inertial survey traverses employ the same computational mechanism as the GLSC tie grid triangulation with image variant self-calibration of the finite element techniques of array algebra, as shown in (Rauhala, 1986, '87).

3.4.2 On-Line DTM Validation

The on-line DTM validation algorithm grew from the experimental development of array algebra and loop inverse filtering. Rauhala (1972-78) and Rauhala and Gerig (1976) report several simulations of the loop inverse interpolators:

$$K = AA_0^{-1} \text{ and filters } H = (K^TK)^{-1} K^T,$$

for extremely efficient data snooping and least squares solution technique of array algebra signal processing. Some of these new techniques were implemented in the DTM validation experiments of Rauhala (1980a ,c ,d).

The AR solution technique of the 1980 finite element DTM study opened the practical GLSC solution of the linearized array correlation formulation of Rauhala (1977). The global solution part of the GLSC algorithm was tested during 1980-82 on an experimental 3-D and 4-D finite element reseau correction technique of CRC-1 camera of STARS. It was also applied in the grid triangulation simulations of Rauhala (1982, '84).

The solution of GLSC is very closely related to the finite element DTM AR of progressive sampling. Therefore, the 1984-88 research effort of the integrated CPS concept started from a refined experimental implementation of the on-line DTM validation and progressive sampling of the AR algorithm. The technique and the results of the extensive experiments are reported in Rauhala (1986, '87) and Rauhala, Davis and Baker (1988). The automated on-line integration of the product validation into the measurement process significantly improved the quality , speed and economy of the DTM generation as expected. An experiment provided the output rate of 533 nodes/sec of the automatically validated DTM with a reliability exceeding that of an experienced operator.

3.4.3 Global Least Squares Correlation

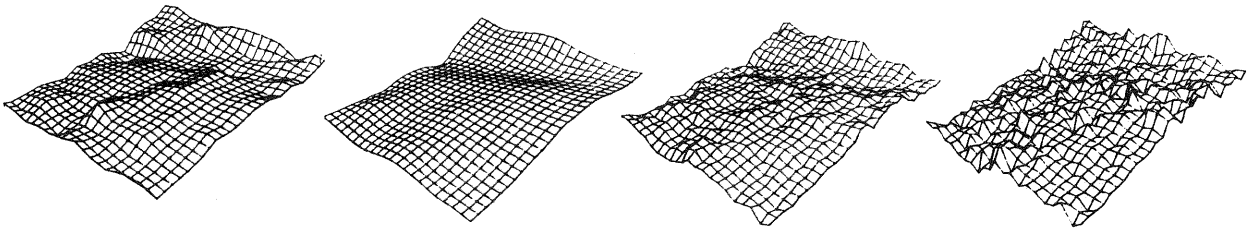
The report (Rauhala, Davis and Baker, 1988) summarizing the on-line DTM validation experiments contains over 50 pages so the more extensive experiments of GLSC have to be greatly condensed as follows:

Micro-topography Correlation:
The critical flaws of all single-point correlation techniques (including multi-ray LSC) is removed by GLSC allowing high quality correlation of an object's micro-topography. The result is illustrated in Fig. 1 at the 8x8 pe node density of "South America" (SA) ISPRS test data. The process of GLSC was initiated from the best available pull-in parallax values of Fig. 1a provided by the manually editted/filled-in values of conventional cross-correlation at 16x16 pe intervals and interpolated to the 8x8 pe density. The finite element model of the x-shifts had overly tight continuity constraints in the first iteration (Fig. 1b). The constraints were loosened in each iteration to illustrate the capability of GLSC to capture the detailed micro-topography (Fig. 1c) with the optimal continuity weight.

Fig. 1d illustrates the inherent weakness of the single-point correlation techniques corresponding to zero continuity weight of the global parallax model. Although the previous iterations yielded the ultimate pull-in and reshaping values the solution diverges at several "adverse" areas that through the incorrect reshaping values start polluting the neighboring "good" points. We may have found an explanation to the frequent failures of the conventional single-point correlation as illustrated in Fig. 3. Fig. 2 shows another example where the single-point technique converges by the help of the refined GLSC pull-in process.

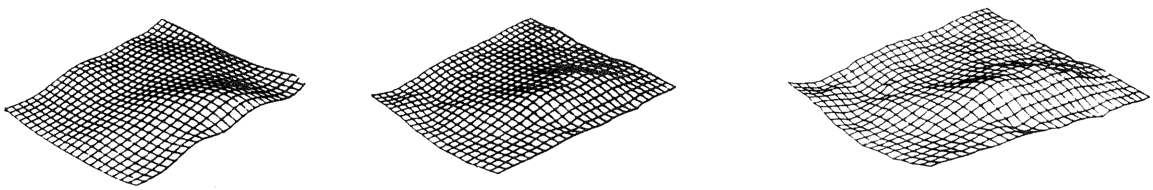
Rigorous Modelling of Radiometric Bias in GLSC:
The above GLSC experiment of 7x7 sample windows at all nodes of 8x8 pe spacing (no progressive sampling) was repeated with 7x7, 5x5 and 3x3

EXPERIMENTAL RESULTS



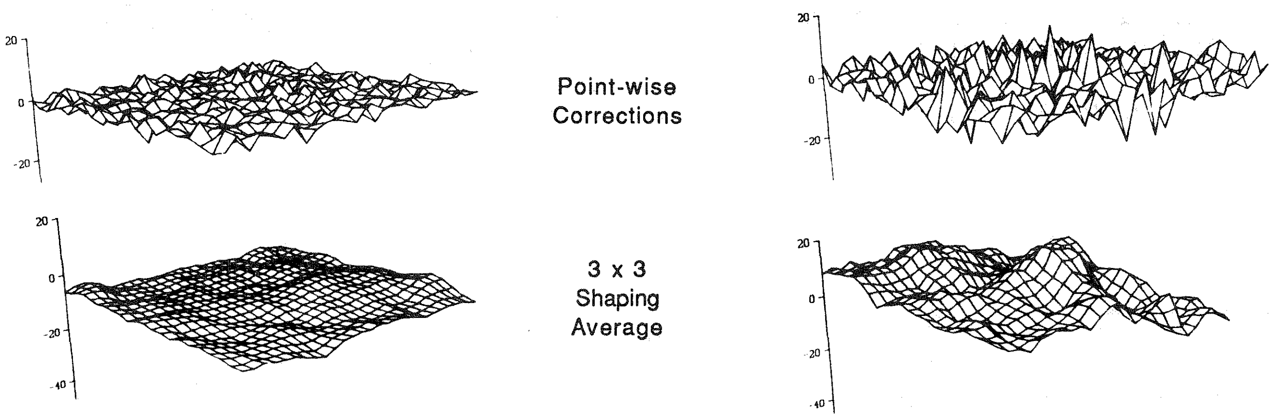
a) Pull-In Shifts b) Over-Constrained c) "Best" Solution d) Under-Constrained

FIG. 1 Over- to Under-Constraint of South America Data



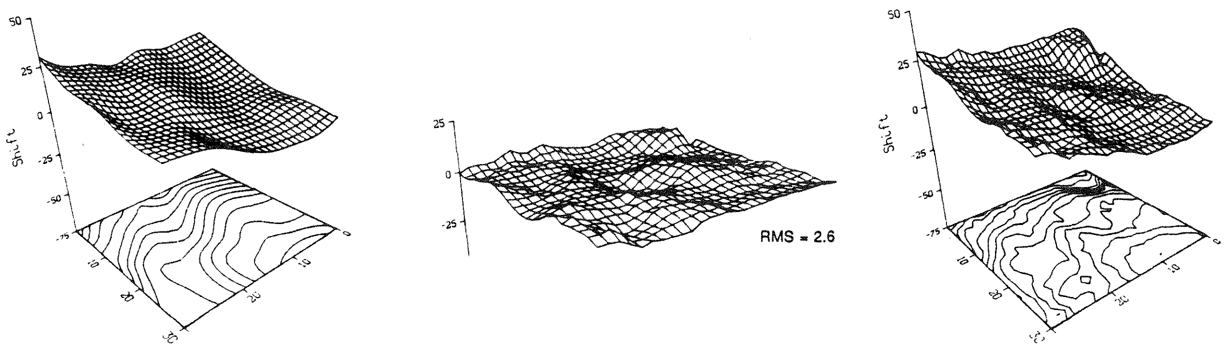
a) Pull-In Shifts b) Begin Constraint Release c) Converged Solution

FIG. 2 Car I Data



Car I Data South America Data

FIG. 3 Radiometric Bias Modelling



a) Pull-In Shifts b) Pull-In Errors c) Converged Solution

FIG. 4 Island Data

sample windows of "Car I" stereo model. The automated cross-correlation succeeded at all attempted pull-in points. The difference r.m.s. of the GLSC and the cross-correlation results was smallest (0.4 pe) in the over-smoothed case of GLSC. As the continuity weight is released the actual shape of x-shifts resolvable by the 8x8 pe node spacing becomes evident. The r.m.s. difference to cross-correlation is only slightly increased but local deviations up to 1.1 pe are found. This time, the GLSC iterations with decreased continuity weights stabilized to a unique point-wise LSC solution, even with the small windows. The iteration process of GLSC gradually refines the reshaping and local pull-in values until the fine LSC samples become possible as illustrated in Fig. 2c.

The initial values of Fig. 2a GLSC test were found by an efficient 1-D GLSC process and 2-D interpolation scheme of array algebra yielding accuracies in the order of a few pixels, (Rauhala, 1986, '87). The solution converged exactly to the same unconstrained point-wise LSC x-shifts as before with the initial pull-in values from cross-correlation. The results of these tests with Car I data confirm those of SA data that the conventional point-wise (or rather area) cross-correlation is capturing only the smoothed (tight continuity weight) or averaged shift values perhaps because of the habitual use of too large windows to avoid the breakdown. But why such large windows to avoid the breakdown? The rigorous modelling of the radiometric biases of GLSC can perhaps explain the fact that Car I image data allowed the unconstrained LSC solution while SA GLSC solution diverges at several "adverse points" if the neighborhood constaints are released too much.

Fig. 3 shows the radiometric bias terms of shaping and correction processes of Car I and SA data. The shaping terms are 3x3 averages of the accumulated bias corrections used for reshaping in each iteration. The correction terms are a part of LSC sample solutions where their magnitude is properly constrained without the rigorous global continuity constraint. The bias shaping and correction terms of Car I data are very smooth and small in comparison to the SA data. These terms reflect the image quality and correlatability of stereo images together with the standard error of unit weight, s_0 , and standard errors for the LSC samples. Car I had s_0 = 4-5 gray values and SA data had low s_0 only at the "good" points.

The main reason for the breakdown of point-wise LSC with SA data is the fact that the effect of radiometric bias cannot be separated from the effect of the shift on point-wise basis at small windows. SA data has poor contrast such that the reduced normal equation for the shift parameter after the elimination of the bias correction becomes ill-conditioned for small windows even if the bias correction is allowed to move only within the noise level. The global continuity constraints of GLSC for the x-shifts and the refined reshaping/pull-in process have the strength of overcoming this and other shortcomings of single-point correlation.

<u>Large Pull-in Range of GLSC:</u>
The concept of global linearized math model (parallax DTM) of array algebra function theory in Rauhala (1977) was prompted by the intuitive idea of computing refined pull-in values to "bridge small sub-spaces" of hidden surfaces or adverse areas. The gradual pull-in of the iterative linearized GLSC process of non-linear correlation was, also intuitively, conceived as the mechanism of increasing the overall pull-in range of a complete frame (1K x 1K pe). This is far beyond the restricted local pull-in range of LSC samples. This may explain why the LSC technique alone could not immediately capture the enthusiasm of correlation experts (Helava, 1987). This limited pull-in of LSC also explains the rather

discouraging reliability and overly optimistic error propagation of the well known tests of the Stuttgart and Stockholm Photogrammetric Institutes.

Both of the above intuitions turned out to be right. Our experiments have demonstrated the capability of GLSC to bridge over a sharp blunder or a break-line using the poor SA data. A gradual automated fill-in of GLSC covered an "adverse area" of a sub grid given over 20 pe blunders. Similar tests were made for the overall pull-in range of SA data by introduction of over 20 pe blunders in some initial values that by the initial pull-in process contaminated a large region of the frame. The correct robust solution of micro-topography was recaptured although the number of iterations was increased. However, the GLSC formulation of array algebra has some superior computational qualities allowing a large number of iterations as discussed next. Fig. 4 is shown as an example of the pull-in range of GLSC with respect to "Island" imagery of the ISPRS test data.

Speed and High Quality of GLSC Is Economy:

The LSC sample process of GLSC requires:

- o Reshaping of only few pixels of a small window of a slave image vs. large windows of both images of traditional cross-correlation,
- o Product sums of a small window vs. large windows,
- o No repetition of the above processes for 8-16 trial centers of the non-linear cross-correlation. This means that the refined sample process and the linearized global solution can be iterated several times in the fashion of automated progressive sampling of Rauhala, Davis and Baker (1988) to avoid over- or under-sampling. The iteration process can be accelerated by some common sense strategies or by the exciting new technique of non-linear least squares of Blaha (1987).

The sample process of one GLSC iteration requires an order of magnitude less arithmetic operations than any other known technique. The "impossible dream" of a general purpose microprocessor implementation is thereby realized for the digital photogrammetric correlation problem. We have experienced in the order of 100-1000 times higher software solution speeds in comparable computers than most of the other emerging LSC related techniques found in the literature; Rosenholm (1986), Grun and Baltsavias (1986, '87), Helava (1987), Ebner et al. (1987), Wrobel (1987), Barnard (1987), Shibasaki and Murai (1986). As shown in Rauhala, Davis and Baker (1988) the main advantage of GLSC is its high quality which improves the overall production timeline by orders of magnitude. The increase in reliability of 2-ray correlation from 50 - 80% to the order of 99% would reduce the manual bottleneck by a factor of 50 - 20. The implementation of the multi-ray and multi-layer GLSC of Rauhala (1986) will further improve the quality, speed and economy of CPS making the digital photogrammetry competitive with the conventional mapping systems.

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