EMPIRICAL ACCURACY OF POSITIONS COMPUTED FROM AIRBORNE GPS DATA

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1. Introduction

The NAVSTAR Global Positioning System (GPS) is a satellite based navigation system designed especially for real-time threedimensional position and velocity determination. Real-time performance, however, is, except for navigation purposes, not necessarily required for the majority of the positioning tasks. Post-processing of the recorded raw GPS data is, therefore, the evaluation technique mainly applied, also by utilizing GPS in geodetic disciplines. Numerous investigations in static geodetic positioning (refs. 3, 14, 15) have meanwhile demonstrated a high accuracy potential of the GPS. Relative positioning accuracies in the order of centimetres are typical when using GPS carrier phase observations in the static mode.

In this presentation attention is concentrated solely on kinematic positioning, i.e. the determination of the trajectory of a moving object, whereby an aircraft is the subject of the investigations. Many airborne scientific applications such as laser bathymetry, airborne gravimetry, airborne laser profiling and aerial triangulation (refs. 1, 2, 6) would be facilitated if the trajectory of the aircraft could be determined within a certain degree of precision. GPS is an appropiate positioning system for these missions, as previous investigations in kinematic positioning (refs. 5, 8, 9, 11, 12, 16) have demonstrated.

At present, the accuracy characteristics of the positions computed from airborne GPS data are of main interest. For this reason a controlled photo flight with simultaneous GPS data registration over a test area in the Netherlands has been carried out. The Survey Department of Rijkswaterstaat Delft was in charge of the planning, preparation and realization of the project (ref. 4).

The aim of the test is the investigation of the accuracy of kinematic positioning with GPS in high dynamic applications, such as airborne applications. In order to guarantee a proper control of the positions derived from the GPS observations, the test flight was performed in combination with a large scale aerial triangulation. The aerial triangulation provides the coordinates of the projection centres of the aerial camera, which can be used for comparison with the GPS antenna positions.

The evaluation of the test flight data has not yet been completed. Therefore, this paper presents only first results of the analyses of the carrier phase observations with respect to the accuracy and systematic effects of the positions computed therefrom.

2. Test description

The test flight itself took place on 10.06 and 12.06.1987. The complete flight comprises 16 parallel strips, with a length of approximately 4 km each. The average speed of the aircraft during this mission was about 240 km/h, thus the flight duration of each of the strips was approximately 60 sec.

The utilized GPS instrumentation consisted of two Sercel receivers : one NR52 receiver stationarily placed at a known reference point (Fig. 1) and one TR5SB receiver on board the aircraft. Both are 5 channel, L1 C/A-code receivers providing pseudorange and carrier phase observations and additionally the broadcast satellite navigation message.

During the entire flight duration pseudorange and carrier phase observations were carried out and recorded simultaneously with an observation rate of 0.6 sec with each receiver, at the reference point and on the aircraft, respectively. In addition, aerial photographs were taken along the 16 parallel strips approximately every 3 sec, which resulted in a total of 360 aerial photographs. The exact time of exposure of the individual aerial photographs was registered on the time scale of the GPS aircraft receiver, in order to establish the reference between the results of the GPS positioning and those of the aerial triangulation.

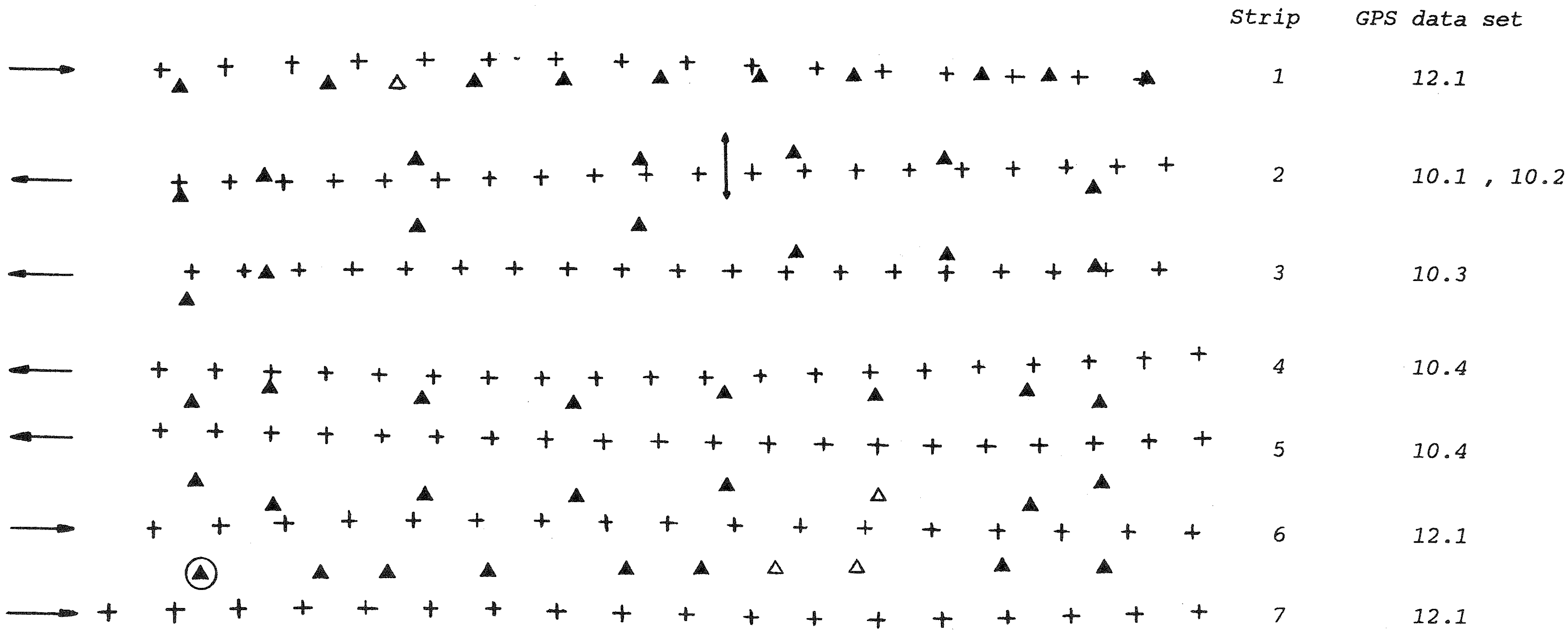
2.1 GPS data

The preliminary investigations presented in this paper are based on a part of the block covering seven flight strips. The seven strips with a total of 130 photographs are chosen in such a way that the requirements with regard to a precise aerial triangulation and consequently for a precise determining of reference points for the GPS solution, are fulfilled. The aerial triangulation of these seven strips is described in the next chapter.

The GPS observations corresponding to the seven photo strips are subdivided into five independent data sets, due to data registration conditions. Each data set (tab. 1) contains pseudorange and carrier phase observations of both the stationary and the aircraft receiver. The observations were carried out at the L1 frequency simultaneously to five satellites (6,8,9,11,12), except for the first part of strip 2 (data set 10.1), where satellite 11 lost lock and thus only 4 satellites were tracked (tab. 1)

Date	Data Set	Strip	Satellites
10.06.87	10.1 10.2 10.3 10.4	2.1 2.2 3 4,5	6,8,9,12 6,8,9,11,12 6,8,9,11,12 6,8,9,11,12
12.06.87	12.1	7,6,1	6,8,9,11,12

Tab. 1 : GPS data sets



▲ - *X,Y,Z ground control point*

△ - *unused ground control point*

+ - *camera projection centre*

⟶ - *flight direction*

Ⓐ - *fundamental point of the geodetic horizon system*
- *stationary GPS receiver*

Fig. 1 : Test block configuration

2.2. Aerial triangulation

The technical data of the photogrammetric block (fig. 1) can be summarized as follows :

block size	: 7 strips with a total of 130 aerial photographs
aerial camera	: Wild RC 10
focal length	: f = 213.67 mm
forward overlap	: p approx. 70 %
side overlap	: q approx. 50 - 60 %
flight altitude	: approx. 800 m
image scale	: M_B = 1 : 3800
number of points	: 47 signalized X,Y,Z ground control points
	236 signalized tie points
	117 natural tie points
	477 marked tie points

The coordinates of the ground control points were determined using both conventional geodetic methods and also GPS. The evaluation of the GPS observations has already been completed and the coordinates of the ground control points resulting therefrom were available with a precision of ± 2 to ± 3 cm with reference to the geocentric coordinate system WGS 84 (ref. 4). These coordinates were transformed into a local geodetic horizon system (ref. 13), with the location of the stationary receiver as the fundamental point (fig. 1).

The aerial triangulation was then performed by the bundle method with additional parameters for compensation of systematic effects using the bundle adjustment programm PAT-BS. The results of major interest of this bundle adjustment are the coordinates of the camera projection centres, which are used for the testing of the GPS antenna coordinates.

Simulations with the above described photogrammetric block were performed in order to testify the accuracy of the projection centre coordinates. Hereby a standard deviation of the image coordinates of $\sigma_x = \sigma_y = \sigma_o = \pm 5\ \mu m$ (± 2 cm in the terrain) was taken into account, according to the σ_o achieved in the bundle block adjustment of the real data set. The simulations have shown that the accuracy of the coordinates of the camera projection centres can be indicated by ± 4 cm in X, Y and ± 2 cm in Z.

3. GPS data processing

The basic observation equations for pseudorange and carrier phase measurements can be written (refs. 7, 15, 17) as :

$$P_r^S = R_r^S + \delta R_r^S + c\cdot(\delta T_S - \delta T_r) + (\delta R_r^S)_{ion} + (\delta R_r^S)_{trop} + \varepsilon_P$$

$$-\lambda\Phi_r^S = R_r^S + \delta R_r^S + c\cdot(\delta T_S - \delta T_r) + (\delta R_r^S)_{ion} + (\delta R_r^S)_{trop} + \lambda N_r^S + \lambda\,\varepsilon_\Phi$$

whereby :

$$R_r^S = \sqrt{(X_S - X_r)^2 + (Y_S - Y_r)^2 + (Z_S - Z_r)^2}$$

P_r^S , pseudorange observation from receiver r to satellite S
Φ_r^S , phase observation from receiver r to satellite S

X_r, Y_r, Z_r	,	unknown coordinates of the antenna phase centre of receiver r
X_s, Y_s, Z_s	,	coordinates of satellite S, calculated from the satellite broadcast ephemeris
δR_r^S	,	range error due to satellite ephemeris uncertainties
δT_s	,	clock error of satellite S
δT_r	,	clock error of receiver r
$(\delta R_r^S)_{ion}$	,	range error due to ionospheric refraction
$(\delta R_r^S)_{trop}$	,	range error due to tropospheric refraction
N_r^S	,	initial ambiguity parameter
c	,	speed of light
λ	,	wavelength of the satellite signal
ε_P	,	pseudorange observation error
ε_Φ	,	phase observation error

These equations include all biases caused by certain systematic effects, which have to be removed by modelling or differencing to achieve unbiased positions. There are several possibilities for removing or rather reducing the biases using linear combinations of the original observations. The method applied in the present investigations is the following :

The satellite clock errors were computed from the clock parameters included in the satellite navigation message. The range errors due to satellite ephemeris uncertainties, ionospheric and tropospheric refraction were first of all neglected. This results in :

$$\begin{aligned} P_r^S &= R_r^S - c\cdot\delta T_r + \varepsilon_P \\ -\lambda\Phi_r^S &= R_r^S - c\cdot\delta T_r + \lambda N_r^S + \lambda\,\varepsilon_\Phi \end{aligned} \qquad (2)$$

The initial ambiguity parameters N_r^S of the individual data sets of the stationary receiver were calculated using the known coordinates of the reference point. In contrast to this, the initial ambiguity parameters of the aircraft receiver data sets were computed with the help of initial positions derived from a pseudorange solution. The resulting observation equation for the corrected carrier phase observations can thus be written as :

$$-\lambda\Phi_r^S = R_r^S - c\cdot\delta T_r + \lambda\varepsilon_\Phi \qquad (3)$$

This observation equation was then used as a basis for the determining of single epoch positions of the receiver antennae at the reference point and on the aircraft. At each epoch the four unknowns, the three antenna coordinates and the receiver clock error, of both the stationary and the aircraft GPS receiver were calculated by independent least squares solutions. Carrier phase observations to five satellites (except for strip 2.1) for each receiver and each epoch were available. The known coordinates of the reference point were taken as approximate values of the unknown coordinates of both the stationary and the aircraft receiver antennae. The approximate value of the receiver clock error was assumed to be zero.

As a result of this process one obtains for each observation epoch i, i.e. every 0.6 sec, the coordinates of the stationary receiver antenna and of the moving antenna in the geocentric coordinate system WGS 84. The variance-covariance matrices of all epochs were also obtained :

$$X_{ref}(t_i) \;;\; Q_{ref}(t_i) \quad \text{and} \quad X_{air}(t_i) \;;\; Q_{air}(t_i) \qquad (4)$$

where :

$X_{ref}(t_i)$, position vector of the GPS antenna at the reference point at observation epoch t_i in the WGS 84
$X_{air}(t_i)$, position vector of the GPS antenna on the aircraft at observation epoch t_i in the WGS 84
$Q_{ref}(t_i)$, corresponding variance-covariance matrices
$Q_{air}(t_i)$

These single epoch positions are influenced by the above neglected systematic effects. Assuming that the systematic effects on the positions of both receiver antennae are identical or rather similar, these effects can be reduced by calculating relative positions of the aircraft GPS antenna with respect to the stationary GPS antenna.

$$X_{relair}(t_i) = X_{ref} + [\, X_{air}(t_i) - X_{ref}(t_i) \,]$$

$$^{*} \quad Q_{relair}(t_i) = Q_{ref}(t_i) + Q_{air}(t_i) \qquad (5)$$

where : X_{ref} ,nominal position vector of the stationary receiver antenna
$X_{relair}(t_i)$,relative position vector of the GPS aircraft antenna in the WGS 84

$Q_{relair}(t_i)$,corresponding variance-covariance matrices

* The single epoch positions of the stationary and the aircraft receiver are assumed to be independent.

The relative positions $X_{relair}(t_i)$ of the GPS aircraft antenna (existing every 0.6 sec) were interpolated onto the times of exposure of the individual aerial photographs and then transformed into the local geodetic horizon system, in order to achieve GPS aircraft antenna positions which are comparable to the camera projection centres obtained from the aerial triangulation.

4. Analyses of the GPS positions

The relative positions of the GPS aircraft antenna interpolated onto the exposure times of the aerial photographs are the subject of the following considerations. As mentioned above, an independent control of these GPS antenna positions is given by the comparison with the positions of the camera projection centres. Therefore, a reduction of the antenna centre positions onto the camera projection centres would, as a matter of fact, be necessary, considering the spatial separation of the antenna phase and camera projection centre.

In this presentation, however, the coordinate offsets, i.e. the differences dx, dy, dz between the GPS antenna coordinates and the projection centre coordinates are computed and analysed. In addition, the spatial distances ds between the antenna phase centre and the projection centre at the individual exposure times are calculated. The r.m.s. values of the coordinate offsets and the spatial distances ds with respect to their mean values are presented in table 2.

GPS data sets	Strip	rms dX [m]	rms DY [m]	rms DZ [m]	rms DS [m]
10.1	2.1	1.970	5.818	1.168	6.238
10.2	2.2	0.072	0.047	0.050	0.047
10.3	3	0.065	0.098	0.049	0.053
10.4	4	0.061	0.126	0.048	0.117
	5	0.056	0.098	0.052	0.088
12.1	7	0.113	0.084	0.069	0.085
	6	0.063	0.073	0.069	0.073
	1	0.062	0.079	0.053	0.074
quad. mean *		0.073	0.089	0.056	0.080

* without data set 10.1 (strip 2.1)

Tab. 2 : Accuracy of the coordinate offsets between the camera projection centres and the GPS antenna centres

At a first glance, the results summarized in table 2 indicate a position accuracy of better than ± 10 cm for kinematic positioning with GPS in high dynamic (240 km/h) applications. The poor accuracy obtained in the first part of strip 2 occurred due to the fact that satellite 11 lost lock which results in a very unfavourable satellite geometry. The PDOP, a criterium for the assessment of the satellite geometry (PDOP= $\sqrt{Q_{xx} + Q_{yy} + Q_{zz}}$; position dilution of precision) was about 40 during the observation period of strip 2.1, in comparison to the PDOP values of the other observation epochs which were about 3.5 .

The position accuracy really obtainable in GPS kinematic positioning is even better than it appears at the first glance, because the coordinate offsets are influenced by at least two effects. On the one hand, the coordinate offsets are not constant, due to the inclination variations of the aircraft between consecutive positions. This effect is not taken into account in this presentation. But its influence is obvious if one compares the r.m.s. values of the coordinate offsets with the corresponding r.m.s. values of the distances. The distances ds are independent of the aircraft inclinations and are for this reason more appropriate as criteria for the position accuracy.

On the other hand, the coordinate offsets include all unmodelled, and in spite of relative positioning remaining systematic effects of the GPS observations. These effects also cause variations of the coordinate offsets. The method of linear regression was therefore applied to analyse the coordinate offsets with regard to a linear time dependent drift. The results of this investigation are listed in table 3. The drift values are given in mm/sec. It is obvious that the unfavourable satellite geometry during the observations of strip 2.1 causes an extreme drift in the coordinates. The drift of the coordinate offsets of the other strips varies between 0 and 6 mm/sec, which result for the worst case in 36 cm/strip.

GPS data sets	Strip	drift DX [mm/sec]	drift DY [mm/sec]	drift DZ [mm/sec]
10.1	2.1	- 237	+ 700	-141
10.2	2.2	+ 6	+ 1	+ 4
10.3	3	+ 2	+ 5	+ 2
10.4	4 5	+ 0 + 0	+ 6 + 4	- 2 - 1
12.1	7 6 1	- 4 - 1 - 1	+ 3 + 1 + 0	- 3 - 3 - 2

Tab. 3 : Linear drifts of the coordinate offsets between the camera projection centres and the GPS antenna centres

The r.m.s. values of the coordinate offsets calculated after removing the linear drifts are presented in table 4. These results demonstrate that, when using a refined model for processing the GPS carrier phase observations, which considers all systematic influences in such a way that no drift in the coordinates remains, a kinematic position accuracy of ± 5 cm or even better can be obtained. If one recalls that the check positions, the projection centres of the aerial camera, were calculated with a precision of ± 4 cm in X,Y and ± 2 cm in Z, a considerable part of the r.m.s. values of the drift corrected coordinate offsets (tab. 4) would thus arise from the accuracy of the coordinates of the camera projection centres. This fact is confirmed by the results of ARI-modelling (autoregressive integrated processes) and variance component estimation (ref. 10). The estimated standard deviations of the relative GPS antenna coordinates are summarized in (tab. 5) varying between ± 0.7 cm and 3.8 ± cm.

GPS data sets	Strip	rms dX [m]	rms DY [m]	rms DZ [m]	rms DS [m]
10.1	2.1	0.045	0.073	0.033	0.037
10.2	2.2	0.041	0.045	0.031	0.028
10.3	3	0.057	0.056	0.035	0.034
10.4	4 5	0.061 0.056	0.055 0.057	0.032 0.045	0.032 0.031
12.1	7 6 1	0.066 0.058 0.055	0.045 0.072 0.078	0.042 0.045 0.045	0.036 0.034 0.039
quad. mean		0.055	0.061	0.039	0.034

Tab. 4 : Accuracy of the coordinate offsets between the camera projection centres and the GPS antenna centres after removing linear drifts

Strip	σ_X [cm]	σ_Y [cm]	σ_Z [cm]
1	2.7	1.1	3.8
2	-	-	-
3	2.0	0.9	2.6
4	2.4	0.9	2.7
5	2.0	1.3	2.4
6	1.4	1.1	1.4
7	1.2	0.7	1.0

Tab. 5 : Accuracy of the GPS antenna positions derived from variance component estimation

Strip	r_{XY}	r_{XZ}	r_{YZ}
1	0.01	0.58	0.20
2	-0.57	0.61	-0.70
3	-0.53	0.59	-0.65
4	-0.47	0.56	-0.51
5	-0.39	0.54	-0.36
6	-0.13	0.56	0.06
7	-0.23	0.55	-0.08

Tab. 6 : Cross-correlation coefficients of the GPS antenna coordinates

	X	Y	Z
r(1d)	0.46	0.69	0.52
r(2d)	-0.06	0.11	-0.01
r(3d)	0.06	-0.15	-0.10

Tab. 7 : Auto-correlation coefficients r of the GPS antenna coord. (d=0.6 sec)

The accuracy analyses of the GPS aircraft positions has not yet been completed, thus not all questions of interest could be investigated up to now. Of interest are, for example, the correlations of the coordinates of consecutive positons, especially for the further utilization of the GPS positions in the evaluation processes of the abovementioned airborne scientific applications. Preliminary results of such analyses (ref. 10) are summarized in table 7. They indicate that the correlations of the coordinates of consecutive positions are negligible. The cross-correlation of the GPS antenna coordinates, derived from the variance-covariance matrices of the GPS solution are summarized in table 6. The variation of the cross-correlation coefficients are due to the continuous changes of the satellite geometry.

5. Conclusions

The presented results of first analyses of the GPS test flight data have demonstrated the high accuracy potential of kinematic positioning with GPS in high dynamic applications. Although a simple processing technique was applied for the calculation of the positions from the GPS carrier phase observations, a coordinate accuracy of better than ± 10 cm was nevertheless directly reached.

The results indicate as well that the potential kinematic position accuracy of GPS is at the centimetre level. This accuracy level is attainable when using a refined processing technique for the GPS carrier phase observations e. g. double differencing or triple differencing (refs. 15, 16,17), in order to reduce or even to eliminate the influence of the systematic effects of the GPS observations. Subsequent investigations will be devoted to these points.

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