CARTOGRAPHIC DISTORTIONS OF REMOTE SENSED IMAGES

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1. Intoduction

Images taken from satellites or other high flying carriers can be considered to be sort of cartographic representation of Earth surfice. Geometric fidelity of each type of cartographic representation is characterized through the cartographic distortions in a point B_i. The principal geometry of a central projection is shown in Fig. 1. The figure is representing the vertical plane going through the projection center and being perpendicular to the flight path, if large format cameras or modified television cameras are used. The figure is good enough as well for one line taken by a scanner.

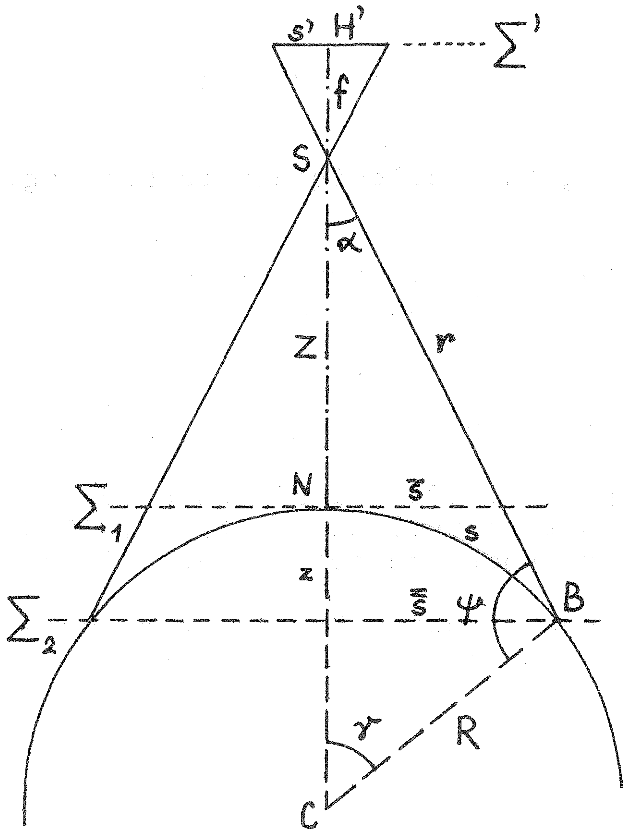

Fig. 1

Some quantities in Fig. 1 can be considered as known: R is radius vector of the Earth sphere, Z is hight of the satellite S, f is the focal length of camera, s' can be measured inthe image and is known as well.
The planes Σ', Σ_1 and Σ_2 in Fig. 1 are parallel. The arc s is projected onto thouse planes as s', $\bar{s}$ and $\bar{\bar{s}}$ respectvely.
Observing the figure some relations and equations can be put down:

$$\operatorname{tg} \alpha = \frac{s'}{f} \tag{1}$$

$$\gamma = 180^{\circ} - (\psi + \alpha) \quad \text{where} \quad \sin\psi = \sin\alpha \frac{R + Z}{R} \tag{2}$$

$$s = R\frac{\gamma}{\rho''} \qquad (3) \qquad\qquad \bar{s} = Z\ \mathrm{tg}\,\alpha \qquad (4)$$

$$\bar{\bar{s}} = R \sin\gamma \qquad (5) \qquad\qquad \bar{\bar{s}} = (Z + z)\ \mathrm{tg}\,\alpha \qquad (5a)$$

where z is the hight above the chord $2\bar{\bar{s}}$ and equals

$$z = R\left(1 - \cos\gamma\right) \qquad (6)$$

2. Differential Increments of Lengths and Areas

2.1 Increments of length in radial direction

Differentially small increments $\delta\bar{s}$ of $\bar{s}$ corresponds to differentially small increment $\delta\alpha$ of the angle α and it is

$$\delta\bar{s} = \frac{Z}{\cos^2\alpha}\,\delta\alpha \qquad (7)$$

For the angle $\alpha \rightarrow 0^{\circ}$ it is $\delta s \approx \delta\bar{s}$ and also $\delta\bar{\bar{s}} : \delta\bar{s} = (Z + z) : Z$. There is

$$\delta s \approx \frac{Z}{\cos^2\alpha}\,\delta\alpha \quad (7a) \qquad\qquad \delta\bar{\bar{s}} = \frac{Z + z}{\cos^2\alpha}\,\delta\alpha \qquad (7b)$$

If the angle α is not small, the equation (7a) is not valid. When the equation (3) is differentiated the relation between the increments δs and $\delta\gamma$ can be shown. Before that, it is necessary to find the explicit relation between the increments $\delta\gamma$ and $\delta\alpha$, which could be done when the equation (2) is applied:

$$\delta\psi = \frac{R + Z}{R}\,\frac{\cos\alpha}{\sqrt{1 - \left(\frac{R + Z}{R}\right)^2 \sin^2\alpha}}\,\delta\alpha$$

then

$$\delta\gamma = \left(\frac{R + Z}{R}\,\frac{\cos\alpha}{\sqrt{1 - \left(\frac{R + Z}{R}\right)^2 \sin^2\alpha}} - 1\right)\delta\alpha$$

Finally we can find out that

$$\delta s = \frac{R}{\rho''}\left(\frac{R + Z}{R}\,\frac{\cos\alpha}{\sqrt{1 - \left(\frac{R + Z}{R}\right)^2 \sin^2\alpha}} - 1\right)\delta\alpha \qquad (8)$$

2.2 Increments of lengths in perpendicular direction

Referring to Fig 1, which shows the vertical projection plane, we can think to have the plane K going through the radius r and perpendicular to the projection plane. The angle deviation $\delta\beta$ in this plan K causes deviations $\delta s'_k$ in the plane Σ', $\delta\bar{s}_k$ in the plane Σ_1, and $\delta\bar{\bar{s}}_k$ in the plane Σ_2. The situation is shown in Fig. 2, which is the horizontal projection plane. Then according to the pictures in the vertical and horizontal planes (Fig. 1 and Fig. 2) it is

$$\delta s'_k = \frac{f}{\cos\alpha}\,\delta\beta \qquad \delta\bar{s}_k = \frac{Z}{\cos\alpha}\,\delta\beta \qquad \delta\bar{\bar{s}}_k = \frac{Z + z}{\cos\alpha}\,\delta\beta$$

For the differentially small $\delta\beta$ it is possible to equal $\delta s_k = \delta\bar{s}_k$. Further more it is always possible to consider the increments $\delta\beta$ and $\delta\alpha$ to be $\delta\beta = \delta\alpha$. Then it is

$$\delta \bar{s}_k = \frac{Z}{\cos\alpha}\,\delta\alpha \quad (9) \qquad \delta s_k = \frac{Z + z}{\cos\alpha}\,\delta\alpha \qquad (10)$$

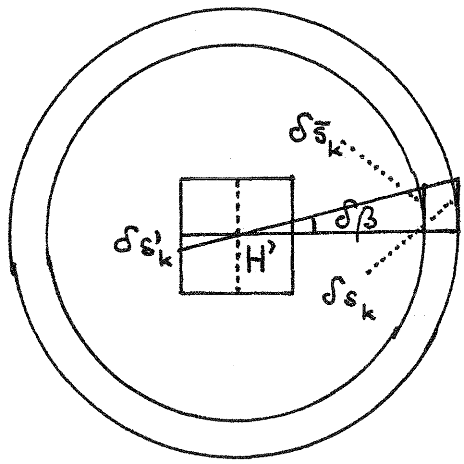

Fig. 2

2.3 Deformation in the infitely small area

A/ Length abreviation
in the radial direction $$\delta\Delta s = \delta s - \delta\bar{s} \qquad (11)$$

in the perpendicular direction $$\delta\Delta s_k = \delta s_k - \delta\bar{s}_k \qquad (12)$$

B/ Plane abreviation $$\delta\Delta p = \delta s \cdot \delta s_k - \delta\bar{s} \cdot \delta\bar{s}_k \qquad (13)$$

3. Distortion of Azimuths

Angles which are measured in the tangent plane τ /horizontal angles measured by theodolite/ are distorted when they are projected from center S to the plane Σ'. It was already stated that the planes Σ' , Σ_1 and Σ_2 are parallel. Distortion of horizontal angles is the same for all the three planes. The easiest way to analyze the angle distortion is to do it in the plane Σ_2, which is going through the point B, where the horizontal angles are measured. The line of intersection p of the planes τ and Σ_2 does not change when projected from the point S. Therefore it is useful to measure angles /azimuths/ starting from the line p. The measured azimuths σ , when projected from S, change to $\bar{\sigma}$. The present author, when applying the laws of spherical trigonometry, has derived the relation between the angles σ and $\bar{\sigma}$:

$$\operatorname{cotg}\bar{\sigma} = \operatorname{cotg}\sigma \frac{\cos\alpha}{\cos(\alpha+\gamma)} \qquad (14)$$

as well as it is

$$\bar{\sigma} = \arccos \frac{\cos\sigma\cos\alpha}{\sin^2\sigma\cos^2(\alpha+\gamma) + \cos^2\sigma\cos^2\alpha} \qquad (14a)$$

The distortion of azimuth then is

$$\Delta\sigma = \bar{\sigma} - \sigma \qquad (15)$$

4. Distortions of Central Projections

4.1 Cartographic distortions

When applying the equations (7), (8), (9), (10) and (15) we can define distortion of lengths, areas and azimuths.

A/ Lengths distortion in the radial direction

$$m = \frac{\delta s' \, \frac{Z}{f}}{\delta s} \quad (16a) \quad \text{or also} \quad m = \frac{\delta \bar{s}}{\delta s} \qquad (16)$$

in the perpendicular direction

$$m_k = \frac{\delta s'_k \, \frac{Z}{f}}{\delta s_k} \quad (17a) \quad \text{or also} \quad m_k = \frac{\delta \bar{s}_k}{\delta s_k} \qquad 17$$

B/ Plane distortion

$$p = \frac{\delta \bar{s} \cdot \delta \bar{s}_k}{\delta s \cdot \delta s_k} = m \cdot m_k \qquad (18)$$

C/ Azimuth distortion

$$\Delta \sigma = \bar{\sigma} - \sigma \qquad (15)$$

4.2 Deformations within defintely small area

In some cases it is important to know the distortion of the remote sensed picture within definitely small area of the reference point B_i of informations collected. Such distortios may be defined when equations derived above are integrated.

For the length abreviation in radial direction within the infinitely small area of the point B_i is

$$\delta \Delta s_i = \frac{1}{\rho''} \left\{ R \left(\frac{R+Z}{R} \frac{\cos\alpha}{\sqrt{1-\left(\frac{R+Z}{R}\right)^2 \sin^2\alpha}} - 1 \right) - \frac{Z}{\cos^2\alpha} \right\} \delta\alpha \quad (19)$$

Within the definitely small area of the point B_i the length abreviation in radial direction is

$$\Delta s_i^j = \int_i^j \frac{1}{\rho''} \left\{ R \left(\frac{R+Z}{R} \frac{\cos\alpha}{\sqrt{1-\left(\frac{R+Z}{R}\right)^2 \sin^2\alpha}} - 1 \right. - \frac{Z}{\cos^2\alpha} \right\} \delta\alpha \quad (20)$$

In the perpendicular direction the length abreviation is /within the infinitely small area/

$$\delta \Delta s_{ki} = \frac{1}{\rho''} \left\{ R \left(\frac{R+Z}{R} \frac{\cos\alpha}{\sqrt{1-\left(\frac{R+Z}{R}\right)^2 \sin^2\alpha}} - 1 \right) - \frac{Z}{\cos\alpha} \right\} \delta\alpha \quad (21)$$

Then the length abreviation /within the definitely small area/ in the perpendicular direction is

$$\Delta s_{ki}^{\,j} = \int_i^j \frac{1}{\rho''} \left\{ R\left(\frac{R+Z}{R} \frac{\cos}{\sqrt{1-\left(\frac{R+Z}{R}\right)^2 \sin^2\alpha}} - 1\right) - \frac{Z}{\cos\alpha} \right\} \delta\alpha \qquad (22)$$

The distortion of azimuths /derived in chapter 3./ is the same within infinitely and definite small areas. Then the equation is again

$$\Delta\sigma = \bar{\sigma} - \sigma \qquad (15)$$

5. Further Factors of Distortion

The presented paper shows the analysis of imageries when the Earth sphere is imaged on a plane. This case is of course only a very idealized one. Remote sensed images are influenced by many physical and geometrical factors. Many of those factors have been analyzed by other authors and can be studied in literature. It would be very usefull to compile all those influences and to try to derive equations comprising all distortions of remote sensed imageries. The whole work should aim to recommendation for cartographic representations the most suitable for analog comparing of maps and remotely sensed imageries.