Production of the highly accurate digital-terrain-model and its application to the topographic analysis

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ABSTRACT

A new highly accurate, gridded digital terrain model (HA-DTM) production system based on digitized contour data whas been developed and involves linear interpolation and extrapolation, spline function interpolation, and quadratic polynomial interpolation. This paper presents the basic concepts and argorithm of the HA-DTM production system and deals with the experiments of topographic analysis based upon the HA-DTM. Based on the experimental results, it has been confirmed quantitatively that the HA-DTM is highly effective in producing the topographic characteristics of the objective area.

Keywords: Digital terrain model/DTM, DTM production system, Highly accurate/High accuracy, Topographic analysis

INTRODUCTION

Digital terrain models (DTMs), especially gridded ones, are very useful means of handling the mountainous topographic information on timber-harvesting, road-network plans and so on. Recently, a new highly-accurate, gridded DTM (HA-DTM) production system was developed (Kitagawa 1987b, 1987c). This paper presents basic concepts and argorithm of the new system (HA-DTM system) and deals with its accuracy on topographic analysis. Main data processing in this system is performed by using the large-frame computer, FACOM M-780 at Nagoya University Computation Center.

BASIC CONCEPTS AND ARGORITHM OF THE HA-DTM SYSTEM

In the HA-DTM production system, the digitized contour data (represented by X and Y coordinates with their associated contour-line elevations), one of the DTMs, are produced by first using aerial photogrammetric mapping or by hand-tracing contour lines from a topographic map with a digital cursor. Then, these contour-line data are converted by the program to a gridded elevation matrix (Kitagawa 1980; Twito et al. 1987).

The HA-DTM production system is constructed upon the following basic concepts: 1) Digitized contour-line data closer to each grid point are conducive to heighten the accuracy of the elevation calculation at the grid point; 2) The topography changes along a water falling line through a grid point represented by a broken line which consists of perpendiculars from the grid point to the two contour-lines on both sides; 3) A time saving data processing technique is to collect contour-line intersections with as few gridlines as possible in the elevation calculation.

Processing methods of digitized contour-line data

The HA-DTM production system uses two kinds of data packing methods for effectively storing and searching for digitized contour-line data on a computer buffer: the bit-plane method and the plural data packing-in-one-word (PIOW) method.

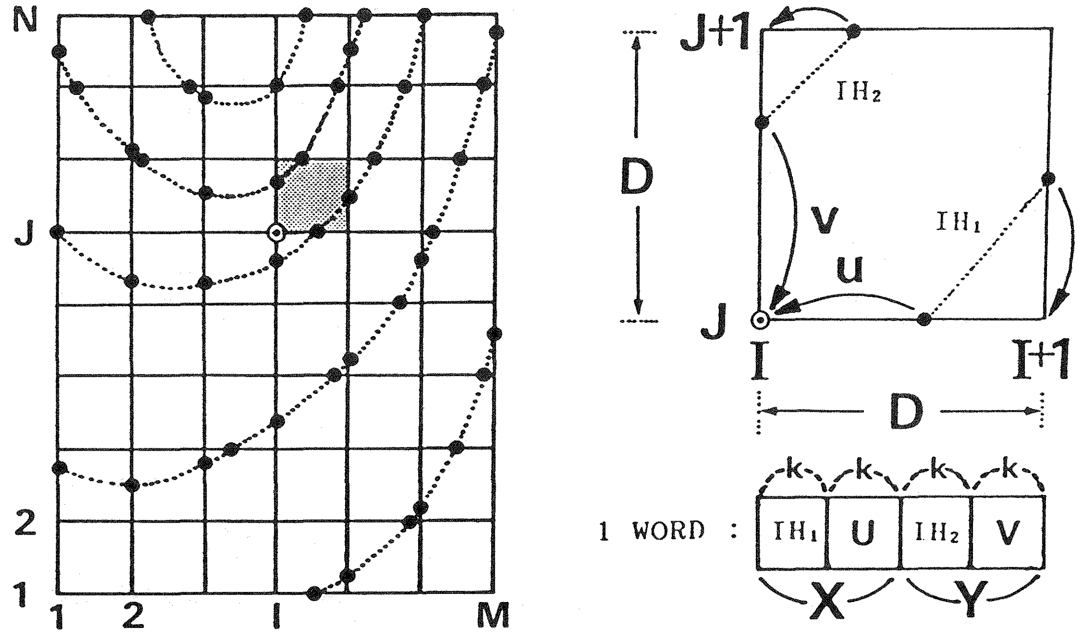
1. Surface information control matrix by the bit-plane method

The bit plane is a two-dimensional data storage for the two-valued data presented by 0 (=OFF) or 1 (=ON). If one word of a computer, a unit data storage, consists of 'K' bits, up to 'K' different species of the two-valued surface information can be stored in each element of the 2-dimensional array corresponding to grid points on the map.

The HA-DTM production system introduces the surface information control matrix (SICM) by the bit-plane method to check whether there are plural contour-line intersections on each gridline between two adjacent grid points. The SICM is also used for checking whether there is an intersection between each gridline segment and the boundary line of the objective area. Based upon the latter information, each element of the array on the computer buffer corresponding to grid points is divided into two parts, namely, the inside and outside of the boundary line for limiting the HA-DTM production area (Kitagawa 1980).

2. Contour data storing by the PIOW method

The PIOW method packs plural digital data in one word, a unit data storage of a computer, by the unitage of some bits. A square grid of a size M x N is set over the objective area. The grid interval is 'D' (m) on the ground. The digitized contour-line data are converted to contour-line intersections with the uniformly spaced, horizontal or vertical gridlines through calculation along with the elevation of the contour-



(a) Intersections between square gridlines and contour lines

(b) Conter data storing rule in the grid cell

Figure-1 Contour data storing method

line (Fig.1(a)). Values of these intersections are converted to the integer type and stored in elements of the array on the computer buffer corresponding to the left lower corner of each grid cell with each height code by the PIOW method (Fig.1(b)).

By transforming an arbitrary point P(X,Y) on the ground coordinate system to P(u,v) on the grid cell coordinate system of which the left lower grid point is P(I,J), the relations between (X,Y) and (u,v) are as follows:

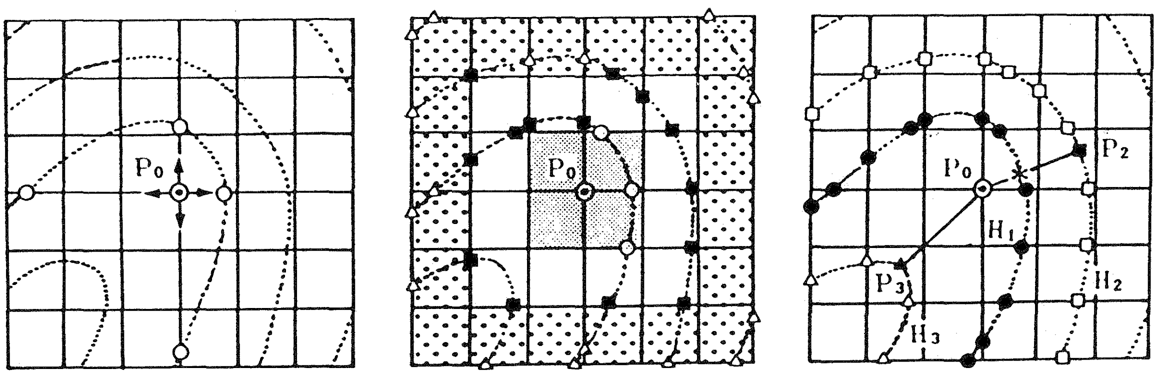
$$X = (I + u)\cdot D \ ; \ (0 \leqq u < 1),$$
$$Y = (J + v)\cdot D \ ; \ (0 \leqq v < 1).$$

If the same 'k' bits are assigned to each datum packed in one word with 'K' bits, the integers U and V corresponding to u and v are represented by the following equations: $U = [2^k \cdot u]$ and $V = [2^k \cdot v]$, where [ ] means the Gaussian symbol.

Next, to find the point P(X,Y), the set of I and J can be used as an index. The height code 'IH' can represent 2^k species of contour lines because by using 'k' bit positions, 2^k different characters can be coded. The position error of the intersection is smaller than $2^{-k}\cdot D$. At least one of a pair, u and v, of the contour-line data stored in the array is always set to zero, so the coordinate information of the contour data can be reduced to either the horizontal coordinate or the vertical one. Then, plural contour-line intersections with gridlines can be stored efficiently in one word by setting different bit positions for the horizontal coordinate data and the vertical ones (Kitagawa 1980).

3. Quick output of contour-line data around the grid point

In order to calculate the elevation at the grid point P(I,J), it need to output plural contour-lines including two contour-lines adjacent to P(I,J). Because contour-line intersections with gridlines have been packed all in elements of the array on the computer buffer corresponding to grid points by the PIOW method in the former step, they can now be locally and quickly outputted around P(I,J). In the HA-DTM production system, the procedure for outputting contour-line intersections with grid-lines around the grid point has three steps (Fig.2).



(a) First step (b) Second step (c) Third step

Figure-2 Contour-line data outputting arround the grid point

The first step is to output the two contour-line intersections (straddling the grid point) on the vertical gridline and the two contour-line intersections (straddling the grid point) on the horizontal gridline. If two neighbouring elevations with different values are outputted, it represents that the grid point P(I,J) is between these two elevations. If a grid point lies in the area with a bad topographic condition, only one contour elevation is outputted in this step.

In the second step, many contour-line intersections with grid-lines are systematically collected around the grid point P(I,J), and the three distances (s_1, s_2, and s_3) from P(I,J) to three contour-lines including the nearest contour-line H_1 and its neighbouring ones, H_2 and H_3 are calculated. In the distance calculation, the contour-line is restored to the nearly original condition by the curve fitting program based upon some contour-line data chained on both sides of the contour-line intersection nearest to the grid point P(I,J) on the same contour line.

The third step is the final determination of all factors required for the elevation calculation. If only one contour elevation H_1 is outputted at the first step, the second contour line is determined between the two contour-lines H_2 and H_3 by distinguishing which contour-line has no intersection with the H_1 contour-line on each minimum length segment from P(I,J) to the contour-line.

Argorithm of elevation calculation on grid points

In order to heighten the accuracy of the elevation calculation on grid points, the HA-DTM production system introduces a new argorithm which consists of four kinds of elevation calculating methods; linear interpolation, linear extrapolation, spline function interpolation, and quadratic polynomial interpolation (Kitagawa; 1987c).

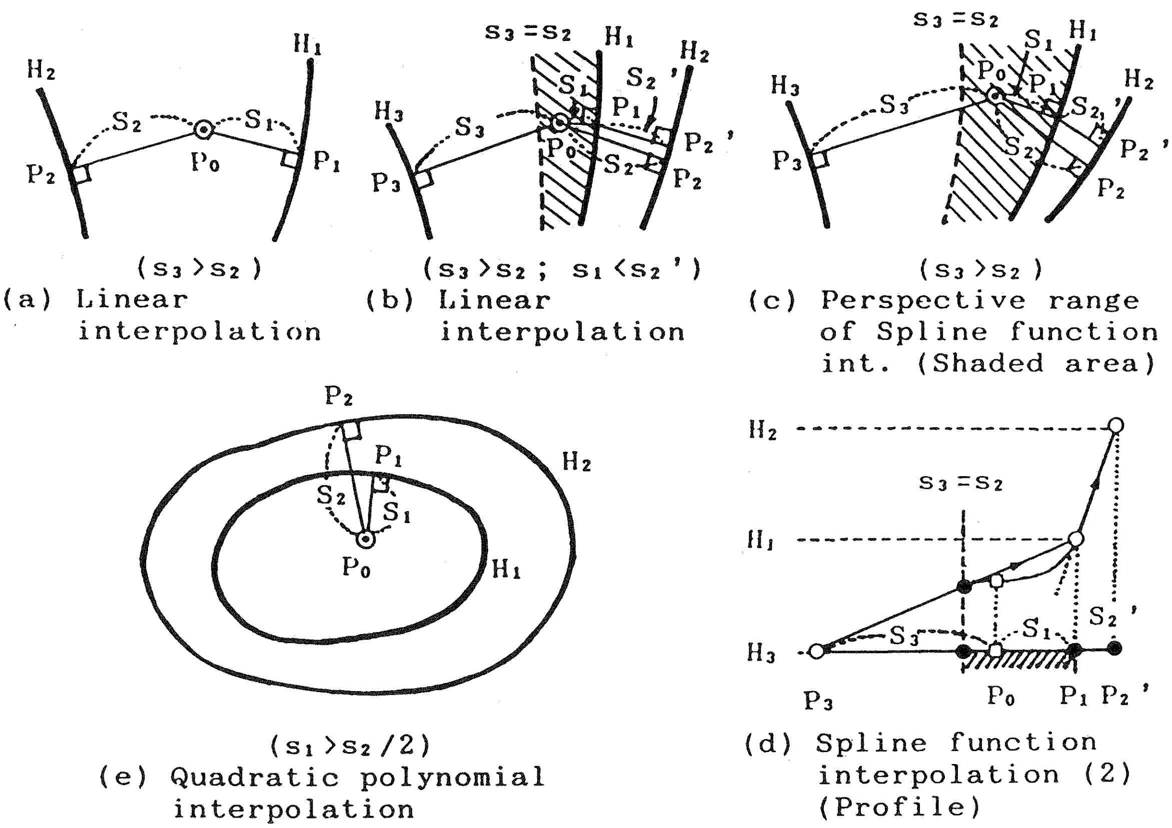
In Figure-3, the three contour lines H_1, H_2, and H_3 are selected in order of distance from the grid point P_0. When the following subscript 'i' varies from 1 to 3, P_i is the foot of the perpendicular from the grid point P_0 on the contour-line H_i, and s_i is the distance between P_0 and P_i. Similarly, P_2' is the foot of the perpendicular from P_1 on the contour-line H_1, and s_2' is the distance between P_1 and P_2'.

Now, assume the contour-line name H_i to is also its elevation. The elevation at the grid point calculated by the linear interpolation method is given by the following equation (Fig.3(a)):

$$H_0 = (H_1 \cdot s_2 + H_2 \cdot s_1)/(s_1 + s_2)$$

However, the elevation at grid point calculated by the linear extrapolation method is given by the following equation:

$$H_0 = \{ H_1 \cdot (s_1 + s_2') - H_2 \cdot s_1 \} / s_2'$$



($s_3 > s_2$)
(a) Linear interpolation

($s_3 > s_2$; $s_1 < s_2'$)
(b) Linear interpolation

($s_3 > s_2$)
(c) Perspective range of Spline function int. (Shaded area)

($s_1 > s_2/2$)
(e) Quadratic polynomial interpolation

(d) Spline function interpolation (2) (Profile)

Figure-3 Four types of elevation calculation methods

The area available for the extrapolation method is restricted within the range in which s_2 is shorter than s_3, and s_1 is shorter than s_2' (Shaded part in Fig.3(b)). Additionally, the linear extrapolation method is applied only when the grid point P_0 is the end point of the gridline segment on which there is an intersection of the contour line H_1.

In Figure 3(c) and 3(d), the shaded part is regarded as the perspective range of the spline function interpolation. The following two requirements are a necessary prerequisite to the application of the spline function interpolation (Kitagawa; 1987a).

(1) For the elevation calculation at the grid point P_0, each one point (P_1, P_2', and P_3) on the three contour-lines (H_1, H_2, and H_3) is used respectively.

(2) The elevation calculation is performed along the water falling line through the grid point P_0 within the shaded range in Figure-3(d), and by the third order polynomial interpolation of which the differential coefficient at the point of being $s_2 = s_3$ corresponds to the slope between P_3 and P_1, the differential coefficient at the point P_1 corresponds to the slope between P_1 and P_2', and the slope increases or decreases uniformly within the shaded range.

The quadratic polynomial interpolation method is used when grid points are in a bad position, causing the linear interpolation or extrapolation methods to be unapplicable (Fig.3(e)). This interpolation method requires the collection of six or more digital contour line data which involve two or more contour elevations, and expresses the topography around the grid point by the following quadratic polynomial:

$$Z = a_1 \cdot x^2 + a_2 \cdot xy + a_3 \cdot y^2 + a_4 \cdot x + a_5 \cdot y + a_6 .$$

Where, Z is the elevation at the (x,y) point around the grid point. x and y are the horizontal element and the vertical element of the relative distance from the grid point, respectively.

EXAMPLE

Figure-4 shows an example of the HA-DTM based upon the map of the Tsukigataira area in Nagoya University Forests on a scale 1:5,000. The program-based contour lines reproduced from the HA-DTM and the original contour lines digitized from the map are drawn on the map. As in Fig. 5, it is evident that the areal discrepancy between both contour lines is very small.

The contour-line data collected on the map covered a mesh size of 125 x 145 whose interval was 2 mm (10 m on the ground). The number of grid points by which elevations were calculated was gridded DTM are close to the original contour lines on the map, the accuracy of the DTM can be regarded as high. By measuring the whole length L of the original contour lines, and the whole areal discrepancy A between the original contour lines and the reproduced ones through the DTM within the objective area, the calculated result W (= A/L) provides the average areal discrepancy width of the objective area. The accuracy of a DTM is estimated by the index W, and several DTMs by different production systems can be compared to each other quantitatively.

Figure-5 represents the experimental result concerning to the average areal discrepancy width of different types of DTM production systems including the HA-DTM production, a linear interpolation, a triangular plane interpolation, a quadratic polynomial interpolation system , and the third-order polynomial interpolation. The values of W by the HA-DTM production system are always the lowest among several DTM production methods, and the average value of the standard deviation by the HA-DTM system is also the lowest (Kitagawa 1987b).

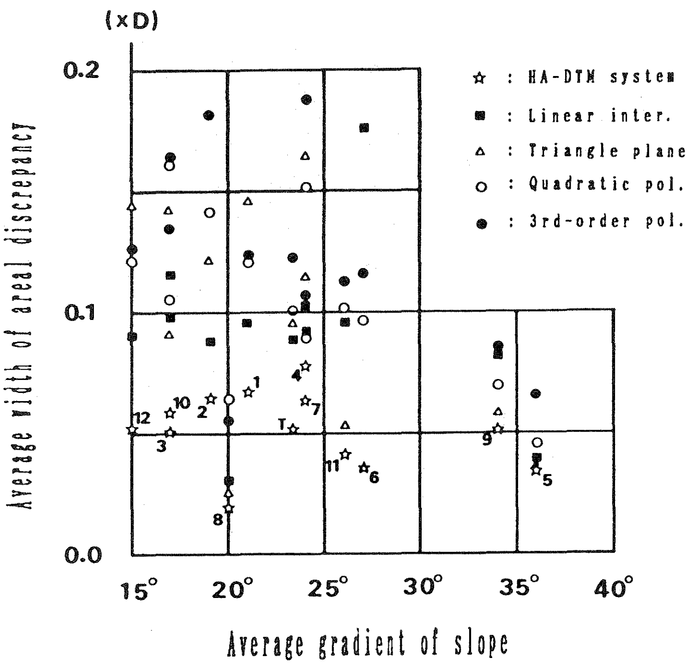


Figure-5 Comparison of DTMs produced by different systems in respect of the areal discrepancy

Estimation of the slope through the HA-DTM

To estimate the slope by the DTM, set Z_{xy} to the elevation at an arbitrary point (x,y). The maximum gradient is generally given by the following equation:

$$\tan\theta = \sqrt{R_x^2 + S_y^2},$$

where, $R_x = \partial Z/\partial x$, and $S_y = \partial Z/\partial y$.

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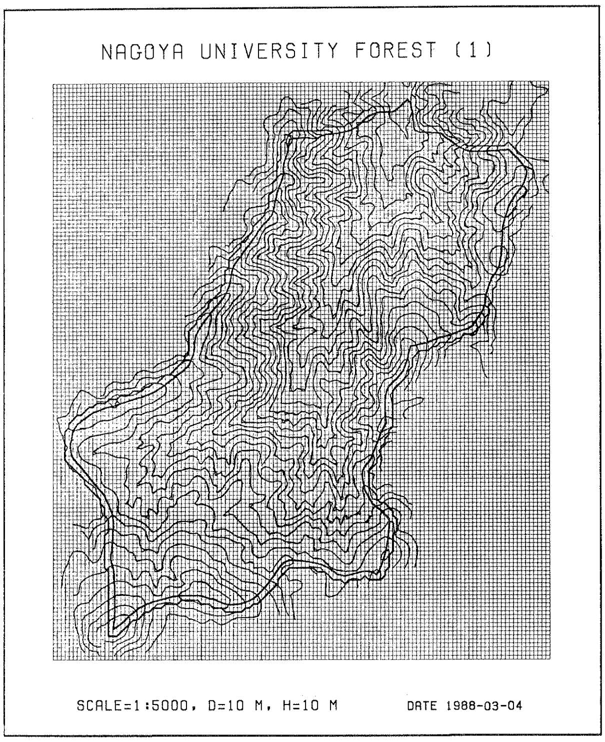

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$$\tan\theta = \sqrt{R_x^2 + S_y^2},$$
where, $R_x = \partial Z/\partial x$, and $S_y = \partial Z/\partial y$.

It has been confirmed through the experimental study about the accuracy of the slope factors estimated by the DTM (Kitagawa 1976) that the approximate method of slope based on the quadratic equation is sufficient in slope estimation. Then, set the following equations to the horizontal and vertical slopes on the grid point P(I,J):

$$R_{ij}=(Z_{i+1,j} - Z_{i-1,j})/2D$$
$$S_{ij}=(Z_{i,j+1} - Z_{i,j-1})/2D$$

Figure-6 shows the accuracy of the gradient estimated by the HA-DTM system. Compared with the other curves (Kitagawa 1976) produced by the linear interpolation DTM system on the same graph, it is evident that the HA-DTM system is superior to the latter system in slope estimation, too.

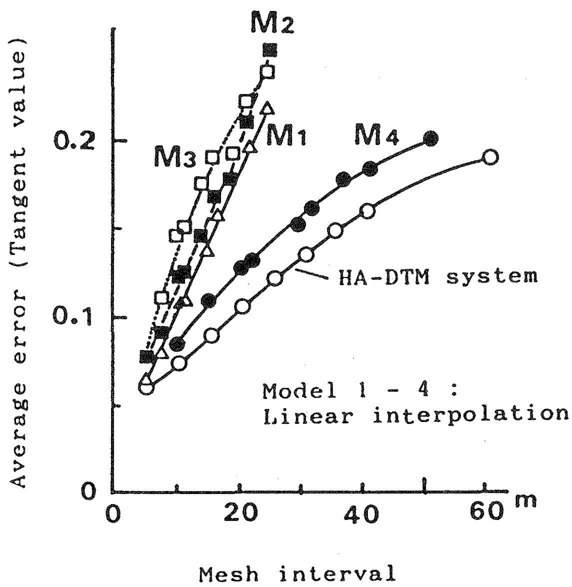

Figure-6 Average error in estimated gradient

DISCUSSION

The HA-DTM production system has been created by improving on the former DTM production system (Kitagawa 1980, 1985) based upon the linear interpolation. It is certain that the difference of the accuracy between them has been brought mainly by linear extrapolation, not by spline function interpolation, as shown in the example of the DTM production.

The HA-DTM system has the following desirable characteristics; 1) high accuracy of the created DTM which can reproduce contour lines very close to the original ones by the figure drawing program, and can represent slope more precisely than by the former methods, 2) good efficiency regarding man-power and time required for creating gridded DTMs, 3) few restrictions on digitizing contour lines, 4) capability in handling the sporadical auxiliary contour lines, and 5) flexibility in limiting the area collecting contour line data within an arbitrary closed boundary line on the topographic map.

CONCLUSION

Through some comparative experiments concerning topographic analysis, it was confirmed quantitatively that the HA-DTM is highly effective for reproducing the topographic characteristics of the objective area. Hereafter, the more detailed topographic analysis based on the HA-DTM system should be studied.

The HA-DTM production system has been registered as one of the library programs of the Nagoya University Computation Center for public use this year. It is expected that the HA-DTM will be conducive to the development of planning technology in wider areas of application as the basic means.

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* : Only in Japanese
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