IMAGE ANALYSIS USING FUZZY ENTROPY AND POSSIBILITY FUNCTION

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ABSTRACT

Analysis of grey tone images is a promising area for using Fuzzy set theory because the image pixels have inherent ambiguity, rather than randomness, due to the multi-valued level of brightness. Fuzzy Entropy, a measure of information content, helps in identifying image regions of high level information. This reduces the search time for recognising the objects in the picture. When the picture composition is conceptually inadmissible, image pixels are substituted with their counterparts having high possibility to compose a picture of acceptable confidence level.

INTRODUCTION

An image is represented by elementary geometrical shapes, called primitives. The extraction of primitives from a given image has been a subject of considerable interest to many researchers [1-5]. In a recent paper [4], Basu and Fu introduced a syntactic approach for identifying edges, lines and corners as image primitives. The minimum distance structure-preserved error-correcting tree algorithm, employed in the paper for the detection of image primitives, involves a laborious task of tree grammer generation. This difficulty is proposed to be overcome in this paper by using the concept of fuzzy entropy which is a measure of information content in vagueness and uncertainty, commonly found in images having pixels of multivalued grey levels in the presence of corrupting noise. This new method is expected to save the computer time for image analysis.

BASIC CONCEPT OF FUZZY ENTROPY AND POSSIBILITY FUNCTION

The theory of fuzzy set was originally developed by Zadeh [6] to deal with ill-defined objects, where possibility of their characteristics, rather than the randomness of occurence, is of vital concern. Following the concept of shannon's entropy, Di Luca and Termini developed the mathematical background of Fuzzy Entropy functional [7] as a Fuzzy measure of information.

It is given by

$$H_j = -\frac{1}{r}\sum_{i=1}^{r}\left\{\mu_{ij}\, p_{ij} \log_e \mu_{ij}\, p_{ij} - (1-\mu_{ij}\, p_{ij}) \log_e (1-\mu_{ij}\, p_{ij})\right\}$$

.......(1)

Where

μ_{ij} = Fuzzy membership of i th attribute of j th object.

p_{ij} = Probability of the occurence of the ith attribute of j th object

r = No. of attributes characterising the object.

Fuzzy entropy functional has been found very useful in decision theory and pattern recognition [8].

Possibility function [9-10] provides an answer to the following question - Given the features x_1, x_2, - - - - - - - -, x_n of an object $f(x_1, x_2, \text{---------}, x_n)$, what is the possibility that f is F i.e. how closely the given object f resembles another object F ? In Fuzzy terminology, the possibility $\pi(F/f)$ is membership degree of f to F. It can be evaluated using equation (2).

$$\pi(F/f) = 1 - \frac{|f-F|}{|f|}$$

.... (2)

Where

$\frac{|f-F|}{|f|}$ is a normalised measure of the deviation of f from F.

IMAGE PRIMITIVES

Image primitives are simple geometrical shapes used for the description of two - dimensional images. A set of image primitives, used in this paper, is given in fig.1. While there can be no unique choice of the image primitives, yet the important guiding principle is that the set of primitives should be small, yet comprehensive enough to describe a large class of images.

Fig.1. Set of image primitives.

It may be pointed out that the primitives shown in Fig.1(a) have a darker background, and those shown in Fig.1(b) have a brighter background.

In order to extract the primitives, the image is divided in the form of an array (usually 256 X 256 or 512 X 512) of pixels. Each pixel is characterised by its position and grey level. The image is scanned by a window of k X k dimension.

It may be pointed out that the pixels in the window may have different grey levels and their distribution will, in general, yield a fuzzy, instead of a sharp, primitive. Following procedure is used to extract a primitive from the knowledge of grey levels of the pixels and their relative positions in the window :-

Step I :- A comprehensive set of ideal-shaped primitives, as shown in fig.1 is collected. The grey levels of the pixels, used for this purpose will either be zero (fully bright) or one (full dark). It may be remarked that the comprehensive set should include all possible ideal shaped primitives. For example, a straight horizontal line primitive can occur in any one of the three rows of 3 x 3 array.

Step 2 :- Determine the entrpy of each position of the pixels, using equation (1). The pixel position which yields the maximum value of the entropy, corresponds to such a position which furnishes maximum information for the classification of primitives. The reading of window pixels should start from this position.

Step 3 :- Read the grey level of the pixel selected in Step.2 and form a set of all k x k window frames having this grey level. It may be noted that for this idealistic situation, the grey level will either be zero or one. Repeat step 2 for the set so formed.

Step 4 :- Repeat step 3 for the other grey level reading of the pixel.

Step 5 :- Continue repeating steps 2 to 4 untill the sub-division of the set is reduced to one of elementary primitives shown in fig. 1.

The above procedure ensures that a primitive can be extracted by reading less than k x k pixels of the windows. For a 3 x 3 window, it was found that about 60% of the primitives were extracted by reading 5 out of 9 pixels and about 95% of the primitives were recognised by reading 6 out of 9 pixels.

POSSIBILITY FUNCTION EVALUATION OF PRIMITIVES

In actual practice, the grey level of pixels will be multi-valued, unlike binary, as assumed in the idealistic primitives.

This causes fuzziness of the primitive in the scanning window. The degree of membership of fuzzy primitive with respect an ideal primitive can be determined by the following algorithm:-

(a) Idealise the grey level values of the pixels by equating grey levels equal to 0.5 or more to unity, and those less than 0.5 to zero.

(b) Count the no. of pixels required to be changed from zero level to unity or vice-versa so that the distribution of pixels matches with the jth ideal primitive. Let it be δ_j.

(c) Determine the average value of the original grey levels of the pixels constituting the primitive. Let it be λ_{1j}.

(d) Determine the averge value of the original grey levels of the pixels constituting the background. Let it be λ_{2j}.

(e) Then the possibility that the primitive x in the window is jth ideal primitive is given by

$$\pi(j/x) = \left(1 - \frac{\delta_j}{k^2}\right)\left(\frac{|\lambda_{1j} - \lambda_{2j}|}{\lambda_{1j} + \lambda_{2j}}\right) \qquad \text{..........(3)}$$

The above procedure is repeated for all the members of the ideal primitives set.

(f) Select the highest value $\pi(l/x)$ and second highest value $\pi(p/x)$ from step (e).

If $\pi(l/x) \geq 0.6$ ....(4)

and $\pi(l/x) - \pi(p/x) > 0.1$ ....(5)

then the primitive in the window belongs to type l.

It may be remarked that the equation (5) provides a measure for a clear confidence level.

CONCLUSION

An extremely general algorithm, based on fuzzy entropy and possibility function, has been presented in this paper for the recognition of primitives in an image. The method was tried on a variety of images and it was found that the method provides very encouraging results for the analysis of synthetic aperture radar (SAR) images having low signal - to - noise ratio and coherent noise.

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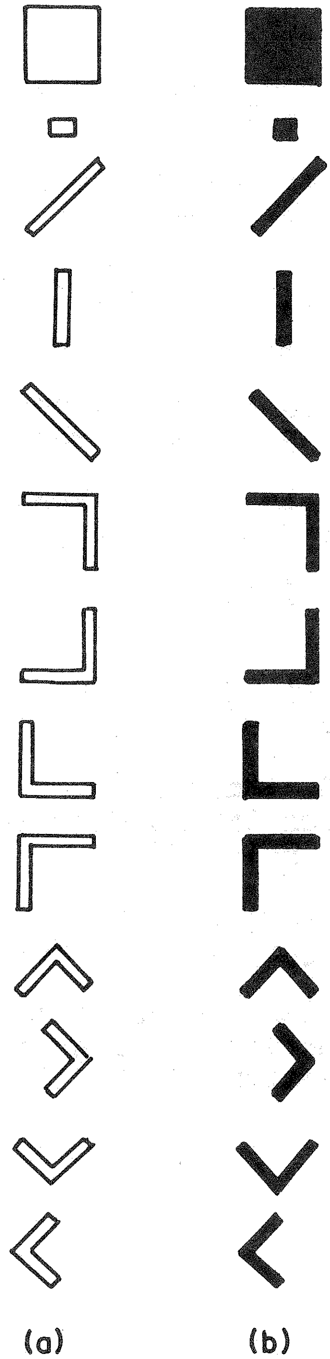

FIG. I SET OF IMAGE PRIMITIVES