ON THE RELIABILITY OF ADDITIONAL PARAMETERS

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ABSTRACT

In order to compensate the possible systematic errors in observations it is necessary to perform a self calibrating adjustment with some suitable additional parameters. These parameters, besides being able to describe the existing systematic errors properly, should have also a good reliability. Based upon the extended reliability theory the internal and external reliability (i.e. determinability and sensitivity) of additional parameters are investigated in this paper. The determinability and sensitivity factors are calculated and analysed for a series of self calibrating bundle block adjustments with different geometric configurations. The results show that the now commonly adopted measures of significance and determinability tests are not quite adequate for the correct selection of additional parameters. It is only their sensitivity test that gives a most efficient measure for such a purpose.

1. INTRODUCTION

It is well known in photogrammetry and in geodesy that the self calibrating adjustment with additional parameters can compensate the systematic errors existing in observations. But up to now a satisfactory method for selecting additional parameters has not been found. The reason is that in many cases, especially in weak blocks with sparse ground control, there is a certain contrary between the complete compensation of systematic errors and the stability of the model. To solve this contrary two different treatments are usually used /Li, 1985/.

In the first treatment the stability of the adjustment model or the numerical stability are ensured by improving the block geometry or by treating additional parameters or best all unknowns as weighted observations without any deletion of additional parameters from a comprehensive parameter set. The another treatment is to select additional parameters from a set of candidates during the adjustment in order to ensure the stability of the model. In this case the statistical tests are of caurse required / Foerstner, 1980 and Li, 1981 /. For instance, in PAT-BS program the significance test and determinability test are introduced for the selection of additional parameters, while in BLUH program besides the significance test a correlation analysis of additional parameters is carried out. Based upon the use of Qxx and Qvv matrices in a principle component analysis or a stepwise regression analysis Sarjakoski has proposed two special measures for controlling the geometrical quatity of the model extension / Sarjakoski, 1984 /.

However the most important thing for the self calibrating adjustment is not whether the additional parameters are significant or determinable, but how great the effect of non-determinable additional parameters on the adjusted results. In this sense it is valuable to investigate the external reliability of additional parameters and to compare it with their internal reliability in order to find a most efficient measure for selecting additional parameters.

2. DETERMINABILITY AND SENSITIVITY OF ADDITIONAL PARAMETERS

According to an extension of the baarda theory /Foerstner, 1983 and Li, 1986/ the fundamental equations for studying onto reliability of additional parameters can be found.

2.1 Determinability of additional parameters

The self calibrating adjustment starts from an extended Gauss-Markov-Model and has the error equations;

$$v = A\hat{x} + H\widehat{\nabla s} - l, \qquad P \tag{1}$$

where $\hat{x}$ -- estimator vector of the unknowns of basic model,
$\widehat{\nabla s}$ -- estimator of additional parameter vector,
A and H -- the correspondent coefficient matrices and
l -- observation vector with weight matrix P.

The lower bound $\nabla_0 s$ for additional parameter vector, which are determinable with a probability $> \beta_0$ under a test with the significance number α_0 , can be derived;

$$\nabla_0 s = \sigma_0 \cdot \delta_0'(s) \cdot s \qquad (|s|=1) \tag{2}$$

with

$$\delta_0'(s) = \delta_0(s)/\sqrt{s^T(p_{ss})s} \tag{3}$$

in which,

$$p_{ss} = H^T P Q_{vv} P H \tag{4}$$

$$Q_{vv} = P^{-1} - A Q_{xx} A^T \tag{5}$$

The value $\delta_0'(s)$ represents the size(with unit σ_0) of determinable additional parameter in direction s of p-dimensional space in which p is the number of additional parameters. Therefore it can be used as a measure of internal reliability of additional parameters.

$\delta_0(s)$ is the non-centrality parameter in direction s and is a function of statistical parameter α_0 and β_0 If $\delta_0(s)=\delta_0$ is chosen independently from s, eq.(2) describes the ellipse of boundary values. If additional parameter vector falls into the ellipse, it can not be statistically determined by the self calibration.

Thus, we can obtain determinability factors for single and multiple additional parameters as following;

$$\delta'_{0,b_i} = \delta_0/\sqrt{(p_{ss})_{ii}} \tag{6}$$

and

$$\delta'_{0,s_i} = \delta_0/\sqrt{\lambda_{s_i(p_{ss})}} \tag{7}$$

where $\lambda_{s_i(p_{ss})}$ is the eigenvalue of matrix Pxx in the direction of eigenvector s_i .

2.2 Sensitivity of additional parameters

The effect of non-determinable parameter vector (eq.(2)) onto the estimated unknowns is called as the external reliability of additional parameters. It is usually determined by the lenghth $\bar{\delta}_0(s)$ of this influence vector. We define this value as sensitivity factor of additional parameters;

$$\bar{\delta}_0(s) = \delta_0(s)\sqrt{\frac{s^T(\bar{p}_{ss} - p_{ss})s}{s^T p_{ss} s}} \tag{8}$$

in which

$$\bar{p}_{ss} = H^T P H \tag{9}$$

Also assuming that $\delta_0(s)$ is independent from the direction s we obtain the sensitivity factors of the adjusted results with respect to non-determinable parameters as following:

$$\bar{\delta}_{0.b_i}=\delta_0\cdot\sqrt{\frac{(\bar{p}_{ss})_{ii}}{(p_{ss})_{ii}}-1} \tag{10}$$

and

$$\bar{\delta}_{0.t_i}=\delta_0\sqrt{\frac{1}{\lambda_{t_i}(\bar{p}_{ss}^{-1}p_{ss})}-1} \tag{11}$$

The vectors t_i here are the solutions of the general eigenvalue problem:

$$(\bar{P}_{ss}-P_{ss})\ t=(\bar{\delta}_c^2/\delta_0^2)\ P_{ss}\ t \tag{12}$$

3. EXPERIMENTAL COMPUTATION

At the present time only in PAT-BS program packet the determinability values for individual additional parameter can be computed. It has been proved in practice that this measure is more important than other statistical tests for selecting additional parameters / Foerstner, 1980 /. Therefore, a systematic investigation on internal and external reliability of additional parameters unter different geometry configurations must be necessary and practically useful.

The computations are done by the use of simulation data which has the block sizes of 2x5, 4x9, 6x13 and 8x17 photographs. Each photograph has 9 tie points. The control point distributions are shown in Fig. 1 . Regarding different flight arrangements the computation versions are summerized in Tab.1. The author has developed a program which is connected with PAT-BS for the computation of matrices P_{ss} , $\bar{P}_{ss}$ and reliability factors of additional parameters. All computations were done in Harris H100 at the Institute of Photogrammetry, Stuttgart University.

4. RESULTS AND CONCLUSIONS

According to eq.(6),(7) and (10),(11) the determinability and sensitivity factors are calculated for different versions in Tab.1 and different block sizes(see Tab.2 --- Tab.5). The orthogonal parameter set from /Ebner, 1976/ is used:

$$\begin{aligned}\Delta x&=b_1x+b_2y-b_3(2x^2-4b^2/3)+b_4xy+b_5(y^2-2b^2/3)+\\&\quad+b_7x(y^2-2b^2/3)+b_9(x^2-2b^2/3)y+\\&\quad+b_{11}(x^2-2b^2/3)(y^2-2b^2/3)\\\Delta y&=-b_1y+b_2x+b_3xy-b_4(2y^2-4b^2/3)+b_6(x^2-2b^2/3)+\\&\quad+b_8(x^2-2b^2/3)y+b_{10}x(y^2-2b^2/3)+\\&\quad+b_{12}(x^2-2b^2/3)(y^2-2b^2/3)\end{aligned} \tag{13}$$

From these results of simulation computations the following conclusions can be drawn:

a) The internal and external reliability of additional parameters depends essentially on the flight arrangement. The widely used single block with 20 percent sidelap is not beneficial for the determination of many parameters (such as b_4-b_5 , b_7 , b_8).
A better determinability of additional parameters can not be obtained through increasing the side overlap to 60 percent. Only when using the cross flights (version K20) all parameters can have much better determinability than those

at other flight versions under any control distributions.

b) The improvement of the reliability of additional parameters with the decrease of control interval is not so significant. For instance, there are always 3-4 non-determinable parameters in commenly used single block with 20% side overlap while the control interval varies from 2 to 12 basis. Only when at the ends of each flight derection line a horizontal and vertical control point is added (version 2a), the non-determinability problem of additional parameters can be solved. On the contrary, if using cross flights, the very good determinability of additional parameters can be obtained independently from the control point distributions.

c) Contrary to the reliability of photogrammetric observations, the reliability of additional parameters has a very complex relation with the size of the blocks. Fig.2-Fig.4 show the dependences of determinability and sensitivity factors on the block sizes, from which we can find the determinability of parameters are always improved with the increase of the block size, because the bigger the size of the block, the more photogrammetric information can be used to determine additional parameters much better. Nevertherless it must be noted, the effects of non-determinable additional parameters on adjusted results are not always decrease with the increase of the block size. In some cases, such as in the case of Fig.2, the bigger the block, the worse sensitivity additonal parameters have. The dependences of the maximum values of δ_0' and $\bar{\delta}_0$ on the size of blocks are shown in Fig.4. Only in the blocks with cross flights (version k20) or in the blocks with 60 percent sidelap and the same flight derection (version D 60 B) the sensitivity of additional parameters will be improved with enlargement of block size. The reason, why the relationship between the reliability of additional parameters and the size of the blocks is so complex, can be found in appendix of Li, 1987, the discussion here will not go on in detail.

d) The external reliability of additional parameters is always worse than their internal reliability, which is fully reversal to the reliability of gross errors. According to the study on the reliability of block triangulation / Foerstner, 1985 / the sensitivity factors for gross errors are in any cases smaller than their controllability factors. From fig.2, 3 and 4 we can find:
Firstly, the maximum value of sensitivity factors for additional parameters is much greater than the maximum value of determinability factors, i.e.

$$\bar{\delta}_{a,\max} \gg \delta'_{0,\max}$$

Secondly, the differences among the sensitivity factors are also much greater than the differences among their determinability factors.

e) The sensitivity factor of additional parameters should be adopted as a main measure for automatical selecting additional parameters in self calibrating adjustment. The determinability factor is although a good measure for selecting additional parameters as compared with other measures, it is still not a ideal measure. The reasons are:
- The determinability factor gets alwats smaller with the increase of block size, it does not mean the effect of non-determinable systematic errors on the adjusted results also gets smaller;
- The differences of determinability factors is so small that it is very difficult to find a suitable threshold for determinability values by the selection of additional parameters.

When using the sensitivity factor of additional parameters as the measure for their selecting the above described problems can be well solved. For example, when in Fig.2

and Fig.3 we use determinability factor to select additional parameter, all parameters will be determinable by the use of $\delta'_{0,\text{亢}} \leqslant 1$ .But if we let $\delta'_{0,\text{亢}} \leqslant 0.6$ the many additional parameters for the block with 2×5 photographs will be non-determinable. On the contrary, if we use sensitivity factor to select them and choose $\bar{\delta}_{0,\text{亢}} \leqslant 5$ as the threshold, the right decision for selecting additional parameters can be always made.

It can be also proved that the sensitivity factor is better than the relative measure of accuracy Zx from / Sarjakoski, 1984 /, which includes some experince-based heuristic factors and demands more computational efforts. Instead of his relative measure of internal reliability Zv we can use the separability multiplying factor or directly use the correlation coefficient between gross and systematic errors (see / Li, 1986, 1987 /).

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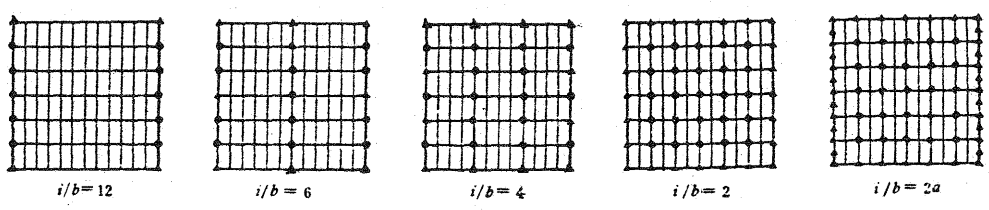

▲ Full control point • Vertical control point

Fig.1 Control point distributions

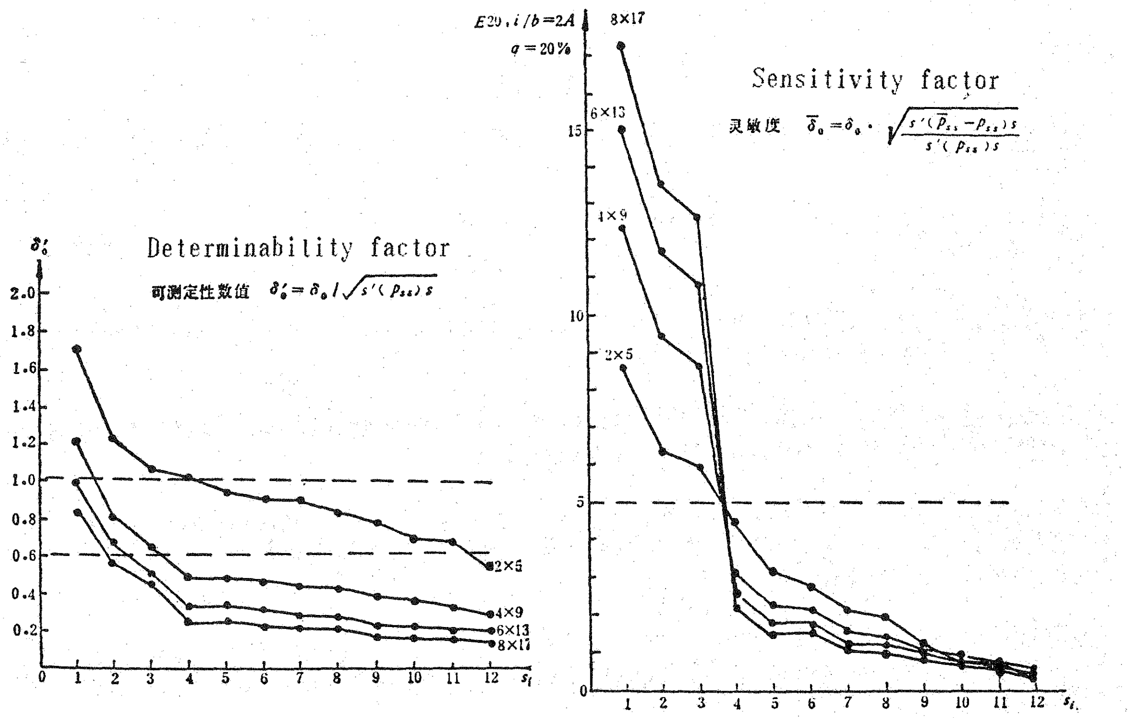

Fig.2 Dependence of reliability of additional parameters on block sizes

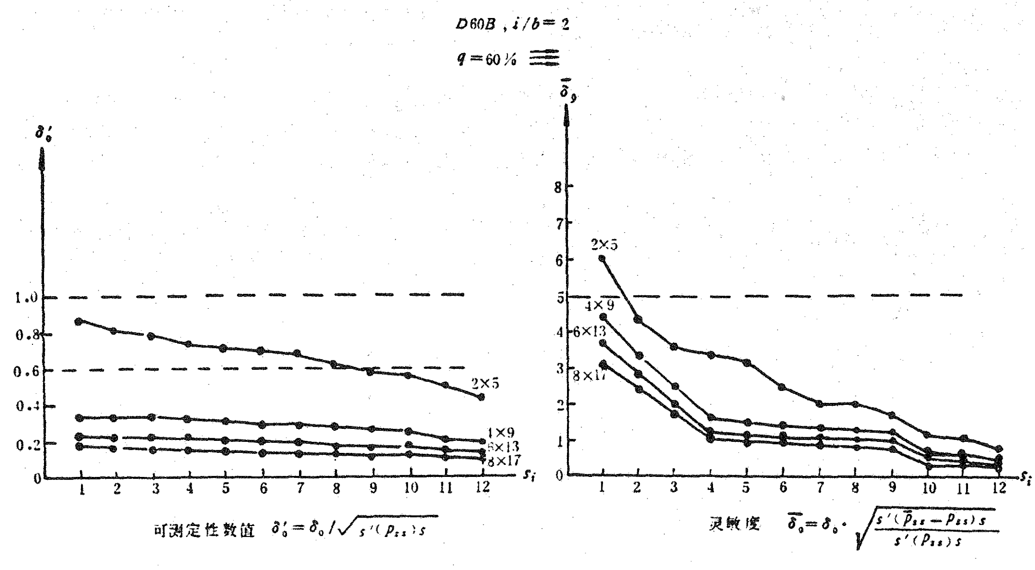

Fig.3 Dependence of reliability of additional parameters on block sizes

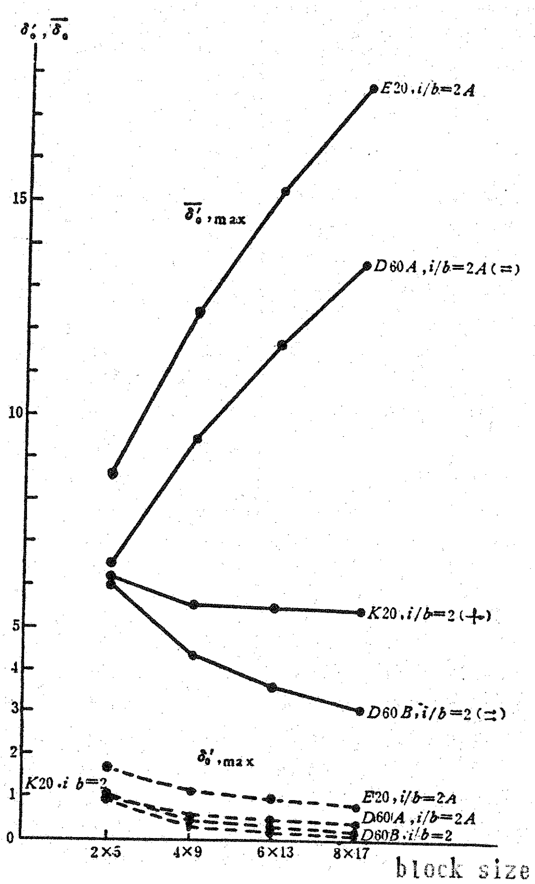

Fig.4 The relationship between the maximum value of δ_0' and $\bar{\delta}_0$ and the block sizes

Tab.1 Versions of simulation computation

Flight arrangement \ ground control		$i/b=12$	$i/b=6$	$i/b=4$	$i/b=2$	$i/b=2a$
Single block $q=20\%$		$E20$-12	$E=20$-6	$E20$-4	$E20$-2	$E20$-2a
Double block	$q=60\%$ ⇄	$D60A$-12	$D60A$-6	$D60A$-4	$D60A$-2	$D60A$-2a
	$q=60\%$ ⇉	$D60B$-12	$D60B$-6	$D60B$-4	$D60B$-2	—
Cross flight, $q=20\%$ ⇸		$K20$-12	$K20$-6	$K20$-4	$K20$-2	—

Tab. 2 Internal and external reliability of additional parameters ($E20, 6\times 13$)

additional parameter	$i/b=12$		$i/b=6$		$i/b=4$		$i/b=2$		$i/b=2a$	
	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$
b_1	2.60	55.77	0.66	13.53	0.39	7.34	0.25	3.50	0.25	3.34
b_2	2.41	51.44	0.54	10.70	0.29	4.70	0.21	2.00	0.21	1.77
b_3	0.19	1.39	0.19	1.37	0.19	1.32	0.19	1.08	0.19	0.89
b_4	∞	∞	∞	∞	∞	∞	∞	∞	0.51	10.81
b_5	∞	∞	∞	∞	∞	∞	∞	∞	0.66	11.56
b_6	0.26	2.52	0.24	1.90	0.24	1.65	0.23	1.21	0.23	1.19
b_7	7.36	112.04	1.86	28.05	1.11	16.38	0.71	10.03	0.60	8.24
b_8	7.26	112.04	0.33	2.76	0.32	2.45	0.30	1.89	0.30	1.88
b_9	0.27	0.65	0.27	0.63	0.27	0.58	0.27	0.48	0.27	0.45
b_{10}	0.30	1.91	0.30	1.85	0.30	1.80	0.29	1.72	0.28	1.25
b_{11}	0.34	1.67	0.34	1.67	0.34	1.66	0.34	1.63	0.32	0.61
b_{12}	0.32	0.85	0.32	0.85	0.32	0.85	0.32	0.84	0.32	0.79
t_1	∞	∞	∞	∞	∞	∞	∞	∞	0.98	15.25
t_2	∞	∞	∞	∞	∞	∞	∞	∞	0.66	11.65
t_3	∞	∞	∞	∞	∞	∞	∞	∞	0.51	10.86
t_4	∞	∞	0.62	11.96	0.38	6.44	0.34	2.59	0.32	2.54
t_5	2.64	56.38	0.55	11.01	0.34	4.90	0.32	2.23	0.32	1.79
t_6	2.41	46.86	0.34	2.76	0.32	2.45	0.30	1.81	0.30	1.78
t_7	0.34	2.52	0.33	1.90	0.31	1.81	0.29	1.80	0.28	1.26
t_8	0.32	1.93	0.32	1.83	0.31	1.72	0.27	1.47	0.27	1.20
t_9	0.30	1.81	0.30	1.79	0.28	1.56	0.24	1.21	0.23	0.88
t_{10}	0.27	1.23	0.27	1.21	0.27	1.07	0.23	0.83	0.23	0.75
t_{11}	0.26	0.84	0.24	0.84	0.24	0.84	0.21	0.82	0.21	0.60
t_{12}	0.19	0.58	0.19	0.56	0.19	0.52	0.19	0.46	0.19	0.42

Tab. 3 Internal and external reliability of additional parameters ($D60A, 6\times 13$)

additional parameter	$i/b=12$		$i/b=6$		$i/b=4$		$i/b=2$		$i/b=2a$	
	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$
b_1	1.41	40.95	0.44	12.27	0.29	7.24	0.19	3.58	0.18	3.18
b_2	1.44	41.85	0.39	10.69	0.24	5.52	0.17	2.74	0.16	2.33
b_3	0.14	1.08	0.14	1.06	0.14	1.05	0.14	1.05	0.14	0.92
b_4	∞	∞	∞	∞	∞	∞	∞	∞	0.35	10.01
b_5	∞	∞	∞	∞	∞	∞	∞	∞	0.49	11.61
b_6	0.16	0.51	0.16	0.48	0.16	0.46	0.16	0.45	0.16	0.43
b_7	0.24	2.80	0.22	1.69	0.21	1.35	0.21	1.05	0.21	0.99
b_8	3.99	82.32	0.24	2.78	0.23	2.48	0.22	1.96	0.22	1.92
b_9	0.20	0.74	0.20	0.70	0.20	0.65	0.20	0.60	0.20	0.49
b_{10}	0.21	1.03	0.21	0.99	0.20	0.93	0.20	0.82	0.20	0.65
b_{11}	0.24	1.29	0.24	1.29	0.24	1.28	0.24	1.26	0.23	0.64
b_{12}	0.23	0.80	0.23	0.80	0.23	0.80	0.23	0.79	0.23	0.68
t_1	∞	∞	∞	∞	∞	∞	∞	∞	0.49	11.70
t_2	∞	∞	∞	∞	∞	∞	∞	∞	0.35	10.08
t_3	446.00	9727.11	0.45	12.45	0.29	7.36	0.24	3.62	0.23	3.24
t_4	1.48	42.96	0.40	10.77	0.24	5.53	0.23	2.74	0.23	2.33
t_5	1.33	36.59	0.24	2.78	0.24	2.47	0.22	1.93	0.22	1.87
t_6	0.24	2.80	0.24	1.67	0.23	1.38	0.21	1.36	0.21	0.98
t_7	0.24	1.38	0.23	1.38	0.23	1.31	0.21	1.04	0.20	0.92
t_8	0.23	1.22	0.22	1.14	0.21	1.07	0.20	0.96	0.20	0.75
t_9	0.21	0.97	0.21	0.95	0.21	0.94	0.19	0.92	0.18	0.69
t_{10}	0.20	0.84	0.20	0.83	0.20	0.83	0.17	0.82	0.16	0.63
t_{11}	0.16	0.45	0.16	0.42	0.16	0.41	0.16	0.39	0.16	0.38
t_{12}	0.14	0.40	0.14	0.38	0.14	0.37	0.14	0.35	0.14	0.33

Tab.4 Internal and external reliability of additional parameters (*D60B*, 6×13)

additional parameter	$i/b=12$		$i/b=6$		$i/b=4$		$i/b=2$	
	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$
b_1	4×10^6	1×10^8	0.44	12.27	0.29	7.24	0.19	3.58
b_2	1.44	41.85	0.39	10.69	0.24	5.52	0.17	2.74
b_3	0.14	1.05	0.14	1.01	0.14	1.00	0.14	0.99
b_4	0.14	1.34	0.14	1.30	0.14	1.26	0.14	1.14
b_5	0.17	1.40	0.17	1.18	0.17	1.10	0.17	0.99
b_6	0.17	1.20	0.17	1.16	0.17	1.15	0.17	1.12
b_7	0.24	2.80	0.22	1.69	0.21	1.35	0.21	1.05
b_8	∞	∞	0.24	2.78	0.23	2.48	0.22	1.96
b_9	0.22	0.74	0.20	0.70	0.20	0.65	0.20	0.60
b_{10}	0.21	1.03	0.21	0.99	0.20	0.93	0.20	0.82
b_{11}	0.23	0.23	0.23	0.23	0.23	0.22	0.23	0.22
b_{12}	0.23	0.41	0.23	0.40	0.23	0.39	0.23	0.38
t_1	∞	∞	0.45	12.45	0.29	7.36	0.23	3.62
t_2	277.75	7081.14	0.40	10.77	0.24	5.53	0.23	2.74
t_3	1.48	42.96	0.24	2.78	0.23	2.47	0.22	1.93
t_4	0.24	2.80	0.23	1.67	0.23	1.31	0.21	1.18
t_5	0.23	1.43	0.23	1.35	0.23	1.30	0.21	1.08
t_6	0.23	1.41	0.22	1.18	0.21	1.11	0.20	1.04
t_7	0.21	1.22	0.21	1.14	0.21	1.10	0.19	1.02
t_8	0.20	1.13	0.20	1.12	0.20	1.07	0.17	0.96
t_9	0.17	1.02	0.17	1.01	0.17	1.00	0.17	0.96
t_{10}	0.17	0.40	0.17	0.38	0.17	0.38	0.17	0.37
t_{11}	0.14	0.39	0.14	0.38	0.14	0.37	0.14	0.35
t_{12}	0.14	0.22	0.14	0.22	0.14	0.22	0.14	0.21

Tab.5 Internal and external reliability of additional parameters (*K20*, 6×13)

additional parameter	$i/b=12$		$i/b=6$		$i/b=4$		$i/b=2$	
	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$	δ_0'	$\bar{\delta}_0$
b_1	0.16	1.77	0.15	1.67	0.15	1.66	0.15	1.65
b_2	0.14	0.37	0.14	0.37	0.14	0.36	0.14	0.36
b_3	0.13	0.72	0.13	0.71	0.13	0.71	0.13	0.36
b_4	0.18	3.68	0.18	3.65	0.18	3.62	0.17	3.45
b_5	0.23	4.17	0.23	4.17	0.23	4.17	0.23	4.15
b_6	0.17	0.90	0.17	0.86	0.17	0.83	0.16	0.20
b_7	0.32	5.21	0.30	4.55	0.29	4.50	0.29	4.43
b_8	0.26	3.24	0.22	1.25	0.21	1.02	0.21	0.60
b_9	0.19	0.36	0.19	0.34	0.19	0.31	0.19	0.20
b_{10}	0.20	0.74	0.20	0.74	0.20	0.74	0.20	0.73
b_{11}	0.23	0.88	0.23	0.88	0.23	0.88	0.23	0.87
b_{12}	0.22	0.34	0.22	0.33	0.22	0.33	0.22	0.30
t_1	0.41	7.36	0.32	5.59	0.32	5.53	0.32	5.46
t_2	0.25	4.20	0.23	4.20	0.23	4.20	0.23	4.17
t_3	0.23	3.70	0.23	3.67	0.22	3.65	0.22	3.47
t_4	0.23	3.11	0.22	1.34	0.22	1.13	0.22	0.91
t_5	0.22	0.95	0.22	0.95	0.21	0.95	0.21	0.82
t_6	0.20	0.90	0.20	0.86	0.20	0.83	0.20	0.78
t_7	0.19	0.80	0.19	0.80	0.19	0.80	0.19	0.52
t_8	0.18	0.72	0.18	0.71	0.18	0.66	0.17	0.33
t_9	0.17	0.56	0.16	0.56	0.16	0.56	0.16	0.17
t_{10}	0.15	0.34	0.15	0.32	0.15	0.32	0.15	0.14
t_{11}	0.14	0.32	0.14	0.32	0.14	0.27	0.14	0.12
t_{12}	0.13	0.14	0.13	0.14	0.13	0.13	0.13	0.12