BLOCK TRIANGULATION AT LARGE SCALE USING FLIGHT VARIANT ZEISS UMK CAMERA PHOTOGRAPHY

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ABSTRACT

Aerotriangulation research was conducted to determine the minimum control configuration that would yield accurate photo control.
In this context, a new mathematical model which accepts photogrammetric and geodetic observations has been developed and investigated for control extension at large scale (1:1500). All parameters of transformation and calibration are recovered simultaneously in a least squares block adjustment Practical experiments have been made with a UMK flight-variant camera mounted on an external photomount of a Cessna 172 aircraft. The ground control points were targeted and their coordinates could be determined from terrestrial observations. The photogrammetric observations were performed with a Wild Aviolyt BC2 analytical stereoplotter and processed subsequently by its softwares.
The analysis of errors and the comparison with the block triangulation by independent models and by bundle adjustment reveals that the proposed method has the same estimated accuracy and offers a theoretical and practical alternative to existing models for control extension.

INTRODUCTION

The photogrammetric theory employed in our study for the least squares adjustment and error propagation of analytical photogrammetric triangulation is based on direct linear transformation. The Projective Transformation Parameters (DLT) are computed by an iterative way using relative control points by considering as unknowns not only the DLT coefficients of each exposure station, but also the X, Y, Z coordinates corresponding to some of the measured images (Barbalata, 1980).
Geodetic measurements, in our case ground distances, provide excellent constraints to the general adjustment. Distances and elevations differences are measured by precise surveying methods and if the number of those complementary measurements is kept to a minimum, no distortions are introduced into the estimated parameters.

THE MATHEMATICAL MODEL OF ANALYTICAL STEREOTRIANGULATION

The fundamental relations between the cartesian space coordinates X , Y , Z of a point j and the coordinates $\hat{x}$, $\hat{y}$ of its photographic image, can be put into the form :

$$\hat{x} = -c \frac{A\Delta X + B\Delta Y + C\Delta Z}{D\Delta X + E\Delta Y + F\Delta Z} , \quad \hat{y} = -c \frac{A'\Delta X + B'\Delta Y + C'\Delta Z}{D\Delta X + E\Delta Y + F\Delta Z} \qquad (1)$$

where $\hat{x}$, $\hat{y}$ are the measured plate coordinates properly corrected for comparator errors and lens distortion , respectively :

$$\hat{x} = \bar{x} + \bar{x}(K_1\bar{r}^2 + K_2\bar{r}^4 + K_3\bar{r}^6 + \ldots) + (\bar{r}^2 + 2\bar{x}^2)P_1 + 2\,\bar{x}\,\bar{y}\,P_2$$
$$\hat{y} = \bar{y} + \bar{y}(K_1\bar{r}^2 + K_2\bar{r}^4 + K_3\bar{r}^6 + \ldots) + 2\,\bar{x}\,\bar{y}\,P_1 + (\bar{r}^2 + 2\bar{y}^2)P_2 \qquad (2)$$

$\bar{x}$, $\bar{y}$ are image coordinates reduced at principal point x_p , y_p :

$$\bar{x} = x - x_p , \quad \bar{y} = y - y_p , \quad \bar{r} = \sqrt{(x - x_p)^2 + (y - y_p)^2} \qquad (3)$$

c is the calibrated principal distance of the i-th exposure,
x , y are the measured image coordinates corrected for comparator errors
K_1, K_2, K_3 are the coefficients for Gaussian symmetric radial distortion.
P_1, P_2 are the coefficients for descentering distortion.

$$\Delta X = X - X^c , \quad \Delta Y = Y - Y^c , \quad \Delta Z = Z - Z^c \qquad (4)$$

where X^c, Y^c, Z^c are object space coordinates of the perspective center of the i-th exposure.

$$T = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix} = \text{the rotation matrix of the i-th exposure} \qquad (5)$$

After some manipulations, equation (1) can be written as :

$$x + \Delta x - \frac{a_1 X + a_2 Y + a_3 Z + a_4}{a_9 X + a_{10} Y + a_{11} Z + 1} = 0, \quad y + \Delta y - \frac{a_5 X + a_6 Y + a_7 Z + a_8}{a_9 X + a_{10} Y + a_{11} Z + 1} = 0 \qquad (6)$$

where :

$$\Delta x = \bar{x}\,(K_1\bar{r}^2 + K_2\bar{r}^4 + K_3\bar{r}^6 + \ldots) + (\bar{r} + 2\bar{x}^2)P_1 + 2\,\bar{x}\,\bar{y}\,P_2$$
$$\Delta y = \bar{y}\,(K_1\bar{r}^2 + K_2\bar{r}^4 + K_3\bar{r}^6 + \ldots) + 2\,\bar{x}\,\bar{y}\,P_1 + (\bar{r} + 2\bar{y}^2)P_2 \qquad (7)$$

and a_1 , a_2,.... a_{11} are eleven parameters of Direct Linear Transformation, respectively :

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & 1 \end{bmatrix} = \frac{1}{V} \begin{bmatrix} -x_p D + cA & -x_p E + cB & -x_p F + cC & R \\ -y_p D + cA' & -y_p E + cB' & -y_p F + cC' & S \\ -D & -E & -F & V \end{bmatrix} \qquad (8)$$

where :

$$R = (x_p D - cA)X^c + (x_p E - cB)Y^c + (x_p F - cC)Z^c$$
$$S = (y_p D - cA')X^c + (y_p E - cB')Y^c + (y_p F - cC')Z^c \qquad (9)$$
$$V = DX^c + EY^c + FZ^c$$

The two condition equations (6) can be expressed by the functions :

$$F_x = x + \Delta x - (m/q) = 0$$
$$F_y = y + \Delta y - (n/q) = 0 \qquad (10)$$

in which :

$$\begin{bmatrix} m \\ n \\ q \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \qquad (11)$$

The equations (6) are the basic equations for DLT, where the eleven parameters are considered as being independent. In order to obtain an exact solution for the calibration, it is necessary to establish two constraints between them, which must be enforced for each station (Bopp, Krauss, 1978)

$$F_1 = Q_1 - Q_2 + (Q_5^2 - Q_4^2)/\,Q_3 = 0, \qquad F_2 = Q_6 - Q_4\,Q_5\,/\,Q_3 = 0 \tag{12}$$

in which :

$$\begin{array}{ll} Q_1 = a_1^2 + a_2^2 + a_3^2\,, & Q_4 = a_1\,a_9 + a_2\,a_{10} + a_3\,a_{11} \\ Q_2 = a_5^2 + a_6^2 + a_7^2\,, & Q_5 = a_5\,a_9 + a_6\,a_{10} + a_7\,a_{11} \\ Q_3 = a_9^2 + a_{10}^2 + a_{11}^2\,, & Q_6 = a_1\,a_5 + a_2\,a_6 + a_3\,a_7 \end{array} \tag{13}$$

The photogrammetric model under consideration is assumed to involve m exposure stations (i=1,2,...,m) and a total of n control points (j = 1,.n) It is assumed that aproximations are known for each of the unknown coordinates of the control points.

According to (10), the linearized condition equations arising from the j-th control point and all m stations can be formulated. The appropriate matrix equation for this ensemble is :

$$v_j + B\,\Delta + \bar{B}\,\bar{\Delta} = E\,, \tag{14}$$

where :

$$v_j = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_m \end{bmatrix}_j, \quad B_j = \begin{bmatrix} B_1' & B_1'' & 0 \ldots & 0 \\ B_2' & 0 & B_2'' \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ B_m' & 0 & 0 & B_m'' \end{bmatrix}_j, \quad \bar{B}_j = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \cdot \\ \cdot \\ \bar{B}_m \end{bmatrix}_j, \quad E_j = \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ E_m \end{bmatrix}_j \tag{15}$$

For calibration the two constraint equations (12) are interpreted as additional observations with a suitable high weight which enforces the zero variances. The linearized constraint equations arising from all m stations are expressed by the system :

$$\overset{o}{v} + \overset{o}{B}\,\Delta = G \tag{16}$$

If all linearized equations arising from all n points and all m stations are gathered, the set of equations can be written in matrix form as :

$$\begin{bmatrix} v \\ \overset{o}{v} \end{bmatrix} + \begin{bmatrix} B & \bar{B} \\ \overset{o}{B} & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ \bar{\Delta} \end{bmatrix} = \begin{bmatrix} E \\ G \end{bmatrix} \tag{17}$$

in which the primary matrices B' ,B" ,B ,B are defined by the Jacobians:

$$B' = \frac{\partial(F_x\,,\ F_y)^o}{\partial(u')^{oc}\ k'=1,2\ldots5}\,, \qquad B'' = \frac{\partial(F_x\,,\ F_y)^o}{\partial(u'')^{oc}\ k''=1,2\ldots11}\,, \qquad E = \begin{bmatrix} -F_x \\ -F_y \end{bmatrix}^o$$

$$B = \frac{\partial(F_x\,,\ F_y)^o}{\partial(u)^{oc}\ k=1,2,3}\,, \qquad B = \frac{\partial(F_x\,,\ F_y)^o}{\partial(u'')^{oc}\ k''=1,2\ldots11}\,,$$

$$F_x^o = x^o + \Delta x^{oo} - (m/q)^{oc} \qquad G = \begin{bmatrix} -F_1 \\ -F_2 \end{bmatrix}^o, \qquad u_1' = K_1 \quad u_2' = K_2 \quad u_3' = K_3\,, \qquad u_4' = P_1 \quad u_5' = P_2 \tag{18}$$

$$F_y^o = y^o + \Delta y^{oo} - (n/q)^{oc},$$

$$\Delta\,\text{distortion} = (\delta K_1 \quad \delta K_2 \quad \delta K_3 \quad \delta P_1 \quad \delta P_2)^T,$$

$$\Delta = (\delta a_1 \quad \delta a_2 \ldots \delta a_{11})^T, \qquad \bar{\Delta} = (\delta X \quad \delta Y \quad \delta Z)^T$$

$$u'' = (a_1,\ a_2,\ldots,\ a_{11})$$

The quantities $x^o + \Delta x^{oo}$, $y^o + \Delta y^{oo}$ represent values of measured image coordinates referred to the approximate principal point and corrected approximately for radial and decentering distortions.

THE MATHEMATICAL MODEL OF GEODETIC OBSERVATIONS

The observations considered in this approach are "p" slope distances "d" measured by precise surveying methods between the points which appear on the photographs. The equations had to be transferred into a Cartesian coordinate system and are given in linearized form (Barbalata, 1981) :

$$v + \overset{\circ}{D}\,\bar{\Delta}d = L \tag{19}$$

where :

$$\overset{\circ}{D} = \begin{bmatrix} \frac{\partial(F_d)}{\partial X_R} & \frac{\partial(F_d)}{\partial Y_R} & \frac{\partial(F_d)}{\partial Z_R} & \frac{\partial(F_d)}{\partial X_L} & \frac{\partial(F_d)}{\partial Y_L} & \frac{\partial(F_d)}{\partial Z_L} \end{bmatrix}^{\circ}$$

$$\bar{\Delta}_d = [\delta X_R \;\; \delta Y_R \;\; \delta Z_R \;\; \delta X_L \;\; \delta Y_L \;\; \delta Z_L]; \qquad L = -(F_d)^{\circ} \tag{20}$$

$$(F_d)^{\circ} = d^{\circ} - \sqrt{(X_R-X_L)^2 + (Y_R-Y_L)^2 + (Z_R-Z_L)^2} = d^{\circ} - d^{\circ\circ}$$

and $d^{\circ\circ}$ is computed using approximate coordinates X°, Y°, Z°.
If $\rho = d/d_m$ represents the scale factor for a distance "1" and $\rho_m = \sum_1^p \rho_i / p$ the average of scale factor, then the approximate coordinates X , Y , Z in (20) may be evaluated multiplying approximate model coordinates x , y , z by the scale factor ρ_m, where :

$$d_m^{\circ} = \sqrt{(x_R - x_L)^2 + (y_R - y_L)^2 + (z_R - z_L)^2} \tag{21}$$

The complete mathemetical model is obtained by combining equations (17) and (19). Using matrix notation, the model may be written as follows :

$$\begin{bmatrix} v \\ \overset{\circ}{v} \\ \bar{v} \end{bmatrix} + \begin{bmatrix} B & \bar{B} & 0 \\ \overset{\circ}{B} & 0 & 0 \\ 0 & 0 & \overset{\circ}{D} \end{bmatrix} \begin{bmatrix} \Delta \\ \bar{\Delta} \\ \bar{\Delta}_d \end{bmatrix} = \begin{bmatrix} E \\ G \\ L \end{bmatrix} \tag{22}$$

The normal equations for a least squares solution are then given by the following expression :

$$\begin{bmatrix} N + \overset{\circ}{N} & \hat{N} \\ \hat{N}^T & \bar{N} + \bar{N}d \end{bmatrix} \begin{bmatrix} \Delta \\ \bar{\Delta} \end{bmatrix} \begin{bmatrix} C + \overset{\circ}{C} \\ \bar{C} + \bar{C}d \end{bmatrix} \tag{23}$$

After some manipulations, the Δ matrix is first solved from a set of reduced normal equations and then matrices inside the matrix are solved for one at a time. The solutions of the normal equations are thus provided by :

$$\begin{aligned} \Delta &= [(N+\overset{\circ}{N}) - \hat{N}(\bar{N}+\bar{N}d)\,\hat{N}^T]^{-1}[(C+C) - N(N+Nd)\,(C+Cd)] \\ \bar{\Delta} &= [\bar{N}+\bar{N}d]^{-1}\,(\bar{C}+\bar{C}d) - [\bar{N}+\bar{N}d]^{-1}\,\hat{N}^T\Delta \end{aligned} \tag{24}$$

Once the vector Δ thus computed in (24), each vector of the X , Y , Z parameters corresponding to the distance "d" can in turn be computed from:

$$\bar{\Delta}_d = [(\bar{N}+\bar{N}d)]^{-1}\,(\bar{C}+\bar{C}d) - [\bar{N}+\bar{N}d]^{-1}\,\hat{N}^T\Delta; \qquad 1 = 1,2, \ldots, p \tag{25}$$

An iterative procedure is used; the iterations are stopped when the corrections in the Δ and $\bar{\Delta}$ matrices become negligibly small. After the solution has been checked to be convergent one, the vector of residuals may be obtained from :

$$v = E ; \qquad \overset{o}{v} = G ; \qquad \bar{v} = L ; \qquad (26)$$

in which E , G , L denote the final discrepancy vectors of the iterative process.
The estimation of the reference variance would be :

$$\sigma_o^2 = \frac{v^T W v + \overset{o}{v}{}^T \overset{o}{W} \overset{o}{v} + \bar{v}^T \bar{W} \bar{v}}{(2mn+2m+p) - (5+11m+6p)} \qquad (27)$$

A computer program (PROJECT) was developed by the author and its formulation is based on the principle of the observation equations as described in the above paragraph.

TEST AND PRACTICAL APPLICATION

The PROJECT program has been tested with fictious data and under operational conditions. The primary purpose of this series of tests was to evaluate the computational accuracy of the mathematical models. Hence, unperturbed image coordinates were used as input data in all tests and the coordinates were assigned a standard deviation of + 5 micrometers.
All of the coordinate control points were weighed with a standard deviation of + 10 cm in X, Y coordinates and + 5 cm in Z.
Unperturbed data were also used for distance measurements but they were assumed to have first-order accuracy. Because the exact object coordinates of all "n" object points were known, the accuracy of the PROJECT solution can be evaluated directly by comparing the computed point coordinates with the corresponding known values.
Various configurations of controls were used for the photography which was generated at various scales. The results show that the PROJECT solution has computational accuracy and sensitivity relative to the accuracy of measured data.
The PROJECT program was applied under operational conditions. The images considered for these tests were acquired using a UMK Zeiss flight-variant camera, f=99.47 mm, mounted on an external photomount of a Cessna 172 aircraft. Three strips B & W at 1/1400 scale and two strips in colour at 1/1800 scale were acquired with a forward overlap of 80 %. The Ground control points were targeted before the flight and their coordinates were determined from terrestrial observations by spatial trilateration. Image coordinates were measured in the Wild Aviolyt BC2 analytical plotter at the photogrammetric laboratory of the Universite de Moncton. Three sets of measurements were taken in monocomparator mode on each frame and subsequently processed to account for the effects of film deformation and lens distortion. Calibration data were obtained from Zeiss-Jena Calibration report.
Models were formed analitically using Wild's program softwares TMO and ATI Different models were formed using 80 %, 60 % and 40 % forward overlap, giving a base/heigh ratio B/H of 0.33, 0.66 and 1, respectively. RMS residual paralaxes for relative orientation varies between 2.2 and 4.7 micro meters.
Aerotriangulation methods used in the tests were :

- Independent model aerotriangulation, program PAT-M-43 (Ackermann et al., 1973). Model coordinates from ATI program were used as input data.

- Bundle adjustment, program BIC (Barbalata, 1981). A set of observed photocoordinates, corrected for film deformation and lens distortion, was used as input.
- Projective transformation, program PROJECT (Barbalata, 1981). A set of observed photocoordinates without any correction, was used as input.

DISCUSSION OF RESULTS

Adjustment of the entire block (B & W) of 36 models was carried out using different control configurations (Table 1).

Table 1. Root Mean Square Error at check points (mm)

Control version i	Number control points	Number check points	Bundle Adjust. X,Y	Bundle Adjust. Z	Indep.Model X,Y	Indep.Model Z	Project.Transf. X,Y	Project.Transf. Z
2	24	68	8.4	15.1	9.7	16.3	10.2	17.1
3	16	70	9.2	18.3	10.5	20.0	11.3	20.7
4	12	82	10.9	22.7	12.4	23.0	13.0	23.9
6	8	95	12.3	25.8	14.6	29.6	15.2	31.4
12	4	95	14.7	30.3	17.9	38.2	18.7	39.2

i = bridging distance (base length).

Comparison between the results of different control versions shows that the Projective Transformation Method combined with distance observations produces comparable accuracies with Independent Models Method. It is evident that larger degradation in vertical accuracies depends on the distribution of control points.

CONCLUSIONS

Because of the concept of simultaneous block adjustment, all photos are directly interrelated, which subsequently increase the band width of the reduced normal equation matrix, especially in Projective Transformation approach. The computing time for solving this system, therefore, could increase beyond acceptable limits in the case of large blocks.
Therefore, it is recommended to prepare a set of image coordinates corrected for lens distortion, atmospheric refraction and earth curvature and then to adjust the corresponding block.
But the obvious advantage of this method is the possibility to use non-metric cameras, reconnaissance cameras, for control extension. No internal camera or external exposure station parameters are needed as input data. The use of straight-line distances as control data enhances the applicability of the method.
Moreover, these tests have shown the outstanding qualities of the Zeiss - UMK system used to take the photos.

REFFERENCES

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