Digital modelling by using the integrated geodesy approach

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1. The integrated geodesy approach

This paragraph is quoted with minor changes from Benciolini et. al (1986). "Since about twenty years it has been realised by geodesists that a theorical scheme allowing for the contemporarary determination of the geometric quantities (e.g. station coordinates) and of the physical quantities (e.g. the anomalous potential) related to geodetic observations, was needed". The same problems accours in the digital modelling of elevations or displacements or other spatial variables (see fig. 1.1), when:

- the surface is represented by a trend too;
- break lines are located inside the area, therefore the correspondent values should be suitably constrained;
- different subareas are processed together;
- the observations are functionally dependent on other non-stochastic parameters.

"The traditional approach to the problem was to write geometrical observation equations where the anomalous field entered only at level of correcting factors computed by some questionable method.

The previous or subsequent least squares adjustment for geometrical parameters was performed on the assumption of uncorrelated observations or residuals. This, however, is not completely correct as the presence of the anomalous field (or of its residual part) creates a statistical correlation between the observed quantities. If that can effect the estimated values of the coordinates, even stronger is known to be its effect on the estimated covariance of the coordinates themselves.

The introduction of the collocation concepts taught how to treat this statistical component as a signal. Whence a complete theory integrating into a unique process the estimate of the geometric and field quantities was proposed by Eeg and Krarup (1975); this was called the integrated geodesy approach.

The theory of the integrated geodesy is schematically based on four points:

1) to write the observation equations and linearize them with respect to all the unknowns, coordinates and field functions;
2) to interpret the anomalous field functions as stochastic processes with some average invariance property with respect to a suitable group of coordinate transformations;
3) to exploit the claimed invariance to estimate covariance and cross-covariance functions of the processes, as well as of all their functionals, via covariance propagation;
4) to apply the second order theory (minimum variance) to estimate unknown parameters as well as the anomalous fields.

On theoretical ground this approach is a global one, aiming at the optimal estimate combining all the available information.

On the other hand, on a pratical computational ground, the number of the estimated parameters connot be very large, since its determination is from a numerical point of view the same as the solution of a least squares problem with a completely filled in covariance matrix." A first improvement can be obtained by using finite covariance functions; however

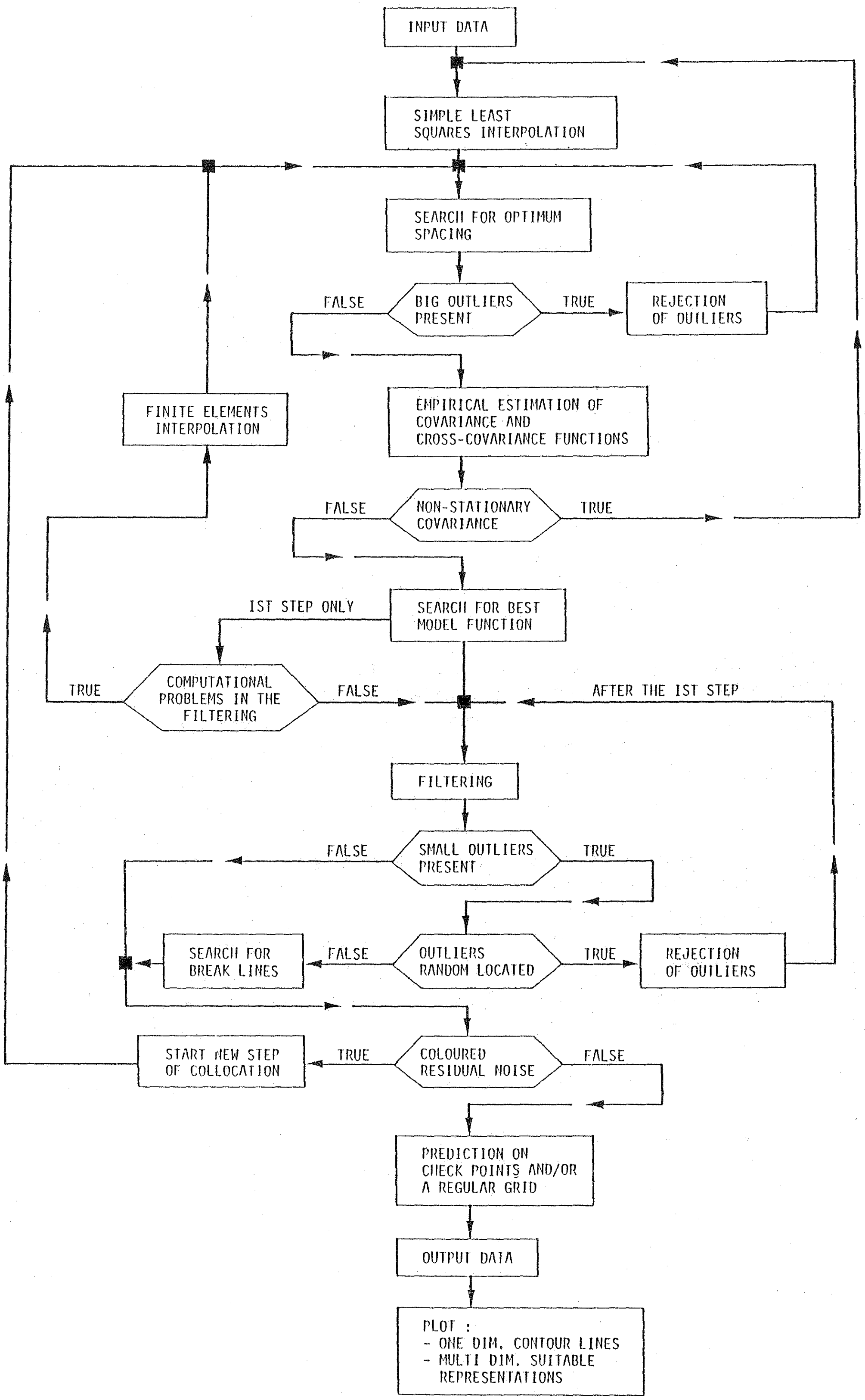

Fig. 1.1 - Flow chart of the system of programs MODEL

the classical solution causes often some computing troubles. A new solution was found by the ITM team and it is first time here discussed and tested; the authors inform that a similar solution was proposed recently by the geodesists of the Universidad Complutense of Madrid (Spain).

2. The least squares collocation method with additional parameters

The observables α are linked to the signal s and to the additional parameters x by a functional model:

$$E(\alpha) = Ax + Bs \tag{2.1}$$

more over a stochastic model connects the observables among the signal, via covariance propagation:

$$D(\alpha) = BC_{ss} B' + \sigma_n^2 P^{-1} \tag{2.2}$$

being C_{ss} the covariance matrix of the signal, P the diagonal matrix of the weights of the observables and σ_n^2 the variance of the noise.
Since the observations α_o are linked to their expected values $\hat{\alpha}$ by the residual noises $\hat{n}$:

$$\hat{\alpha} = \alpha_o - \hat{n} \tag{2.3}$$

the expression connecting the observations α_o to the estimations of the signal $\hat{s}$ and of the additional parameters $\hat{x}$ becames:

$$\alpha_o = A\hat{x} + B\hat{s} + \hat{n} \tag{2.4}$$

This is the classical formulation proposed by H. Moritz (1959). Unfortunately this solution gives some computational troubles. Nevertheless an advantageous trick allows for solving this troubles.
The trick consists in considering the additional parameters as a component of the signal:

$$s = \begin{bmatrix} s \\ x \end{bmatrix} \tag{2.5}$$

Therefore the functional model is redefined according to the new position:

$$B = [\ B \quad A\ ] \tag{2.6}$$

and the stochastic model is consequently modified in the following way:

$$C_{ss} = \begin{bmatrix} C_{ss} & 0 \\ 0 & H \end{bmatrix} \tag{2.7}$$

where H is a diagonal matrix with the principal elements suitably large.
Thus by using the least squares principle:

$$1/2\ [\ \hat{s}' \quad \hat{n}'] \begin{bmatrix} C_{ss}^{-1} & 0 \\ 0 & P/\sigma_n^2 \end{bmatrix} \begin{bmatrix} \hat{s} \\ \hat{n} \end{bmatrix} + \lambda'(B\hat{s} + \hat{n} - \alpha_o) = \min \tag{2.8}$$

being λ a vector of lagrangian multipliers, the estimation of the filtered signal containing the additional parameters too, is achieved:

$$\hat{s} = (B'PB)^{-1} B'P\alpha_o - \sigma_n^2 (B'PBC_{ss} B'PB + \sigma_n^2 B'PB)^{-1} B'P\alpha_o \qquad (2.9)$$

Note that some theorems of linear algebra must be used to obtain this expression.
The expected values of the observations and the residual noises follow according to (2.3) and (2.4) respectively.
In the same way the estimation of the predicted signal is achieved:

$$\hat{s}_p = C_{s_p s} B'PB (B'PBC_{ss} B'PB + \sigma_n^2 B'PB)^{-1} B'P\alpha_o \qquad (2.10)$$

Note that the predicted signal doesn't contain additional parameters, being their cross-covariance matrix identically equal to zero.
All the estimations are unbiased and of minimum variance among the linear estimators.
The covariance propagation furnishes the covariance matrices of the estimation error of the filtered signal:

$$C_{ee} = \sigma_n^2 (B'PB)^{-1} - \sigma_n^4 (B'PBC_{ss} B'PB + \sigma_n^2 B'PB)^{-1} \qquad (2.11)$$

of the estimation error of the predicted signal:

$$C_{e_p e_p} = C_{s_p s_p} - C_{s_p s} B'PB (B'PBC_{ss} B'PB + \sigma_n^2 B'PB)^{-1} B'PBC_{ss_p} \qquad (2.12)$$

of the expected values, considering the stochastic properties of the signal:

$$C_{\bar{\alpha}\bar{\alpha}} = BC_{ee} B' \qquad (2.13)$$

and of the residual noises:

$$C_{\bar{n}\bar{n}} = \sigma_n^2 P^{-1} - C_{\bar{\alpha}\bar{\alpha}} \qquad (2.14)$$

These matrices are important for the analysis of the results.
The variance of the noise can be a posteriori estimated and the resuld can be used for a global judgment:

$$\hat{\sigma}_n^2 = \bar{n}'P\bar{n} \,/\, k \qquad (2.15)$$

where:

$$k = m - n + Tr[\sigma_n^2 P^{1/2} B (B'PBC_{ss} B'PB + \sigma_n^2 B'PB)^{-1} B'P^{1/2}]$$

being:

m the number of the observations;
n the dimension of the signal, including the additional parameters too.

In conclusion of this paragraph the trick can be justified from the numerical point of view.
Indeed by using finite covariance functions, the covariance matrix of the signal becames sparse; moreover the sparseness is conserved in the matrices of the expression (2.9), since these ones don't contain inverse matrices.
Therefore no expression presents computational difficulties, but for the (2.12) one that is often omitted for sake of brevity.

Note that a finite covariance function is obtained by the product of a covariance function and a positive definite spline function.

3. The DEM test example

A program INTGEO was implemented not only to test the new solution, but also as a flexible service program of general interest.
Therefore the program accepts as input files the design matrix , the known vector, the weights vector, moreover the covariance matrix of the signal and the variance of the noise. The solution is performed by using a preconditioned conjugate gradient algorithm; a special subroutine allows for computing the product of three sparse matrices, since finite covariance functions are used. As output files the program furnishes the solution and some information about its precision and accuracy.
A part of the Noiretable test area was chosen as a DEM test example. Its prevalent surface is a plane, moreover both a regular roughness and an irregular one modify the topography. The fig. 3.1 shows the 3D rapresentation of the DEM test example observations.
Its covariance function doesn't look as one of a stationary stochastic process (see fig. 3.2). Therefore a linear trend was removed; a new covariance function was estimated by using the obtained residuals (see fig. 3.3) and the filtering of the signal from the noise was computed.
Since the new covariance function looks as one of a stationary stochastic process, good results are achieved by the filtering. Indeed the shape of the residual noises is quite smooth, as shown in the fig. 3.4 by means of a contour line map.
The new procedure was started at the end of the first one performed in two separate steps. In this plase the previous estimations of the trend parameters and of the signal were added to the original system as new indipendent data of suitably low weights, in order to obtain a better numerical stability. Moreover, since both the filtered signal and the trend parameters must be considered as signal, the stochastic model should be consequently modified by adding suitably large principal elements to the covariance matrix of the signal according to the position (2.7).
Thus the new solution was computed by means of the program INTGEO; in such a way one obtained:
- the filtered signal, containing the additional parameters too, and the standard deviation of the estimation errors;
- the expected values of the observables and their standard deviations, considering the stochastic properties of the signal;
- the residual noises and their standard deviations;
- the a posteriori estimation of the standard deviation of the noise.

The fig 3.5 shows the contour line map of the residual noises. Note that the shape of the residual noises is quite smooth again; moreover the high simularity between the residual noises derived from the classical procedure in two separate steps and the residual noises obtained from the new unique solution is impressive. Indeed the values of the differences between the two samples of residual noises are always close to zero, as shown in the fig. 3.6 by means of a contour line map.
Finally the fig. 3.7 shows the 3D rapresentation of the DEM test example predicted signal, which obviously reproduces the observations but for the residuals noises.
Since good results were achieved, the first test of the new solution should be considered satisfactory; however the numerical sensitivity of the new solution from the arbitrary values of the weights of the previous estimations and of the fictitious variances of the additional parameters is remarkable.

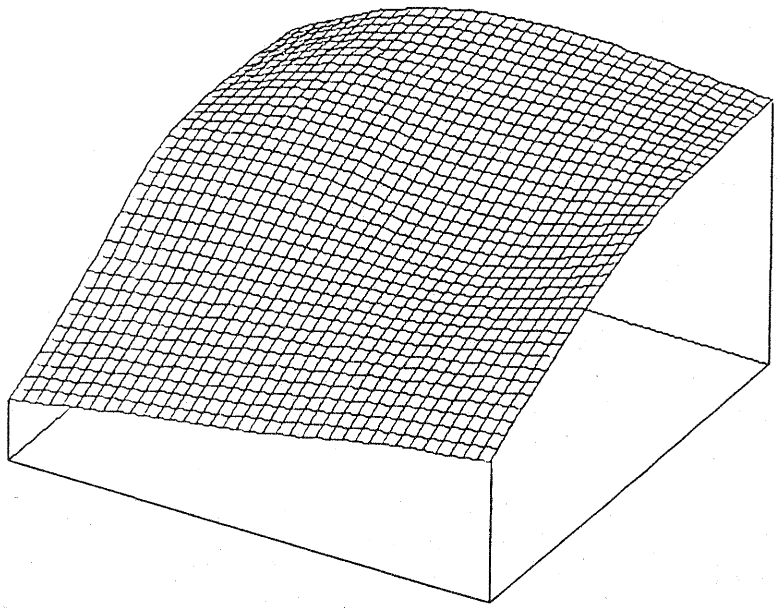

Fig 3.1 - 3D representation of the DEM test example observations

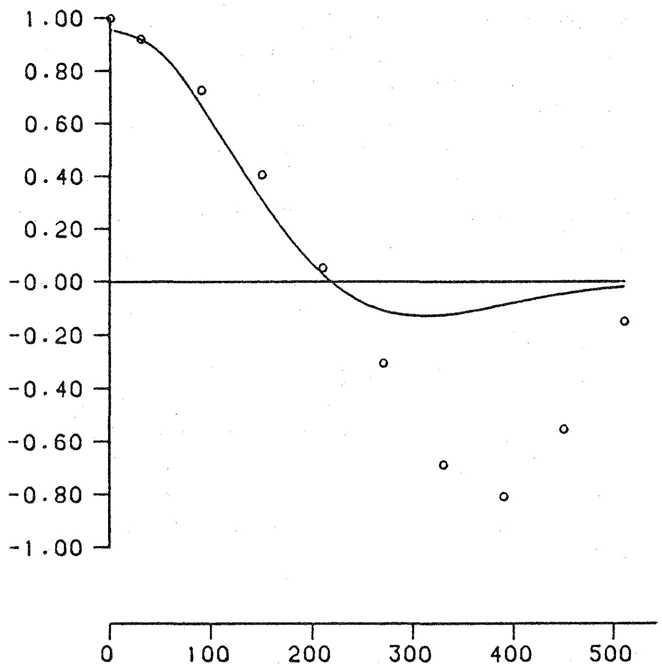

Fig. 3.2 - Covariance function of the observations

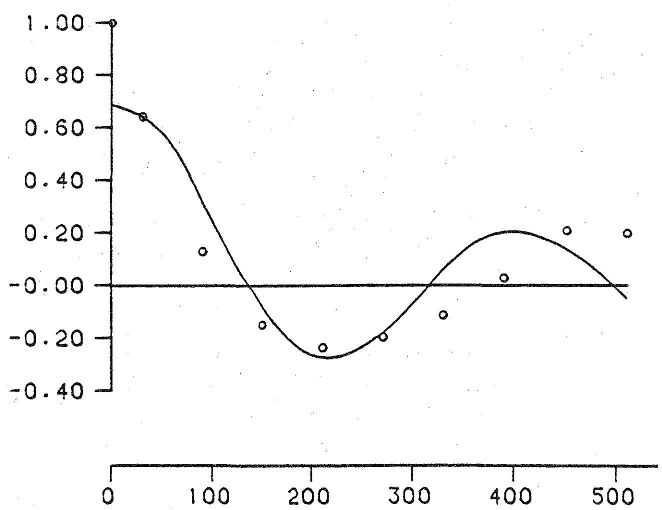

Fig. 3.3 - Covariance function of the residuals after the trend removal

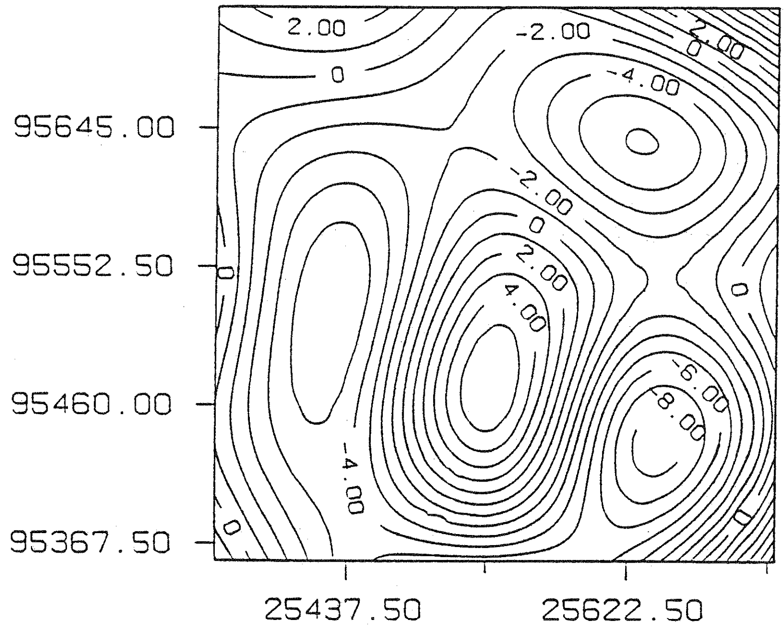

Fig. 3.4 - Contour line map of the residual noises (two steps procedure)

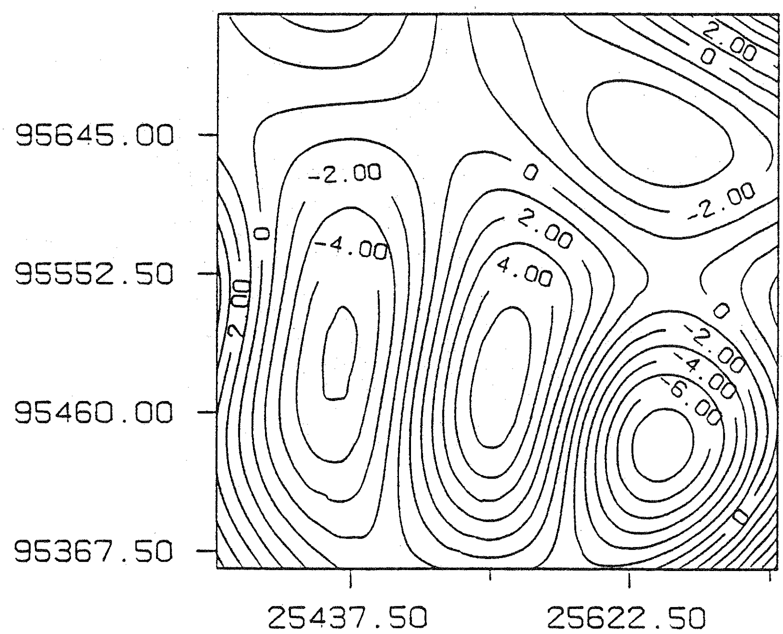

Fig. 3.5 - Contour line map of the residual noises (unique procedure)

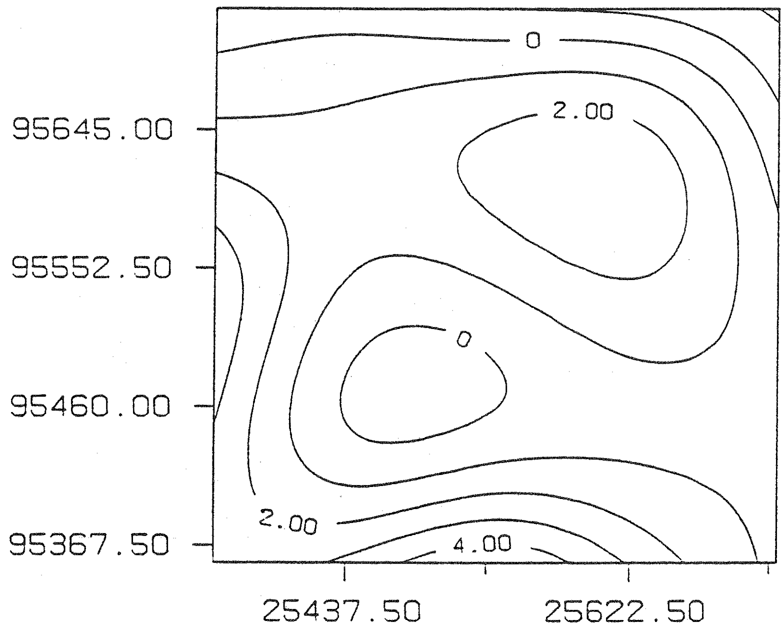

Fig. 3.6 - Contour line map of the differences between the two samples of residual noises

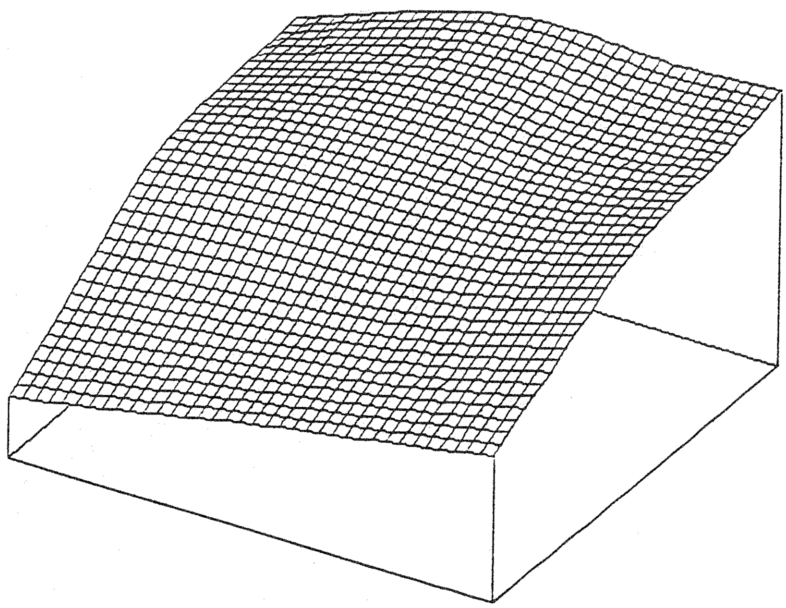

Fig. 3.7 - 3D representation of the DEM test example predicted signal

4. Some general considerations

The good results achieved by using the integrated geodesy approach applied to digital modelling allow for some general considerations.
Where is geodesy going?
What is the role of photogrammetry in the geodetic and cartographic sciences?
The aim of theoretical geodesy is to study the figure of the earth and its time variations both from a geometrical point of view and with respect to the gravity field. Surveying and modern satellite geodesy follow the geodetic developement also for historical reasons; moreover their news suggest new topics to theoretical geodesy.
On the contrary photogrammetry has been for a long time more separated from geodesy, but modern space photogrammetry and remote sensing now ask for closer conctats. Indeed the global reference frame must be derived from geodesy; however not only photogrammetry is calling geodesy, but also geodesy is calling photogrammetry.
In fact only the photogrammetric observations give a permanent documentation and furnish a dense point positioning, which can be used in several dynamic problems.
Last but not least the experience of the ITM team in many fields of the geodetic and cartographic sciences confirms that the advantages of using a unique common methodological background are very big indeed.

References

a) essential bibliography:

Ackermann F. (1973): Numerische Photogrammetrie. Herbert Wichmann Verlag, Karlsruhe.

Blachut T.J., Chrzanowski A. (1979): Urban Surveying and Mapping. Springer Verlag, New York Heidelberg Berlin.

Eeg J., Krarup T. (1975): Integrated Geodesy. In B. Brosowsky and E. Martensen (Eds.), Methoden und Verfahren der mathematischen Physik, Band 13, Wissenschaftsverlag Bibliographisches Institut, Mannheim Wien Zurich.

Heiskanen W.A., Moritz H. (1967): Physical Geodesy. W.H. Freeman and Company, San Francisco London.

Konecny G., Lehmann G. (1984): Photogrammetrie. Walter de Gruyter, Berlin New York.

Meissl P. (1982): Least Squares Adjustment: a Modern Approach. Mitteilungen der geodaetischen Institute der Technischen Universitaet Graz, Folge 43, Graz.

Mikhail E.M., with contributions by Ackermann F. (1976): Observation and Least Squares. IEP-A Dun Donnelley Publisher, New York.

Moritz H. (1959): The Geometry of Least Squares. Pubblication of the Finnish Geodetic Institute, n. 89, Helsinki.

Moritz H. (1980): Advanced Physical Geodesy. Herbert Wichmann Verlag, Karlsruhe, Abacus Press, Tunbridge, Wells Kent.

Moritz H., Suenkel H. (1978): Approximation Methods in Geodesy. Herbert Wichmann Verlag, Karlsruhe.

b) ITM contributions:

Ammannati F., Benciolini B., Mussio L., Sansò F. (1983): An Experiment of Collocation Applied to Digital Height Model Analysis. Proc. of the Int. Colloquium on Mathematical Aspects of Digital Elevation Models; Stockholm, 19-20 Apr. 1983, Dept. of Photogrammetry, Royal Institute of Technology, Stockholm, pp. 1.1-1.9.

Ammannati F., Betti B., Mussio L. (1984): Statistical Analysis of Relevant Terrain Features through Digital Elevation Models. Int. Archives of Photogrammetry and Remote Sensing, vol. XXV, part A3a/b, Rio de Janeiro,

17-29 June 1984, pp. 774-780.
Barzaghi R., Sansò F. (1983): Sulla stima empirica della funzione di covarianza. Boll. di Geodesia e Scienze Affini, n. 4, 1983.
Barzaghi R., Betti B., Sansò F. (1984): Integrated Geodesy: A Purely Local Approach. Atti del 3° Convegno del GNGTS-CNR, Rome, 14-16 Nov. 1984, pp. 197-227.
Barzaghi R., Sansò F. (1984): La collocazione in geodesia fisica. Boll. di Geodesia e Scienze Affini, n. 2, 1984.
Barzaghi R., Benciolini B., Betti B., Forlani G., Mussio L., Sansò F. (1988): An Experiment of Integrated Geodesy in the Large. Bull. Géodésique, (in printing).
Colombo L., Mussio L. (1985): Analisi della subsidenza nell'area milanese. Boll. della SIFET, n. 2, 1985.
Colombo L., Fangi G., Mussio L., Radicioni F. (1986): Further Developments on Digital Models in a Control Problem for the "Ancona '82" Landslide. Proc. of the Symp. from Analytical to Digital, ISPRS Comm. III, Rovaniemi, 19-22 Aug. 1986, Int. Archives of Photogrammetry and Remote Sensing, vol. XXVI, part 3.1, pp. 134-147.
Colombo L., Fangi G., Mussio L., Radicioni F. (1986): Kinematic Processing and Spatial Analysis of Leveling Control Data of the "Ancona '82" Landslide. In H. Pelzer and W. Niemeier (Eds.), Determination of Heights and Height Changes, Duemmler, Bonn, pp. 703-719.
Crippa B., Mussio L. (1987): The new ITM System of the Programs MODEL for Digital Modelling. Proc. of the Int. Colloquium on Progress in Terrain Modelling, Copenhagen, 20-22 May 1987, Technical University of Denmark, pp. 75-87.
Cunietti M., Fangi G., Mussio L., Radicioni F. (1984): Block Adjustment and Digital Model of Photogrammetry Data in a Control Problem for the "Ancona '82" Landslide. Int. Archives of Photogrammetry and Remote Sensing, vol. XXV, part A3a/b, Rio de Janeiro, 17-19 June 1984, pp. 792-802.
Cunietti M., Mussio L. (1984): High Precision Photogrammetry. In M. Unguendoli (Ed.), Proc. of the Meeting on the High Precision Geodetic Measurements, Bologna, 16-17 Oct. 1984, pp. 329-360.
Cunietti M., Bondi G., Fangi G., Moriondo A., Mussio L., Proietti F., Radicioni F., Vanossi A. (1987): La grande frana di Ancona del 13 dicembre 1982 - 5. Misure topografiche ed aereofotogrammetriche. Studi Geologici Camerti, Numero speciale, Dip. di Scienze della Terra - Università di Camerino, Camerino, pp. 41-82.
Forlani G., Mussio L. (1986): The Kinematic Adjustment of Levelling Networks. In M. Unguendoli (Ed.), Proc. of the Meeting on Modern Trends in Deformations Measurements, Bologna, 26-27 June 1986, pp. 265-290.
Forlani G., De Haan A. (1987): Digital Height Variation Modelling by Least Squares Collocation. Proc. of the Int. Colloquium on Progress in Terrain Modelling, Copenhagen, 20-22 May 1987, Technical University of Denmark, pp. 113-126.
Migliacci A., Mirabella Roberti G., Monti C., Mussio L. (1986): Monitoring of Historical Wall Painting. Proc. of JABSE Symp. Safety and Quality Assurance of Civil Engineering Structures, Tokio, 4-6 Sept. 1986, pp. 283-291.
Monti C., Mussio L. (1987): Relevé Photogrammétrique d'une Coupole et Analyse de sa Forme par des Méthodes Statistique Modernes. Sociètè Française de Photogrammétriec et de Télédétection, Bull. n. 105, 1987.
Mussio L., Radicioni F. (1986): High Precision Photogrammetric Measurements to Control the Shape of a Section of Musone River. In M. Unguendoli (Ed.), Proc. of the Meeting on the Modern Trends in Deformation Measurements, Bologna, 26-27 June 1986, pp. 143-163.