FAST IMPLEMENTATION FOR GENERATING EPIPOLAR LINE IMAGES WITH ONE-DIMENSIONAL RESAMPLING

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Abstract

Instead of the conventional two-dimensional resampling methods a simple one-dimensional resampling technique along the epipolar lines directly on digital images by use of the epipolar geometry is presented in this paper. In view of the simplicity and high-speed this technique could be effectively applied to near real-time or real-time image matching.

1. Introduction

The epipolar line is an ancient basic concept in photogrammetry. For the classical or analytical photogrammetry, however, it does not seem as important as for nowaday digital photogrammetry. The obvious advantage of exploiting epipolar geometry for image matching is, that conventional two-dimensional correlation can be completely substituted by one-dimensional correlation along the epipolar lines or one-dimensional search along the epipolar lines within two-dimensional windows /Panton 1978, Wong et al. 1986/. In other words, the epipolar constraint reduces the search required to find matching features from two dimensions to one, thus reducing computational costs significantly. For this reason much attention has been devoted to this concept not only by photogrammetrists /Helava 1972, 1976, Masry 1972/, but also by computer scientists /Barnard et al. 1982, Hannah 1984, Henriksen et al. 1986, Knudsen 1987, Robert et al. 1987 and Smith 1987/. It is somewhat interesting and surprising that the epipolar correlation concept has been highly recommended in the field of photogrammetry during the 70´s, in a time, when computer scientists did not show so much interest in it. Now it attracts more and more interest in the field of computer-science, while on the contrary it does not look encouraging in photogrammetry. One reason could be that resampling for succeeding epipolar lines correlation is generally done in two-dimensional manner, which is quite time-consuming.

In fact, in terms of epipolar the geometry resampling does not need to be performed in two-dimensional manner. In the following we first outline epipolar geometry, then describe a simple technique for one-dimensional resampling along the epipolar lines directly on the original digital images. Since 1982 this method has been applied at the Institute of Digital Photogrammetry (IDP) at Wuhan Technical University of Surveying and Mapping (WTUSM), the P. R. of China /Zhang 1983/. Some resampling examples and benchmarktests are also given in this paper.

2. Epipolar Geometry

From photogrammetry we know, that the epipolar plane of a point contains the basis B, and intersects two images (Fig.1). Each epipolar plane intersects the two image planes along epipolar lines (in the follwing called EpL for short). Obviously, corresponding image points must lie on the corresponding EpL, as all points of an epipolar plane are projected onto one EpL on the left image, and onto the corresponding EpL on the right image. To find a match for a point on an EpL on the left image, all the necessary is to search along the corresponding EpL on the right image. This is the so-called epipolar geometry or epipolar constraint. In practice for EpL correlation one-dimensional search along the EpL is performed within a two-dimensional window in order to exploit more information.

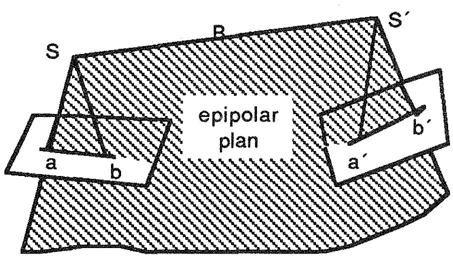

Fig.1

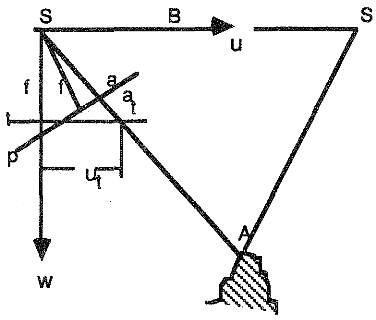

Fig.2

In Fig.2 SS´A is an epipolar plane, where p is the left image, and t is the normal image corresponding to the original image p. From the projective geometry of the photograph, the following relationship exists /Wang 1979/ :

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ -f \end{bmatrix} \tag{1}$$

where u, v, w are spatial coordinates of point (a) in image p; a, b, c are direction cosine, i.e. the functions of the orientation of image p in relation to basis B; x, y are the coordinates of point (a) in image p; f is the focal length of the camera.

The coordinates of point (a) on the normal image t can easily be derived from Fig.2:

$$u_t = u\left(\frac{-f}{w}\right) = -f\frac{a_1x + a_2y - a_3f}{c_1x + c_2y - c_3f} \qquad v_t = v\left(\frac{-f}{w}\right) = -f\frac{b_1x + b_2y - b_3f}{c_1x + c_2y - c_3f} \tag{2}$$

or in the original image by the inversion formula :

$$x = -f\frac{a_1u_t + b_1v_t - c_1f}{a_3u_t + b_3v_t - c_3f} \qquad y = -f\frac{a_2u_t + b_2v_t - c_2f}{a_3u_t + b_3v_t - c_3f} \tag{3}$$

Generally, on image p the EpLs are convergent lines, while on the normal image t all EpLs are mutually parallel (Fig.3). Corresponding EpLs on left and right normal images must have the same ordinate. It is easy, therefore, to determine corresponding EpLs on the normal images, and (3) can be substituted by a simpler expression (3´) :

$$x = \frac{m_1u_t + n_1}{l\,u_t + 1} \qquad y = \frac{m_2u_t + n_2}{l\,u_t + 1} \tag{3´}$$

where coefficients l, m, n can be derived from (3).

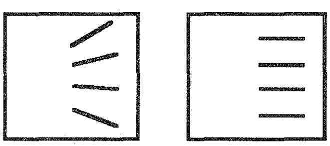

Fig.3

Assuming that the orientations of the camera at the station S and S´ are known, all the points on image p can be projected onto the normal image by formula (2), or vice versa by formulae (3) or (3´) (Fig.2). The latter is the conventional digital rectification. Here it should be emphasized, that these two projection methods rearrange EpLs on the normal image. This means, that it must be calculated point by point, and the graylevel of each pixel on EpL has to be interpolated with its four adjacent pixels. Thus it belongs to two-dimensional resampling. Clearly its computational expenses cost a lot, although only a simple linear interpolation is used.

Actually from epipolar geometry one can easily find a very simple thing : the EpL is a straight line !! By using this characteristic two-dimensional resampling can be simplified to one dimension. This means, that formulae (2), (3) or (3') do not need to be solved point by point any more, and two-dimensional interpolation can be substituted by the one-dimensional method explained as below.

3. One-Dimensional Resampling

The approach to one-dimensional resampling can be divided into two steps : determination of the corresponding EpLs on the two images, and resampling along the EpL.

1) Determination of corresponding EpLs :

The well-known coplanarity condition for three lines (e.g. SS', Sa and Sb of Fig.1) can be written as follows /Wang 1986/:

$$\overline{SS'} \cdot (\overline{Sa} \times \overline{Sb}) = 0$$

or
$$\begin{vmatrix} B & 0 & 0 \\ u_a & v_a & w_a \\ u_b & v_b & w_b \end{vmatrix} = B \begin{vmatrix} v_a & w_a \\ v_b & w_b \end{vmatrix} = 0 \tag{4}$$

From equation (1) and (4) one gets :

$$\begin{vmatrix} b_1 x_a + b_2 y_a - b_3 f & c_1 x_a + c_2 y_a - c_3 f \\ b_1 x_b + b_2 y_b - b_3 f & c_1 x_b + c_2 y_b - c_3 f \end{vmatrix} = 0 \tag{5}$$

Given x_a, y_a, x_b one can solve y_b from (5), thus obtaining the location and orientation of the EpL on the original image.

The direction of the epipolar line can also be expressed by the angle κ between the axis x and the EpL (Fig.4) :

$$\operatorname{tg} \kappa = \frac{y_b - y_a}{x_b - x_a} \tag{6}$$

Similarly, from the coplanarity of lines (SS', Sa, Sa') and lines (SS', Sa', Sb') (Fig.1) one can also determine the location and orientation of the corresponding EpL on the right image (Fig.4).

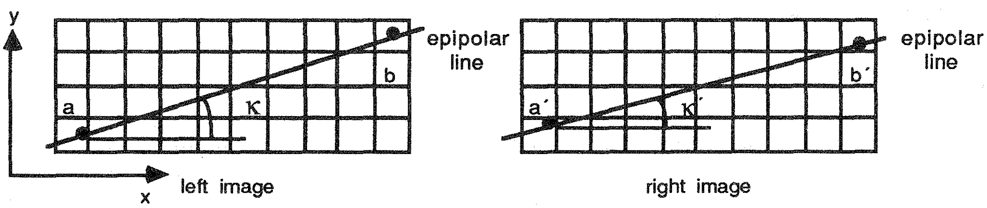

Fig.4

Note: 1) For digitized images, the image coordinate system (x,y) is different from the scanning system. The latter can be transformed into the former in advance, with a few given points such as the fiducials, and a certain transform, e.g. the affine transform. 2) With the absolute orientation a similar formula can be derived.

Summarizing : with any point (e. g. point (a) of Fig.1 or Fig.4) in the original image, both another point (b) on the same EpL of the same image, and the corresponding points (a') (b') on the corresponding EpL of the other image can be found. Then rearrange the graylevels along the EpLs directly on the original image.

2) Resampling along the EpL :

From Fig.5 or formula (5) one can see, that with the method mentioned above the length in x direction is kept, before and after resampling. It follows, that graylevel interpolation on the EpLs is only related to y-direction rather than to both directions. Fig.5 shows two resampling methods for this case: linear interpolation using the closest two pixels, and a neighborhood method with only one closest pixel.

Although it still requires calculation point by point for the linear interpolation, the calculation for interpolation is worked out related to two adjacent pixels only, which can be directly found according to the orientation of the EpL, instead of interpolating with four pixels in the two-dimensional resampling case. The result is that computational costs are much less than in the two-dimensional case. In other words, computational time can be decreased in square.

Another method for resampling is the neighborhood method, in which one can directly take the grayvalue of the pixel, on which the point is projected. Compared with the above linear interpolation method, the neighborhood method is simpler and more economic, as it only transfers data in batches, instead of any computation, e. g. only four groups of pixels have to be read from the original image in Fig.5 (upper-right), compared to 20 pixels and the involved calculation for interpolation by using the method of linear interpolation.

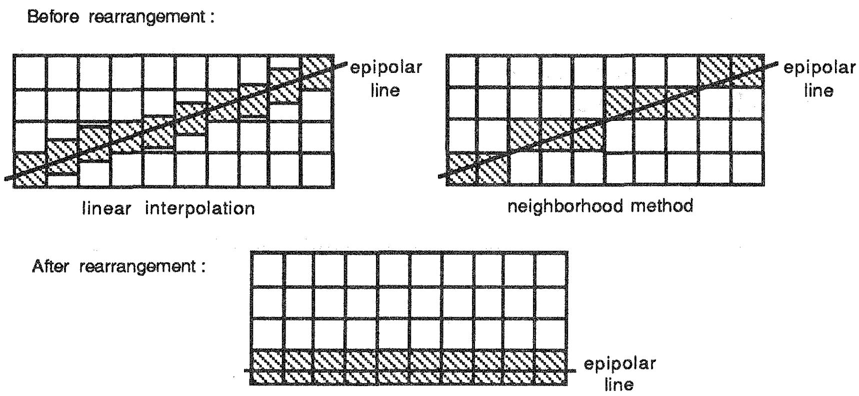

Fig.5 Two kinds of one-dimensional resampling methods

After all relevant EpLs on left and right images have been rearranged sequentially, we obtain the image pair without y parallax, which can be directly used for succeeding EpL correlation /Zhang 1988/.

4. Examples

Fig.6 and Fig.7 give the part of results achieved by using the two resampling methods with the test images distributed by Working Group III/4 of ISPRS /Förstner et al. 1986/. The images obtained with these two methods almost look identical due to the small image size. Fig.8 shows the part of ISP photo 17 in Fig.6 and the part of ISP photo 21 in Fig.7, which are with 8 times larger pixel size. In Tab.1 the two methods are compared. Both methods are programmed in Language C.

It should be pointed out, that for the test data of ISPRS one must do somewhat differently. For the resampling mentioned above, the test images of ISPRS are related to three coordinate systems :

s-System : Sensor-system with rows and columns in pixels;
n-System : Normal-Image System in mm;
w-System : Window of Normal Image in pixels, y=const. are epipolar lines.

In the tested examples different transformation matrices were predefined by the organizers of the test, e. g. Tsn for transfering from n-System to s-System, Tnw for transfering from w-System to n-System, and transformations between the coordinate systems (e.g. a, b of formula (7)). Expressed in homogeneous coordinates one gets :

$$\begin{bmatrix} u \\ v \\ t \end{bmatrix}^a = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}_b^a \begin{bmatrix} u \\ v \\ t \end{bmatrix}^b \qquad \text{with} \quad x = \frac{u}{t} \qquad y = \frac{v}{t} \tag{7}$$

a, b - different coordinate systems.

or explicitely

$$x^a = \frac{t_{11} \cdot x^b + t_{12} \cdot y^b + t_{13}}{t_{31} \cdot x^b + t_{32} \cdot y^b + t_{33}} \qquad y^a = \frac{t_{21} \cdot x^b + t_{22} \cdot y^b + t_{23}}{t_{31} \cdot x^b + t_{32} \cdot y^b + t_{33}} \tag{8}$$

In our case the relationship (Tsw = Tsn • Tnw) should be used to calculate the corresponding coordinates of the four corner points of the EpL image projected onto the s-System, then one can obtain an EpL image with the above mentioned method. For example, for photo 17 we get

$$T_{sn} = \begin{bmatrix} -54.9288640000 & 0.2908541600 & 1229.0422000000 \\ -0.2069664300 & -52.5252230000 & -810.7252200000 \\ -0.0000941288 & -0.0000050574 & 1.0000000000 \end{bmatrix} \qquad T_{nw} = \begin{bmatrix} 0.018 & 0 & 17.653 \\ 0 & 0.019 & -20.146 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{sw} = T_{sn} \cdot T_{nw} = \begin{bmatrix} -0.9887 & 0.0055 & 253.523 \\ -0.0037 & -0.998 & 243.821 \\ 0 & 0 & 0.998 \end{bmatrix} \tag{9}$$

The EpL image gained with (9) is shown in Fig. 8.

Theoretically, the linear interpolation method should be more reasonable and accurate than the other. Tab.1 shows, that although the differences of graylevels gained by the two methods without filtering are quite big, they could be decreased greatly by filtering the original image. It is well known, that a certain filtering is usually done before correlation is performed, in order to eliminate the influence of high frequency noise. Furthermore, because the length in x-direction before and after the resampling is always kept, the difference is caused only by the displacement of y-direction (see step-shape of Fig.8), which maximum is 0.5 pixels (rms = 0.29 pixels). As a result, the neighborhood method should be accurate enough to the succeeding feature based image matching for better approximate values /Zhang 1988/.

From Tab.2 one can see, that the required CPU time with the neighborhood method is far less than with another, e.g. on a SUN 3/280 the averaging CPU time consumed was only 0.85 seconds for 2 images with 240x240 pixels for the neighborhood method, and 15.63 secondsfor the linear interpolation method.

5. Conclusions

This Paper presented two kinds of one-dimensional resampling approaches along the epipolar lines directly on the original images by using epipolar geometry. As the length in x-direction before and after resampling is always kept, 2-dimensional resampling is reduced to one-dimensional, so that the computation time can be save significantly. Especially the neighborhood method on the basis of prior filtering is both very simple, fast, and accurate enough for the succeeding feature based image matching. Therefore it could be effectively applied to near real-time or real-time image matching.

Further research effort is requested to clarify how the accuracy of epipolar line correlation is influenced by different resampling methods for EpLs.

6. Acknowledgement

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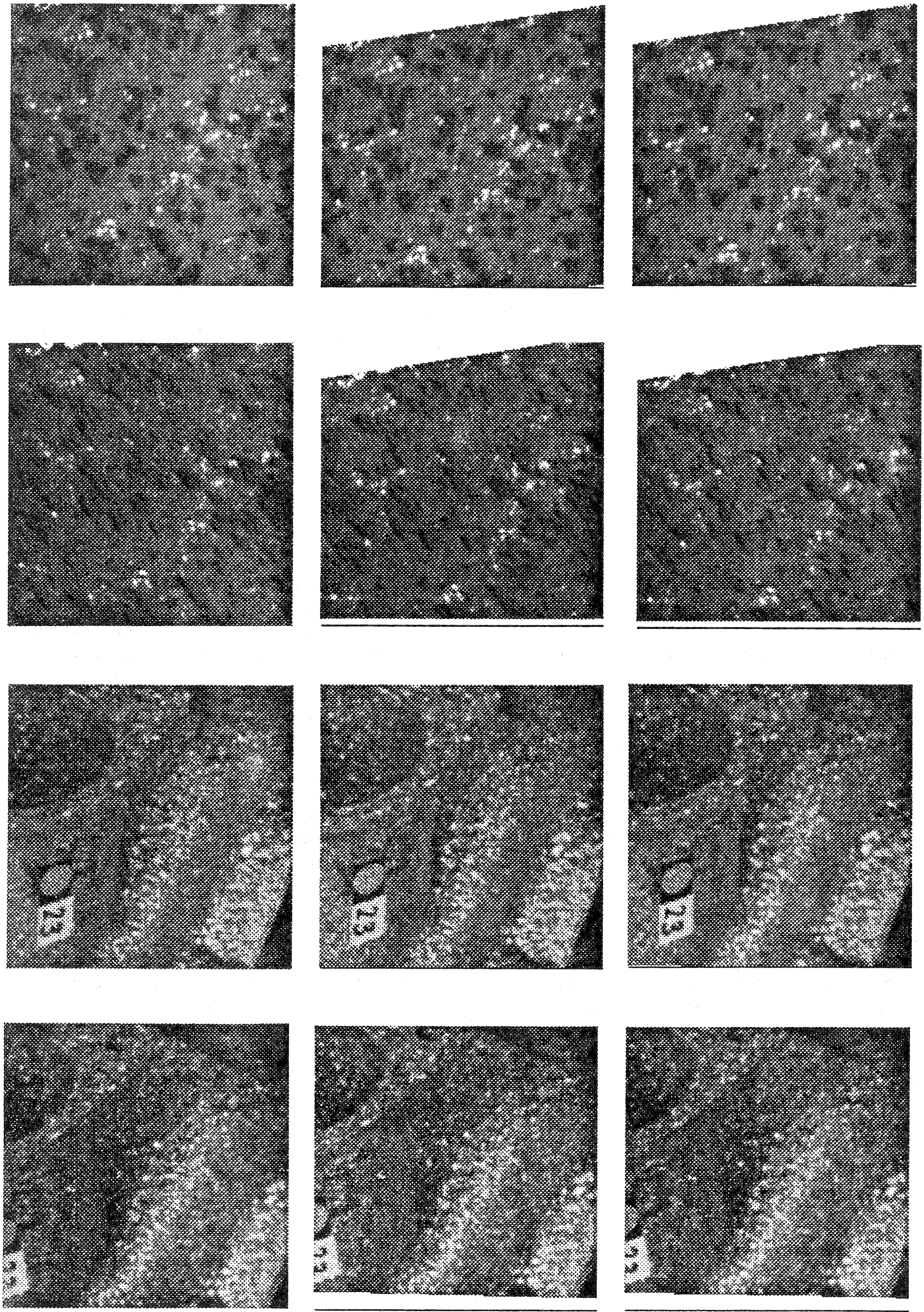

Fig. 6 From upper to below : ISP photo 15, 16, 17, 18 (model 8, model 9)
From left to right : 1) original photo
2) after resampling by the neighborhood method
3) after resampling by the linear interpolation method

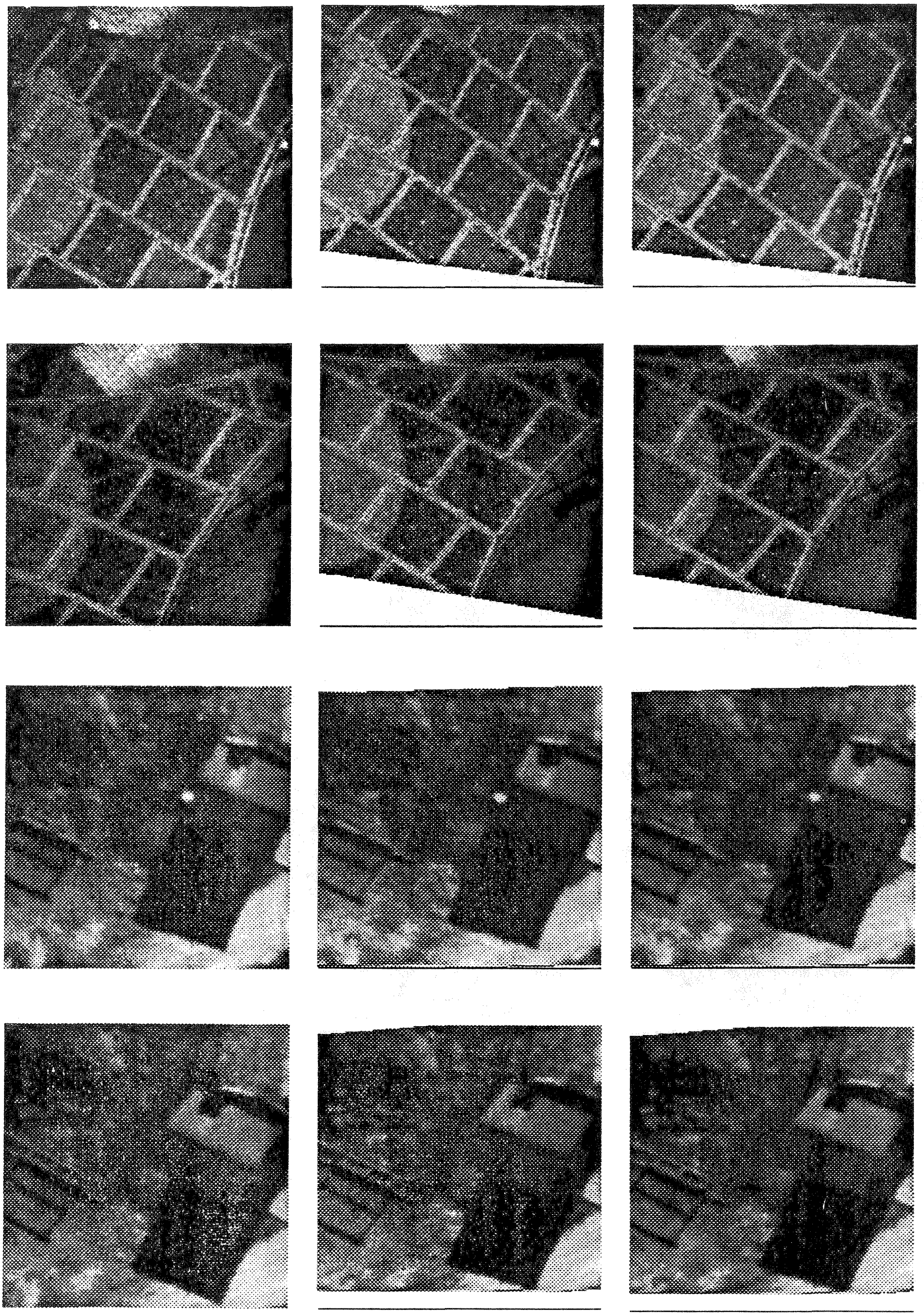

Fig. 7 From upper to below : ISP photo 21, 22, 23, 24 (model 11, model 12)
From left to right : 1) original photo
2) after resampling by the neighborhood method
3) after resampling by the linear interpolation method

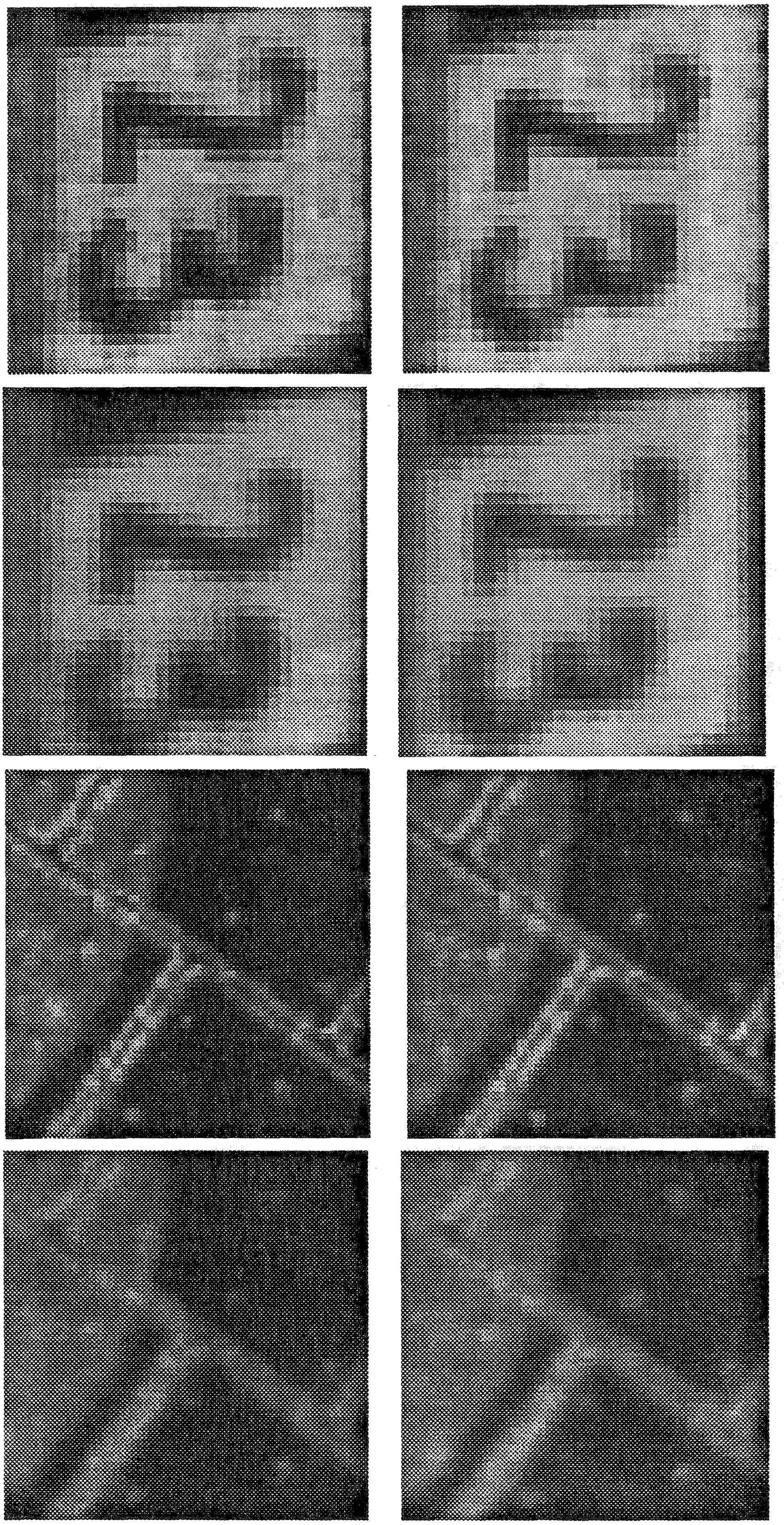

Fig.8 The parts of Fig.6 and Fig.7, which are enlarged 8 times.
left : by neighborhood
right : by linear interpolation

ISP photo 17
without filtering

ISP photo 17
with filtering

ISP photo 21
without filtering

ISP photo 21
with filtering

Tab1. Differences of graylevels between the two graylevel rearrangement methods

No. photo	Numbers of pixels	without filtering			with filtering (3x3 local average)		
		No. of maximum	maximum (graylevel)	rms (graylevel)	No. of maximum	maximum (graylevel)	rms (graylevel)
3	56640	8008	27	3.48	706	24	2.47
4	55920	9387	41	3.48	7970	21	2.23
5	49680	6518	84	5.90	6278	43	3.86
6	46080	25434	116	7.54	36187	41	3.66
9	56640	53445	51	7.14	52967	26	3.40
10	55920	55377	43	6.13	47840	24	2.90
11	56640	55519	44	4.28	26485	28	2.92
12	55920	52029	46	4.70	51786	29	3.26
15	48420	28819	91	4.01	28818	36	2.17
16	47520	40838	91	3.81	39878	34	1.98
17	56400	27972	49	5.27	11163	24	2.99
18	53280	22237	94	5.61	21997	37	3.15
21	49440	28553	118	4.53	28553	65	2.51
22	45840	38915	42	3.39	4168	17	1.72
23	55440	52731	36	2.52	1441	45	2.09
24	51360	11715	40	2.53	11715	23	1.82

Note: photo 1, 2, 7, 8, 13, 14, 19, 20 already are windows of "normal images" /Förstner et al. 1986/

Tab2. Benchmarktests for two graylevel rearrangement methods on SUN 3/280

No. photo	size of photo (pixels)	CPU time (seconds)	
		neighbourhood method	linear interpo. method
3-4	2x240x240	0.64	15.61
5-6	2x240x240	1.18	15.82
9-10	2x240x240	0.66	14.60
11-12	2x240x240	0.56	14.58
15-16	2x240x240	1.26	16.04
17-18	2x240x240	0.70	16.03
21-22	2x240x240	1.08	16.23
23-24	2x240x240	0.72	16.13
average		0.85	15.63

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