ON THE OPTIMAL SORTING IN COMBINED BUNDLE BLOCK ADJUSTMENT

Shan Jie
Department of Photogrammetry and Remote Sensing
Wuhan Technical University of Surveying and Mapping
Wuhan, P.R.China
Commission III

[ABSTRACT]

This paper shortly reviews the sparse matrix technique and its graph theory. With the introduction of optimal sorting, one criterion of optimal sorting in bundle block adjustment is abstracted , which is named as the criterion of object point priority. A new sorting strategy---nested dissection method is develpoed with the dissection of one graph, after the recursive partioning method and the Kruck's method are compared with each other. The new algorithm would be more effective with the use of multi-processor or the distributted computer.

1. INTRODUCTION

The sorting and solving of large scale normal system is one of the most important techniques in (combined) block adjustment. The recursive partioning method (RPM), that was developed by D.C.Brown about 30 years ago, has been widely and traditionally used in photogrammetry. On the other hand, a few photogrammetrists also examined the possibility of sparse matrix technique(SMT) to solve block adjustment system. Recently, Dr. Kruck evolved a new strategy of sorting and solving with graph theory and SMT.

All these methods have their own characteristics. The RPM has nice data structure and program structure. It has shown very high efficiency in practice. The SMT is complex in data and programme structure, and does not take less time expense than the RPM. So it is not very proper to the block adjustment. In Kruck's method, the sorting strategy is different from that in the RPM. With the aid of Jennings' profile storage technique (PST), Kruck's method get quite high computational efficiency. But it requires large main memory and is generally less efficient than the RPM. It accepts only the geodetic observations which do not connect object points with long distances, although it aims at combined adjustment. In combined adjustment with high quality, geodetic observations may often range long distances.

This paper reviews briefly the usual algorithms and graph theory in SMT. The characteristics among various optimal sorting strategies are shortly discussed and compared. According to the dissection of graph, the nested dissection method (NDM)---a new strategy of sorting is developed. This method can not only make full uses of main memory of existing computer but also remain the characters that RPM has. Therefore the newly developed method is more efficient than the RPM and Kruck's method. As for combined adjustment with geodetic observations, the pseudo-knots are introduced. This makes the NDM suitable to combined adjustment where geodetic observations range long distances.

2. REVIEW OF SPARSE MATRIX TECHNIQUE AND ITS GRAPH THEORY

Generally speaking, a matrix is called a sparse one, if its zero elements take about more than 80 percent of all elements. The normal system in large scale adjustment problem often belongs to sparse one.

In SMT there are basically two kinds of algorithms, pure sparse algorithm and envelope or band width algorithm.

The pure sparse algorithm sorts the unknowns according to the criteria of minimum storage requirement or operation. Only the non-zero elements are stored and the data structure is random. The resolution can be performed with high efficiency, because all zero elements are not stored and operated. But this algorithm has quite complicated data and program structure. It also needs a lot of storge cells to record the addresses of non-zero elements and computation of address finding.

The unknowns are ordered according to the criteria of minimum profile or band width in envelope algorithm. It can only avoid the storage and operation of zero elements outside the envelope. But the data and program structure are more simple than the pure sparse algorithm. And because it can be easily handeled with blocked matrix, it is preferable to the pure sparse algorithm in block adjustment.

Graph theory is an importment tool in SMT. The structure of a matrix and its graph are correspondent with each other. As shown in Fig.1, each bundle in photogrammetry is a tree whose root is at the camera station. The photogrammetric meaning of degree is the number of image points on one photo(the degree of camera station), or the number of photos on which an object point is imaged (the degree of object point). In Fig.1, the degree of the camera station is 4, and the degree of each object point is 1. The reduced graph of Fig.1 is shown in Fig.2.

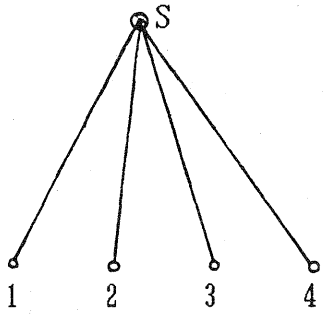

Fig.1 Photogrammetric bundle as a tree.

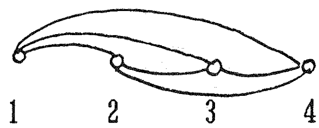

Fig.2 Reduced graph of Fig.1 after the reduction of knot S

3. OPTIMAL SORTING

As stated above, the structure of a normal system and its graph are corresponding with each other. Therefore the structure of the normal system could be improved with the aid of graph theory. This is the optimal sorting of graph.

Assume that graph G(V,E) has n knots and α is a mapping from V to $\{1,2,3,\cdots,n\}$, then the mapping α is called one sorting of V. As for one chosen specific objective function $f(\alpha)$, if one mapping $\bar{\alpha}$ makes it minimum, i.e.

$$f(\bar{\alpha})=\min_{\alpha} f(\alpha) \qquad (1)$$

, the mapping $\bar{\alpha}$ is an optimal sorting under this objective function.

It should be pointed out that an optimal sorting is refered to a chosen specific objective function. Different object functions would have different results of optimal sorting. Objective function can be chosen with respect to different requirement. If we choose

$$f(\alpha)=\max_{i} b_{\alpha(i)} \qquad (2)$$

, where $b_{\alpha(i)}$ is the band width of i-th column, then this will lead to the problem of minimum band width.

In engineering, commonly we have the following optimal sorting strategies

a) Minimum Degree Method;
b) Minimum Surplus Branch Method;
c) Minimum Defective Value Method;

d) Quotient Tree Method;
e) CM-RCM Method;
f) GPS Method.

Among above methods, a),b),c) and d) belong to the pure sparse algorithm, e) and f) are envelope algorithm. The pure sparse algorithm produces minimum operation or fill-in in the process of reduction. And the envelope algorithm makes the band width or profile minimum.

Theoretically speaking, the minimum problem of objective function could be solved with dynamic programming, but it is more complicated than the solving of normal system. Therefore all the optimal sorting strategies do not exactly make the objective function minimum. They are approximately optimal. In block adjustment, one generally optimal criterion of sorting should be found rather than some differences in detail among various algorithms be examined carefully and tediously. Otherwise the sorting and data struction would become quite complex without any practical efficiency in resolution. This is the reason why the exactly optimal sorting algorithms, such as minimum surplus branch method and minimum defective value method etc., are not suitable to block adjustment.

Then, what is the generally optimal criterion of sorting in bundle block adjustment ? With the optimal sorting algorithms aboved mentioned, an optimal sorting criterion in bundle block adjustment is abstracted, which is named as the criterion of object point priority:

> The unknowns of object points should be ordered before the unknown orientation parameters of photos in which these points image. That is because the degree of object point is almost always less than that of camera stations, even in a reduced graph.

This criterion would lead to a sorting with approximately minimum operation and fill-in or profile. Both the RPM and Kruck's method obey this criterion. The RPM is total object point priority, while Kruck's method is local object point priority. As for independent model block adjustment, this criterion may be used similarly.

Practical sorting tests demonstrate that the traditional RPM approximately obeys the minimum degree and minimumm surplus branch criteria, it is also a kind of quotient tree method. Although Kruck's method is developed to obtain minimum profile, it has somewhat the same sorting result as minimum defective value method and minimum surplus branch method which may produce minimum fill-in or operation.

In fact we can show that the RPM and Kruck's method have the same amount of fill-in and operation. The only difference between these two methods is profile. The reason is as follows. If the sorting obeys the criterion of object point priority, there is no fill-in among object points in the reduction procedure. Also because they have the same sequence of photos, the fill-in between object points and photos and the fill-in among photos have no difference within these two methods. Therefore they have the same amount of fill-in. The amount of operation is determined by the related parts between object points and photos and the related parts among photos, so they have the same amount of operation too.

With the above analysis, it is easy too see that the RPM and Kruck's method are both equally optimal theoretically. But the former has better data and program structure than the latter. Therefore the RPM has quite high computational efficiency associated with its storing and solving strategy. Kruck's method can only get high computational efficiency with the aid of profile storing technique.

4. NESTED DISSECTION METHOD

As demonstrated above, the RPM and Kruck's method are both approximately optimal sorting strategies theoretically. The question is whether there is even better optimal sorting strategy. On the other hand, the former optimal sorting strategies don't accept geodetic observations with long distances. Therefore the existing sorting methods are also not directly suitable to combined adjustment.

The principle of newly developed sorting strategy comes from computer science and it has been used in structure analysis with finite element method. The nested dissection method (NDM) tries to produce the normal system with good 'structure', i.e. as much main diagonal blocks as possible. Therefore the reduction of normal system could be performed independently.

In NDM, firstly we find a knot set, with the deletion of which the original graph will become two non-connected sub-graphs. This knot set is called cutset. After that each sub-graph is dissected to even smaller sub-graphs with cutsets. This procedure of nested dissection continues until no cutset exists. One importmant feature of the NDM is that the fill-in will not appear among sub-graphs but only between sub-graphs and cutsets.

The procedure of the NDM is demonstrated with the block in Fig.3, where only two pass points are chosen for simplicity. Obviously, this would not affect the structure of normal system. Fig.4 and 5 are the normal systems with the RPM and Kruck's method.

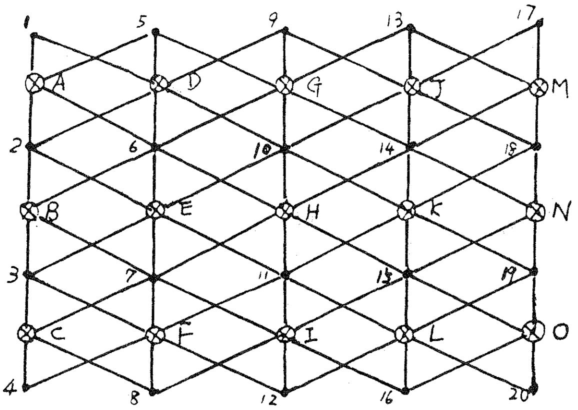

Fig.3 Bundle block with 3×5 photos (Block P)

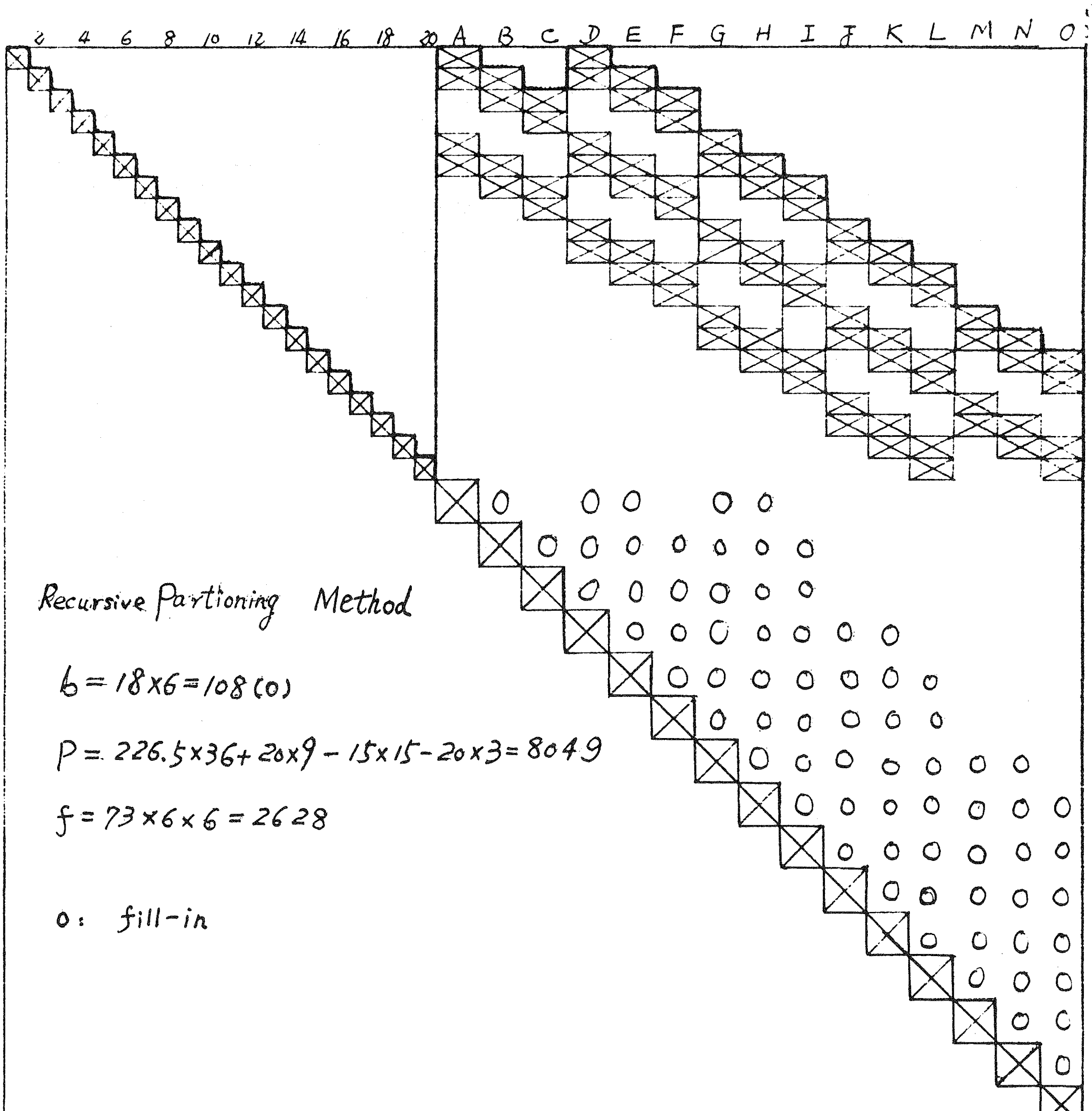

Fig 4. Normal system of Block P by the RPM

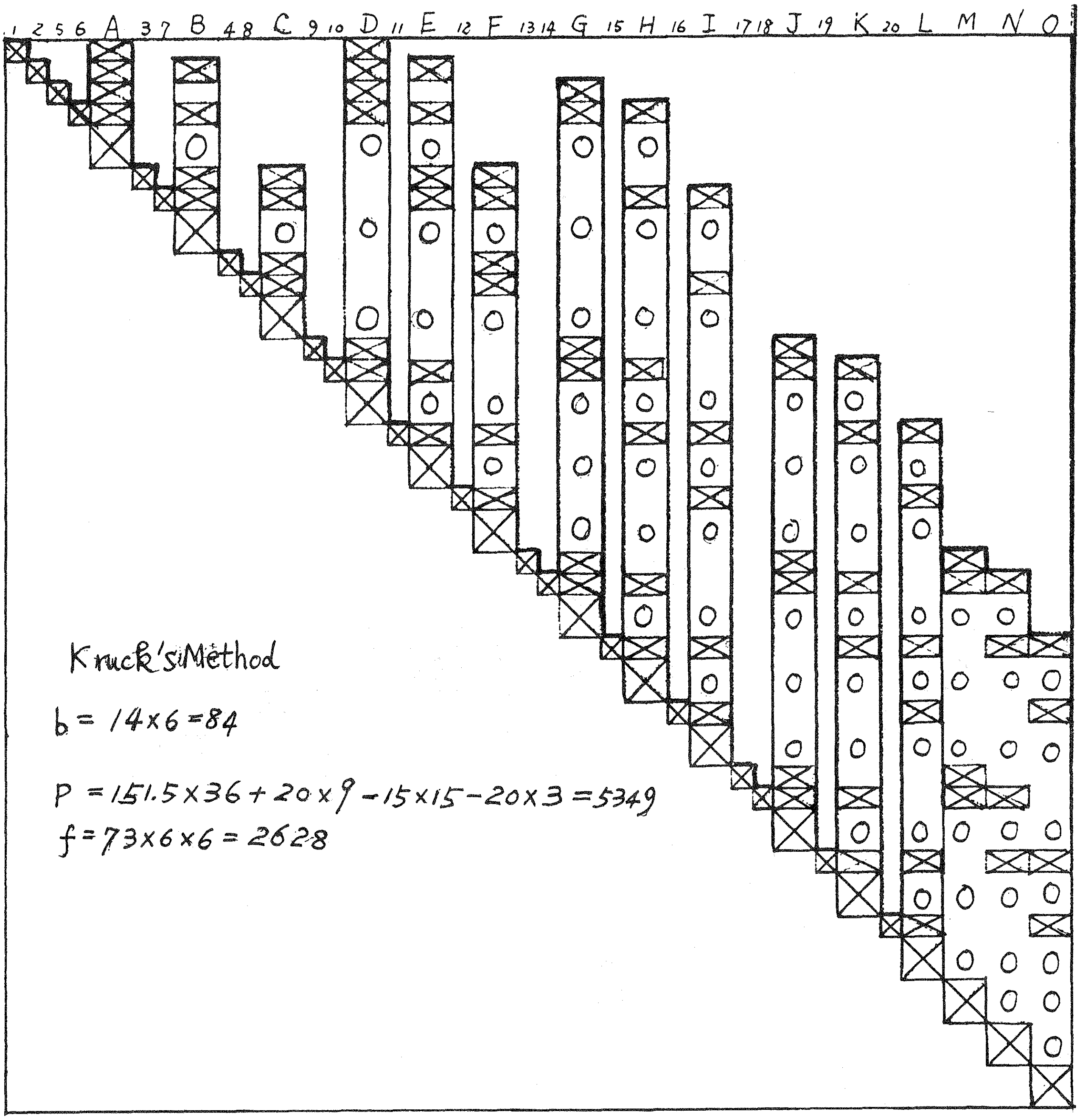

Fig 5. Normal system of Block P by Kruck's method

It should be noted that the pure NDM is not thoroughly proper to block adjustment, therefore it should be combined with the criterion of object point priority. The block is dissected to the following tree

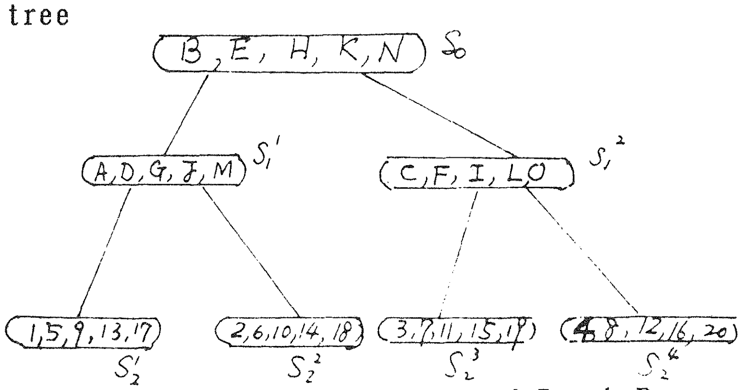

Fig. 6 Nested dissection of Block P

The unknowns are ordered as $S_2^1, S_2^2, S_2^3, S_2^4, S_1^1, S_1^2, S_0$. Because of the large profile of this sorting, the effection area in which fill-in may appear in combined adjustment enlarges also. To avoid this trouble, we slightly alter the sorting sequence in Fig.6. The normal system is shown in Fig.7. Obviously the first two strips can be reduced in parallel or independently. The solving strategy of the RPM can be used to handle each strip. Comparing with Fig.4 and 5, the NDM has less fill-in than the RPM and Kruck's method. With computer network or multi-processors, the NDM would be more efficient than common methods.

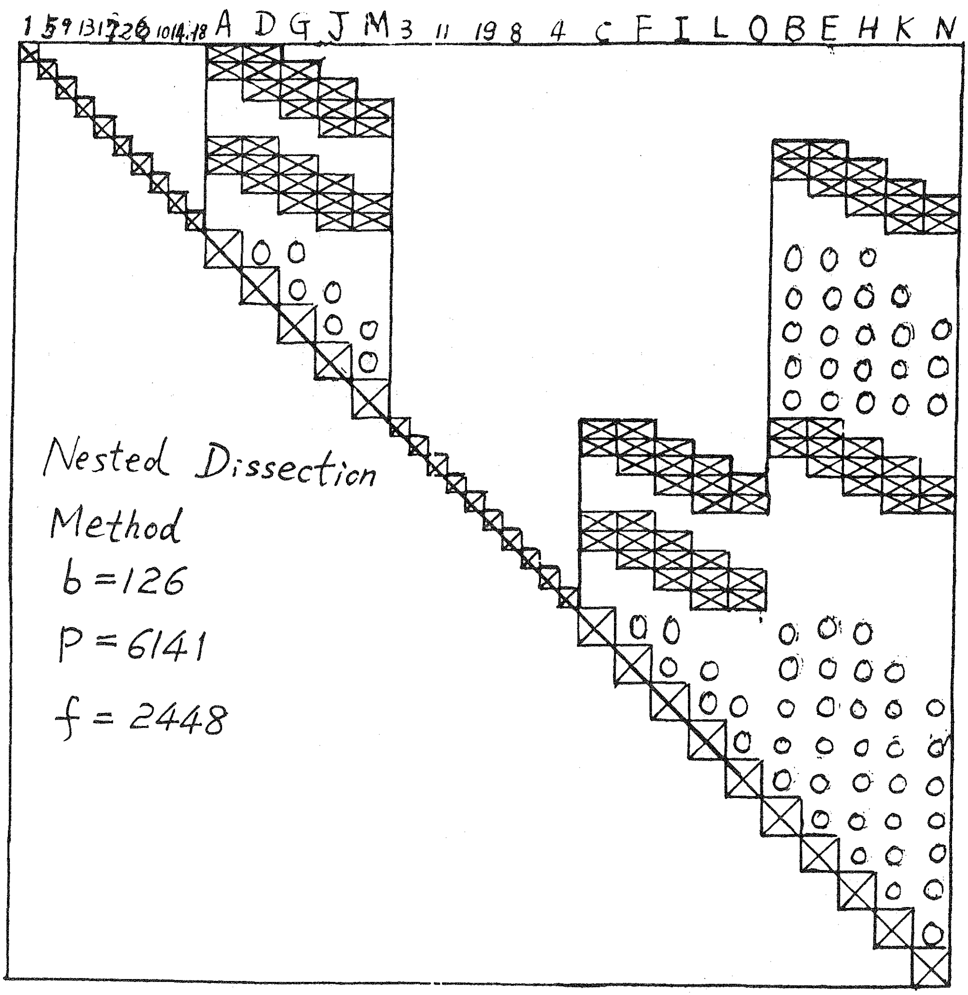

Fig.7 Normal system of Block P by the NDM

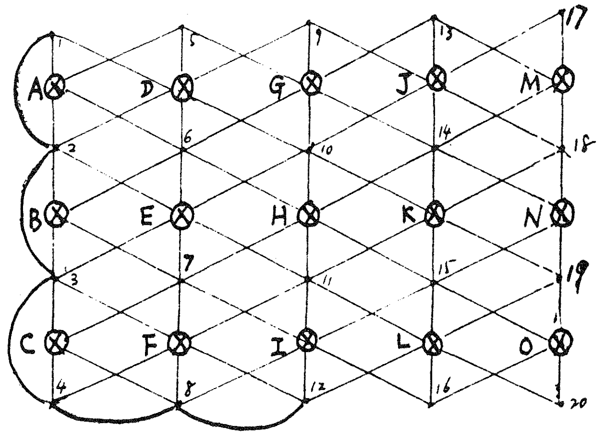

Fig.8 Bundle block with geodetic observations (Block G)

As for combined block adjustment in Fig.8, the easiest and most direct sorting strategy is taking the geodetic points as one cutset and then putting them after other unknowns. In fact, some geodetic observations only appear in one strip. Geodetic points within one strip may be ordered directly after the subblock of this strip instead of after all other unknowns.

Finally, we introduce the concept of pseudo-knot. It is interesting for maintaining the good data structure as well as reducing fill-in and operation.

Imagining that there is a knot P23 between knot 2 and 3, this pseudo-knot makes the direct connection between knot 2 and 3 indirect (see Fig.9). Therefore the pseudo-knot P23 is chosen as a cutset instead of knot 2 and 3. The normal system is demonstrated in Fig.10. The introduction of pseudo-knot leads to less fill-in, profile and band width. It is quite concise especially when geodetic observations range long distances.

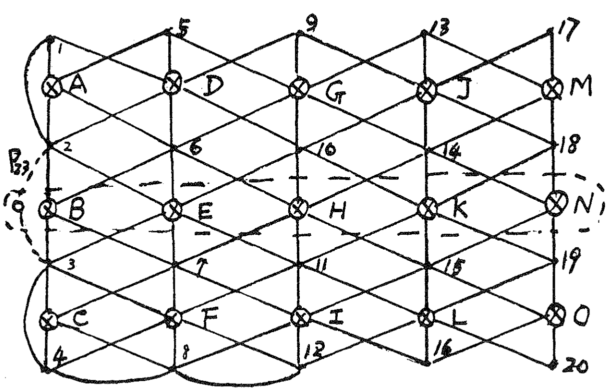

Fig.9 The concept of pseudo-knot

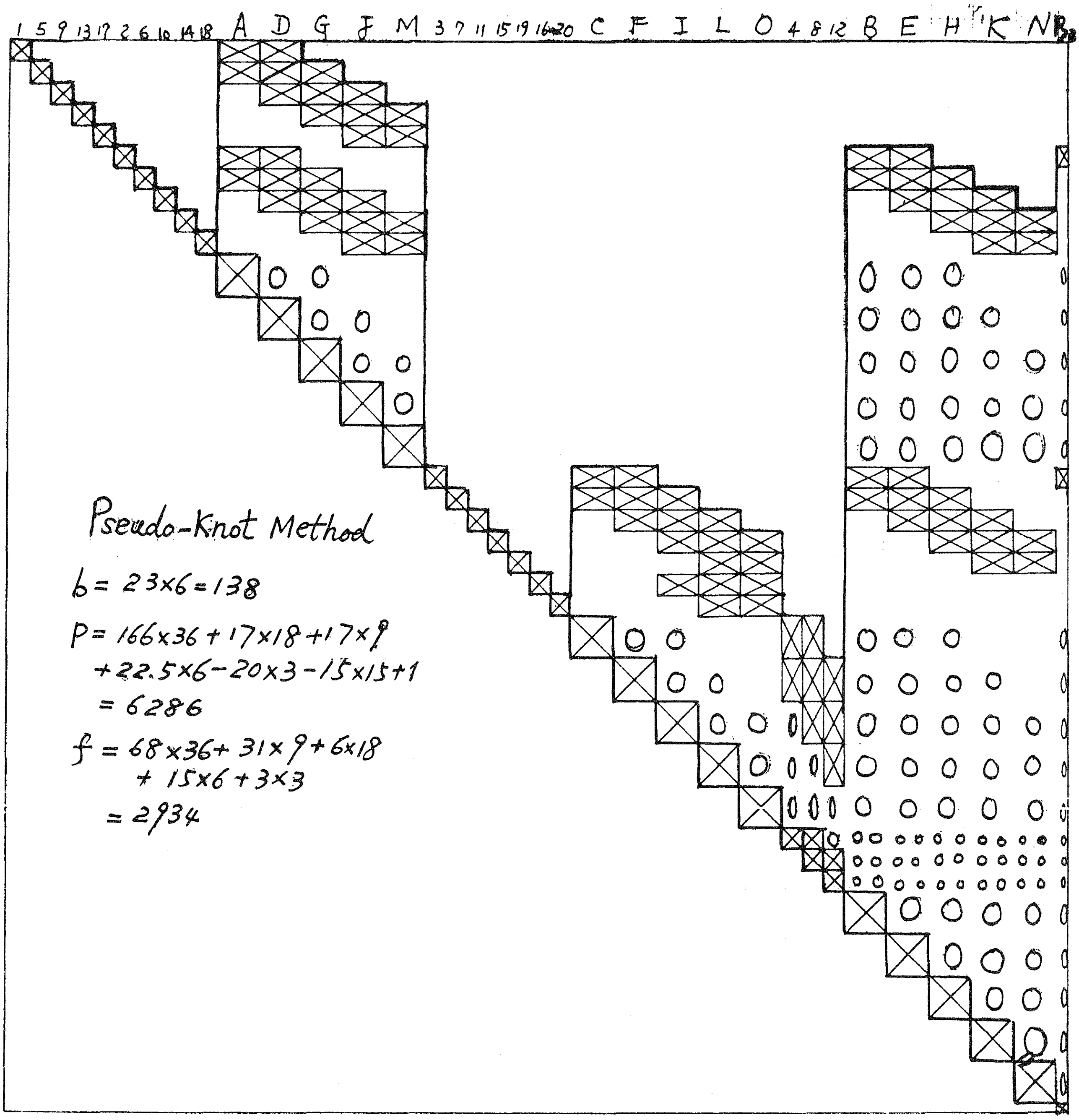

Fig.10 Normal system of Block G with pseudo-knot

It should be pointed out that the pseudo-knot is only a scheme to dissect a graph. It could be proved that introducing one pseudo-knot is equivalent to adding one conditional equation in the adjustment system. Therefore the pseudo-knot strategy is applicable for combined adjustment with geodetic observations, especially when they range long distances.

5. SUMMARY

From the above analysis and discussion, we may conclude the following remarks.

1) The pure sparce matrix technique is not proper to photogrammetric adjustment system because of its complicated data and program structure.

2) The criterion of object point priority would lead to an optimal sorting with approximately minimum operation and fill-in or profile in bundle block adjustment. Both the recursive partioning method and Kruck's method obey this criterion.

3) The newly developed nested dissection method can obtain less fill-in than common methods in bundle block adjustment. With the aid of pseudo-knot, it can also be used in combined adjustment with geodetic observations, especially when they range long distances in the block. It would be more efficient with the use of distributted computer or multi-processor.

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