ANALYTICAL PROCESSING OF SINGLE METRIC TERRESTRIAL STEREOGRAM USING EQUATIONS OF COLLINEARITY AND COPLANARITY

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1. INTRODUCTION

In numerous applications of the terrestrial photogrammetry the stereoscopic observation is an indispensable element of the measuring procedure. Character of the observed object may render identification of the same point on two separate photographs difficult and of low accuracy. On the other hand, signalling of the observed points is not always possible or worthwhile.

The necessity of stereoscopic observation hinders or even makes impossible the use of convergent photos. Only a single one or several separate stereograms can be elaborated. The stereograms should have approximately parallel axes of cameras, and a small base-length ratio is often desired.

The paper presents a discussion of some aspects of the analytical elaboration of a single stereogram.

2. DESCRIPTION OF THE EQUATIONS USED IN THE ELABORATION

2.1. The collinearity equation

The matrix form of the collinearity equation is most often given by:

$$\mathbf{r} = \frac{1}{\lambda} \mathbf{M}^T \mathbf{R} \qquad (1)$$

where:

$\mathbf{r} = [x,\ c_k,\ z]^T$ is the vector joining the projection centre and the image of the observed point,

$\mathbf{R} = [X-X_o,\ Y-Y_o,\ Z-Z_o]^T$ is the vector joining the projection centre and the terrain point,

$\mathbf{M}$ is the transformation matrix [2] given by:

$$\mathbf{M} = \begin{bmatrix} \cos\varphi\ \cos\varkappa - \sin\varphi\ \sin\omega\ \sin\varkappa & -\sin\varphi\ \cos\omega & \cos\varphi\ \sin\varkappa + \sin\varphi\ \sin\omega\ \cos\varkappa \\ \sin\varphi\ \cos\varkappa + \cos\varphi\ \sin\omega\ \sin\varkappa & \cos\varphi\ \cos\omega & \sin\varphi\ \sin\varkappa - \cos\varphi\ \cos\varkappa\ \sin\omega \\ -\cos\omega\ \sin\varkappa & \sin\omega & \cos\omega\ \cos\varkappa \end{bmatrix}$$

λ - scale coefficient

The equation (1) can be converted to the form:

$$\begin{aligned} x-x_o &= F_x = c_k \frac{\mathbf{M}_1^T \mathbf{R}}{\mathbf{M}_2^T \mathbf{R}} \\ z-z_o &= F_z = c_k \frac{\mathbf{M}_3^T \mathbf{R}}{\mathbf{M}_2^T \mathbf{R}} \end{aligned} \qquad (2)$$

After the Taylor's series expansion we obtain the linear observation equations of the form:

$$v_x = \frac{\partial F_x}{\partial(X_1)} d(X_1) + \frac{\partial F_x}{\partial(X_2)} d(X_2) + l_x$$
$$v_2 = \frac{\partial F_z}{\partial(X_1)} d(X_1) + \frac{\partial F_z}{\partial(X_2)} d(X_2) + l_z \tag{3}$$

where:

$X_1 = (X_o, Y_o, Z_o, \varphi, \omega, \varkappa)$ are elements of image orientation

$X_2 = (X, Y, Z)$ are terrain coordinates of the point

$l_x = x^o - x$

$l_z = z^o - z$

x^o, z^o - are approximate image coordinates

x, z - are the observed image coordinates

Some authors [2] advocate the following form of the collinearity equation

$$\mathbf{R} = \lambda \cdot \mathbf{M} \cdot \mathbf{r} \tag{4}$$

which leads to the following linear observation equations:

$$VR = \frac{\partial \mathbf{R}}{\partial(X_1', X_1'')} d(X_1', X_1'') + l_R \tag{5}$$

where:

$X_1' = (X_{o'}, Y_{o'}, Z_{o'}, \varphi', \omega', \varkappa')$

$X_1'' = (X_{o''}, Y_{o''}, Z_{o''}, \varphi'', \omega'', \varkappa'')$

$l_R = R^o - R^G$

R^o - is the approximate vector $\mathbf{R}$

R^G - is the observed vector $\mathbf{R}$, determined by geodetic methods.

The observation equations can be also given in the form:

$$VR = \frac{\partial \mathbf{R}}{\partial(X_3)} d(X_3) + l_R \tag{6}$$

where:

$X_3 = (X_{o'}, Y_{o'}, Z_{o'}, \Phi, \Omega, K, BX, BY, BZ, \Delta\varphi, \Delta\omega, \Delta\varkappa)$

Working with the equations (5) and (6) we deal with observations of different accuracy - proper weighting of these observations should not be forgotten [2].

2.2. The coplanarity condition

$$F = \begin{vmatrix} X_{o''} - X_{o'} & Y_{o''} - Y_{o'} & Z_{o''} - Z_{o'} \\ x_T' & y_T' & z_T' \\ x_T'' & y_T'' & z_T'' \end{vmatrix} \tag{7}$$

where:

$\mathbf{r}_T' = [x_T', y_T', z_T']^T = \mathbf{M}' \mathbf{r}'$

$\mathbf{r}_T'' = [x_T'', y_T'', z_T'']^T = \mathbf{M}'' \mathbf{r}''$

The equation (6) reduces to linear conditions after the series expansion [4,5]. The conditions were applied to points which were not used as control points.

$$\frac{\partial F}{\partial(X_1', X_1'')} d(X_1', X_1'') + \frac{\partial F}{\partial(x_1)} v(x_1) + F^o = 0 \tag{8}$$

where:

$x_1 = (x', z', x'', z'')$

$v(x_1) = [v_{x'}, v_{z'}, v_{x''}, v_{z''}]^T$ is the vector of residuals of the image coordinates

F^o - is the free term computed on the base of the approximate values of unknowns

2.3. Equations introduced by pseudoobservations of the terrain coordinates of control points

Equations of this kind were used by some authors [6]:

$$V(X_2) = d(X_2) + l(X_2) \tag{9}$$

where:

$l(X_2) = X_2^o - X_2^G$

$X_2^o = (X^o, Y^o, Z^o)$ approximate terrain coordinates of control points

$X_2^G = (X^G, Y^G, Z^G)$ pseudoobservations, terrain coordinates obtained from field surveys

$l(X_2) = 0$, as $X_2^o = X_2^G$ was assumed in the investigations

3. SOLUTION OF EQUATIONS

The observation equations (3), (5), (6) were transformed into normal equations and solved with the standard Cholesky algorithm. The Schmidt orthogonalization was also applied to solution of the equations (6).

The set of observation equations (3) with conditions (8) has the general form [3]:

$$A\Delta + BV + L = 0 \tag{10}$$

where:

$A_{m\times n}$ - design matrix of unknowns
$B_{m\times k}$ - design matrix of corrections to observations
Δ_n - vector of unknowns
V_m - vector of corrections
L_m - vector of free terms
m - number of observation equations
n - number of unknowns
k - number of condition equations

Equation (10) can be written in the form:

$$\left[\begin{array}{c} A_1 \\ \hline A_2 \end{array}\right]\Delta + \left[\begin{array}{c|c} B_1 & 0 \\ \hline 0 & B_2 \end{array}\right] \cdot \left[\begin{array}{c} V_1 \\ \hline V_2 \end{array}\right] + \left[\begin{array}{c} L_1 \\ \hline L_2 \end{array}\right] = 0 \tag{11}$$

where "1" pertain to observation equations, while "2" to condition equations.

The weightmatrix has the form $P = \left[\begin{array}{c|c} P_1 & 0 \\ \hline 0 & P \end{array}\right]$

As the matrix $B = -I$ the equations (11) reduce to [1]:

$$\Delta = \left(A^T P_* A\right)^{-1} A^T P_* L \tag{12}$$

where:

$$P_* = \left[\begin{array}{c|c} P_1 & 0 \\ \hline 0 & \left(B_2 P_2^{-1} B_2^T\right)^{-1} \end{array}\right]$$

so the whole problem is simplified to an ordinary parametric case.

Every control point emerges only once in the condition equations (10), so the matrix $B_{2\,m\times k}$ can be transformed to a more condensed form $B_{3\,m\times 4}$, what saves a lot of computer memory.
The vector of corrections V can be computed with the formula:

$$V = \left[\frac{V_1}{V_2}\right] = \left[\frac{A_1 \Delta + L_1}{-P_2^{-1} B_2^T P_3 \left(A_2 \Delta + L_2\right)}\right] \tag{13}$$

The equations (9) were sometimes applied as a supplement to the equations (3) or ((3) + (7)), in order to minimize corrections to the terrain coordinates of control points. Together with them a special matrix of weights, accounting for accuracies of geodetic surveys, eas used.

4. THE INVESTIGATIONS AND THEIR RESULTS

4.1. The aim and the method of the investigations

The investigations were aimed at answers to the three following questions:

a) which form of the collinearity equations: (3), (5) or (6) is more advantageous to the elaboration of a single terrestrial stereogram?
b) which of the two: the collinearity equations (3), or the coplanarity conditions (8), are more advantageous for observed points which are not control points?
c) does the introduction of pseudoobservation equations, pertaining to terrestrial coordinates of control points, bring any and what profits?

Several tests were run in order to find answers to the above questions. Computer-simulated test fields of 100 to 300 points were used. The investigations were limited to stereograms of parallel axes of cameras and base-length ratios between 1:2 and 1:20. The conclusions were drawn on the base of the analysis of:
- true errors of terrain coordinates of newly determined points,
- true errors of orientation parameters of the images,
- convergence of the iterative solution of equations,
- coefficients of correlation between unknowns.

The correlation coefficients were determined with the formula:

$$\rho_{ij} = \frac{\text{cov}\,(i,j)}{\sigma_i \cdot \sigma_j} \tag{14}$$

where:

ρ_{ij} - coefficient of correlation between unknowns i and j

σ_i, σ_j - variances of the unknowns

cov (i,j) - covariance of the unknowns i, j

4.2. Conclusions

4.2.1. Question a

Analysis of the equations (5) and (6), as applied to a single stereogram, showed relatively large correlations between the unknowns. In equations (5) our attention was drawn by correlations between corresponding parameters of orientation of both images: the coefficient ρ ranged between 0.8 and 0.95. In the equations (6) very large correlations occured between $X_{o'}$, $Y_{o'}$, $Z_{o'}$ and the remaining orientation parameters: ρ ranged between 0.5 and 0.95. For comparison: the equations (3), when applied to the same cases, gave substantially lower correlations, with ρ between 0.2 and 0.4, and only exceptionally reaching 0.5 to 0.95.

It is probably due to these high correlations, that the convergence of solution of the equations (5) and (6) was 2 to 5 times slower than in case of equations (3). The true errors of the computed terrain coordinates and of the orientation parameters were, in case of equation (5) and (6), usually several times larger than for equations (3). Only in rare cases the accuracies were comparable.

The equations (5) and (6) give results similar to these given by equations (3), provided that the corrections to the left projection centre ($X_{o'}$, $Y_{o'}$, $Z_{o'}$) are not asked for. In this case also the correlations between unknowns determined with the equations (5) and (6) are significantly smaller.

On the base of these investigations one can say, that equations (3) are much more convenient for elaboration of a single stereogram, than equations (5) or (6). In addition. the equations (5) and (6) can be used only for geodetic control points and only for a single stereogram, what excludes the use of additional, separate photos and creates difficulties when joining separate stereograms into one, simultaneously adjusted block.

4.2.2. Question b

Numerous tests have shown, that the accuracy of computations practically does not depend on whether points other than control points are treated with collinearity equations (3) or with the coplanarity conditions (8). So the use of coplanarity conditions for strengthening of the collinear model, a method recommended by some authors [5] for cases of poor geometry of the model, does not help much.

The economic side of the problem is totally different. With the coplanarity conditions (8) we obtain normal equations containing significantly less unknowns than in case of the exclusive use of equations (3). Large amount of computer memory can be also saved thanks to sparsity of the matrix B_2 (equation (11)).

Let us examine the following case. A stereogram contains 3-5 control points and 50-100 additional check points. Using the equations (3) for all points, we arrive at normal equations containing about 6 times more unknowns than in case of conditions ((3) + (7)). In addition, the profile of the normals generated by equations (3) is about 5 times larger, than in case of conditions ((3) + (7)).

The economic profits brought by the coplanarity conditions diminish with the growth of the number of simultaneously adjusted stereograms.

4.2.3. Question c

The investigations have shown, that the pseudoobservation equations of terrain coordinates of control points are an important addition to the model. Through proper weighting of the equations (9) one can include accuracies of geodetic surveys of the control points into the adjustment.

The terrain coordinates were disturbed by random errors for the analysis. Comparing the results it was found, that errors of the newly determined coordinates, computed in the model, which kept all control points fixed, were about 1.5 times larger, than corresponding errors obtained with the expanded model, which allowed a weight matrix to be associated with coordinates of the control points.

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