Describing Spatial Relation based on Voronoi Diagram in Discrete Space

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[Abstract]

This paper focus on the description of topological spatial relations in the discrete space. First, introducing the method of constructing the raster Voronoi Diagram in the discrete space. Then, a V-framework for description of topological spatial relations is constructed, it can also represent the spatial adjacent relations. At last, two examples are given to explain that raster Voronoi Diagram can be used for description spatial relations ,especially for lateral adjacent relations.

1. Introduction

The planar-based model and raster tessellation-based are the two spatial data models commonly used in designing GIS systems today. The spatial entities are described and represented in the planar graph-based spatial data model as point, line, polygon objects and the topological relationships among these objects(Such as spatial contiguity, connectivity and inclusion) are represented by special topological data structure. However, some other spatially topological relationships (i.e., the lateral spatial adjacency between two objects) are not described and represented in the planar graph-based spatial data model. For instance, the spatial objects of two entities who are spatially adjacency may not have common edges or points, such as boundaries of a playground and the point-like place of a ball falling within the playground. In that case, a polygon should be formed for representing the playground and the line-intersection approach would be used for determining if the lateral spatial adjacency exists between the polygon and the point, which is often time-consuming. Therefore, the ordinary planar graph-based model should be extended for better representing the spatial entities and their spatially topological relationships.

With the development of research about computational geometry, many researchers managed to use the Voronoi Diagram to represent the lateral spatial relations by the Voronoi Diagram of the point, line and polygon objects[Gold, C. M., 1992; Gold and Edwards, 1992]. In fact, spatial objects has its own Voronoi region which defines implicitly the spatial adjacency with the adjacent objects. However, the

Voronoi tessellation defines only the 'influence area' of the spatial objects could not describe and represent the spatial objects. So we want to develop the integrated data model of graph based spatial data model with the Voronoi approach in discrete space. The idea is to divide the discrete space under consideration into Voronoi tessellation according to spatial point, line and polygon objects, and integrate the planar graph-based model and the Voronoi tessellation into a new data model and the Voronoi tessellation into a new data model, i.e, planar graph-based Voronoi spatial data model.

Computational geometry is concerned with the design and analysis of algorithms for geometrialproblems. In addition, other more practically oriented, areas of computer science, computer-aided design, robotics, pattern recognition, and operations research -give rise to problems that inherently are geometrical.. In few years, an increasing interest in a geometrical construct called the Voronoi Diagram. Given some number of points in the plane, their Voronoi Diagram divides the plane according to the nearest -neighbor rule: Each point is associated with the region of the plane closet to it

2. Voronoi Diagram in discrete space.

The Raster Voronoi Diagram is defined as one kind of Voronoi Diagram generated in discrete space. Although it is the same in nature as vector Voronoi Diagram, the algorithms of generating 2D and 3D raster raster Voronoi Diagram is different compared with that in vector space.

2.1 The definition of spatial objects.

In the discrete space the point objects are defined as in the follows:

(1). A={four points around the center objects}

(2). A={only one point}

The boundary and interior define the line-like objects as the strictly 4-neighbor or 8-neighbor.

(1). A={strictly 4-neighbor or 8-neighbor.}

(2). A={strictly 4-neighbor or 8-neighbor.}

The boundary and interior also define the area-like objects as the strictly 4-neighbor or 8-neighbor.

(1). A={not strictly 4-neighbor or 8-neighbor.}

(2). A={not strictly 4-neighbor or 8-neighbor.}

2.2 Voronoi Diagram in discrete space.

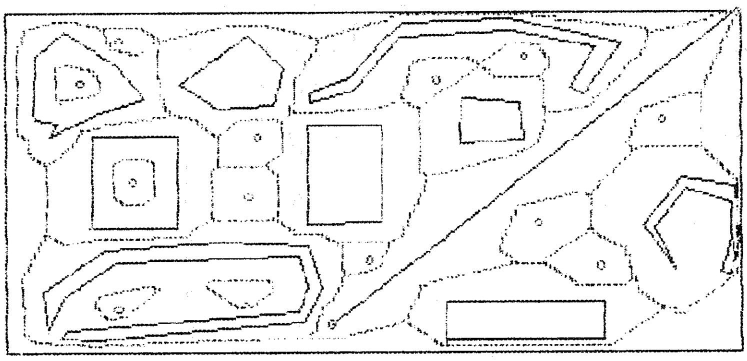
For a boundary 2D space, there exists a subsets of hybrid spatial objects as follows: $P=\{p, p, \ldots p\}$. Pi is called ith generator. For Pi, Pj

$$H_{pipj}=\{X: ||X-P_i||<=||X-P_j||\},\ K_{pipj}=\{X: ||X-P_i||=||X-P_j||\}$$

Hpipj is half plane, Kpipj is boundary. $R_p=\cap H_{pq}\ (q\neq p)$, $R=\{R_p\ |p\in P\}$, R is called the Voronoi Diagram of the spatial objects.

In the discrete space the definition of distance is most important in generating raster Voronoi diagram. The distance of spatial objects is defined as:

$Ds=Min\{\ \|X-Xi\|,\ Xi \in Objects\}$



Raster Voronoi Diagram of spatial objects.

3. The description of topological relations based on Voronoi diagram.

Kainz[1990], Egenhofer and Franzosa[1991] made more systematic investigation on the definitions of spatial topological relations. They proposed a formal framework, as follows in term of the intersections of the boundaries and interiors of two point-sets.

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^{o} \\ A^{o} \cap \partial B & A^{o} \cap B^{o} \end{bmatrix}$$

But this kind of description framework exists drawbacks, some relations can't be represented by them. For conquering the deficients, Chen, Sun[1994] take the complement besides interior and boundary into a count and construct a new framework in terms of the intersections of boundaries, interior and complements of two sets.

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^{o} & \partial A \cap B^{-1} \\ A^{o} \cap \partial B & A^{o} \cap B^{o} & A^{o} \cap B^{-1} \\ A^{-1} \cap \partial B & A^{-1} \cap B^{o} & A^{-1} \cap B^{-1} \end{bmatrix}$$

In nature, the new framework has no considerable changes. The lateral adjacent relations still are not represented. Therefore, we introduce the Voronoi Diagram into the discrete space and modify the description framework as follows:

$$\begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^{o} & \partial A \cap V_B \\ A^{o} \cap \partial B & A^{o} \cap B^{o} & A^{o} \cap V_B \\ V_A \cap \partial B & V_A \cap B^{o} & V_A \cap V_B \end{bmatrix}$$

Va, Vb are raster Voronoi regions of spatial objects. Making use of this framework it is possible for description lateral adjacent relations besides general spatial relations. In the follows two examples are given, the first is to find nearest object near to one location, the second is to find adjacent objects near to the center object in the term of Voronoi diagram adjacency.

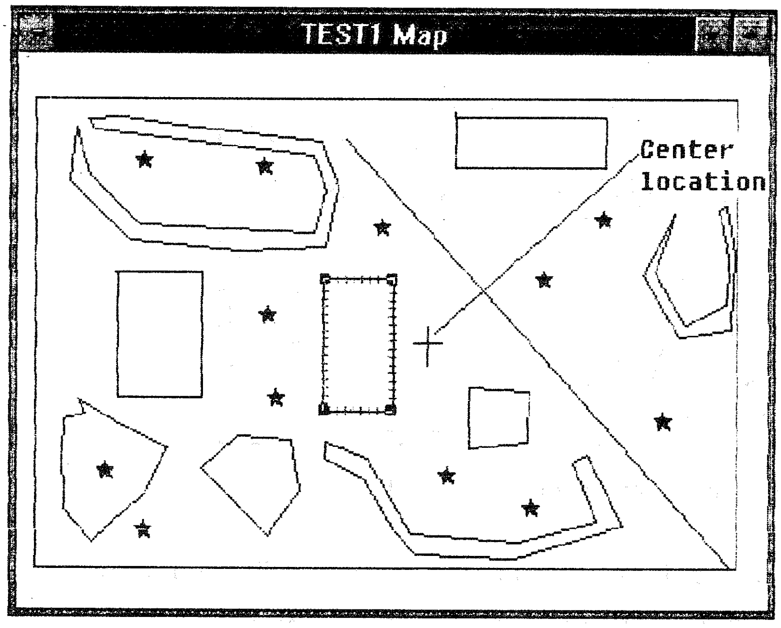


Fig 1 The nearest object to center location

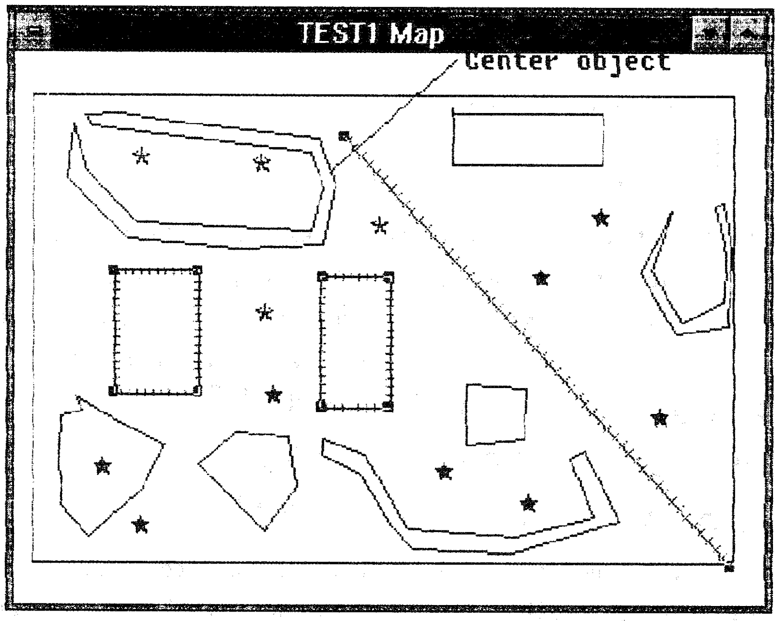


Fig 2. The nearest objects to center objects.

In the above two examples, The "nearest" is in the term of Voronoi Diagram, and find the adjacent objects according to the above definition framework.

4. Conclusion.

We introduce the raster Voronoi diagram into the discrete space for description spatial relations, especially lateral spatial adjacent relations, the proofs in the paper show: firstly, the new framework can describe general spatial relations, such as, overlap, separation, inclusion, etc. secondly, using the Voronoi Diagram can represent the separation, adjacency wonderfully.

Acknowledgments

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References

1. Chen, Jun. 1992. Improving urban planning by integrated utilization of remote sensing and GIS, Proceeding of 17th ISPRS Congress, Vol., pp. Washington, Aug., 1992.
2. Chen, Jun. and C. M. Gold. 1992. Research directions for spatial decition support, Proceeding of the international Colloquium on photogrammetry, Remotesensing and GIS, pp. 1-8, Wuhan, May, 1992.
3. Chen, Jun and Maclaughlin. 1992. Strategic directions of surveying and mapping. Proceedings of 1992 ACSM/ASPRS Congress, Vol. 1, pp. 254-261, Washington, Aug., 1992.
4. Chen, Jun and Zhifeng Tang. 1992. Improving spatial analysis in urban GIS environment, Proceedings of the 3rd Beijing International GIS Workshop, Vol. 1, pp., Beijing, Aug., 1993.
5. Chou, Hsueh-cheng and Yuemin Ding. 1992. Methodology of integrating spatial analysis /modeling and GIS, Proceeding of the 5th International Symposium on Spatial data Handling, Vol. 2, pp. 514-523, 1992.
6. Dutton, G. 1991. Improving spatial analysis in GIS environment, Proceeding of 1991 ASCM-ASPRS Annual Convention, pp. 168-185.
7. Granger, K. J. 1990. Process modeling and GISs: Breathing life into spatial analysis , Mathematics and Computers in Simulation, Vol. 32, pp. 243-247, 1990.
8. Kemp, K. K. 1992. Spatial models for environmental modeling with GIS, Proceeding of the 5th International Symposium on Spatial Data Handling, Vol. 2, pp. 524-533, 1992.
9 Tiang, Suyu. 1993. Simulation of the Submergence of urben flooding based on DTM, Master's thesis of WTUSM, May, 1993.

10. Yuguo Sun, 1993. Complete Topological Spatial Relations: the Framework and the Case in 2D Space, Proceeding of AUSIA'93 on Advanced Urban Spatial infomation and Analysis. pp. 16-25, October, 1993.
11. Abler R, 1987. The NSF National Center for Geographic Information and Analysis in the U.S.A, Inf J. GISs, Vol. 1, No. 1, No. 4, 303-326.
12. Burrough, P., 1992, development of intelligent GISs, IJGIS, Vol. 6, No. 1, 1-11.