SPATIAL RELATIONS BETWEEN UNCERTAIN SETS

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ABSTRACT:

As a part of our serial researches, this paper presents methodologies for modeling spatial relations between uncertain sets. The uncertainty of spatial relations may arise through the fuzzily defined concepts or linguistics, the presence of varying shapes and features of complicated spatial objects, and the imprecise measurements of spatial data. By using fuzzy set theory, Mathematical Morphology, and the dynamic 9-intersection model for integrally representing spatial relations [Chen, et. al. 1995, 1996], a fuzzy 9-intersection model is developed in which the spatial relations are defined in terms of the intersections of the boundaries, interiors and exteriors of two dynamically generated uncertain sets. Then, the presented models are extended for quantitatively deriving the spatial relations between sets in consideration of conceptual and positional uncertainties. Finally, some potential applications of presented theories and the ideas for spatial and temporal reasoning in Geographical Information Systems (GIS) are also suggested.

1. INTRODUCTION

Geographical Information Systems (GIS) have evolved from tools for spatial data management and cartography into sophisticated decision support systems that utilize variety of spatial and tabular analysis to derive new information. These systems are finding a wide variety of applications including: urban and regional planning; environmental and resource management; facilities management; archaeology; and market research. In the field of GIS research and application, one of the most fundamental requirements is to modeling and communicating error in spatial databases. With increased research into error modeling over the past few years, there has been a considerable body of models and techniques available for measurement spatial and temporal database error from researching to real applications [Goodchild, 1989; Hunter and Goodchild, 1995; Shibasaki, 1994; Vergin, 1994]. Spatial relationships (such as distance, direction, ordering, and topology) between spatial objects, as very useful tools for spatial and temporal reasoning in GIS, may be strongly influenced by the uncertainties of original data. The practical needs in GIS have led to the investigation of formal and sound methods for driving spatial relations and their variations with uncertainties [Chen and et.al., 1995, 1996; Egenhofer and Franzosa, 1991; Frank, 1992; Kainz, et.al., 1993; Peuquet and Zhang, 1987]. However, how to derive spatial relations between uncertain sets based on an mathematically well-defined algebra framework is still an open problem up to now. The lack of this comprehensive theory has been a major impediment for solving many sophisticated problems in GIS, such as formally deriving spatial relations between complicated spatial objects, spatial and temporal reasoning in GIS with multiple representations, and generation of the formal standards for transferring spatial relations.

As a part of our serial researches, this paper presents methodologies for modeling spatial relations between uncertain sets. The uncertainty of spatial relations may arise through the fuzzily defined concepts or linguistics, the presence of varying shapes and features of complicated spatial objects, and the imprecise measurements of spatial data. By using fuzzy set theory, Mathematical Morphology, and the dynamic 9-intersection model for integrally representing spatial relations [Chen, et. al. 1995, 1996], a fuzzy 9-intersection model is developed in which the spatial relations are defined in terms of the intersections of the boundaries, interiors and exteriors of two dynamically generated uncertain sets. Then, the presented models are extended for quantitatively deriving the spatial relations between sets in consideration of conceptual and positional uncertainties. Finally, some potential applications of presented models and the ideas for spatial and temporal reasoning in GIS are also suggested.

This paper is structured into three main sections that follow this introduction. Section 2 contains a review of related definitions concerning uncertainty and imprecision for deriving spatial relations. In section 3, after the brief introduction of some fundamental theories, the fuzzy 9-intersection model is developed for integrally deriving spatial relations between uncertain sets. Section 4 contains the extensions of the presented theories for deriving conceptual and positional uncertainties between sets. In the last section conclusions and outlook for further research are given.

2. UNCERTAINTIES OF SPATIAL RLATIONS

Uncertainty and imprecision refer to the degree of knowledge (or ignorance) which we have concerning some domain of interest. Uncertainty is an assessment of our belief (or doubt) in an outcome, based on the available data. Typically uncertainty is modeled by probability theory. Imprecision is a feature of the data itself. It refers to data expressed as a range of possible values. Other descriptions of data are accurate and exact, approximate, and ambiguous. Geographical features, such as spatial relations, may contain many kinds of uncertainties which are often caused by the fuzzily defined concepts and linguistics, the presence of varying shapes and features of complicated spatial objects, and the imprecise measurements of spatial data. In following section, we will briefly introduce these uncertainties of spatial relations.

2.1. Conceptual Uncertainties

Conceptual uncertainties of spatial relations in GIS are mainly caused by the fuzzy linguistic and conceptual variables. For example, we often use the fuzzy concepts (such as near, far, almost west, middle east, and et. al.) to deriving metric relations. In this case, the metric relations are derived by a set of metric values together with their fuzzy memberships. The fuzzy membership is generally a real number on [0,1], where 0 indicates no membership and 1 indicates complete membership. Similarly, the fuzzy concepts of weak connected, strong connected, almost same, big different, and et. al., are caused the uncertainties of topologic and ordering relations. It should be emphasized that topologic relations are generally independent on the geometry, but topologic relations usually are derived from geometric descriptions, so this is also necessary when taking the uncertainty of the geometric entities into consideration which of cause leads to derived quantities of uncertainty of the topologic relations. Some related examples are shown in **Fig.1 (a)-(c)**.

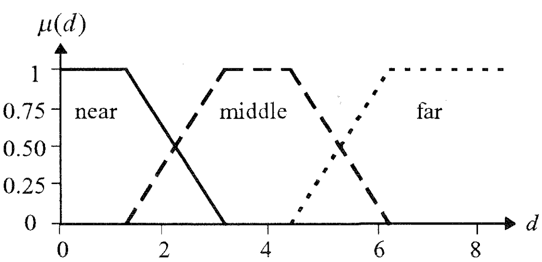

Fig.1(a). Fuzzy distance describing ***near***, ***middle*** and ***far***

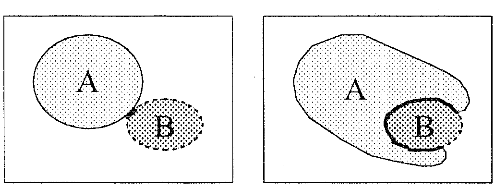

Fig.1(b). Topologically met objects with the ***weak*** and ***strong*** connections

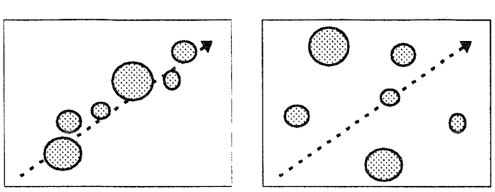
Fig.1(c). Directionally ordered objects with the ***almost same*** and ***big different*** orientations

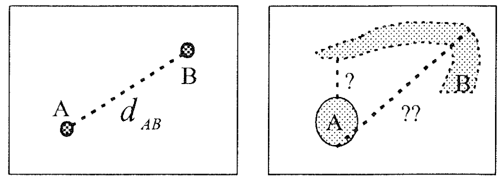

Fig.2(a). Distances between points and objects

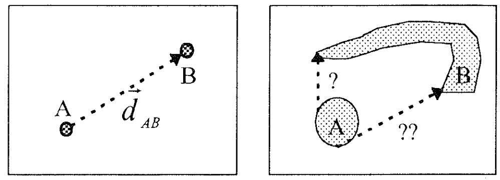

Fig.2(b). Directions between points and objects

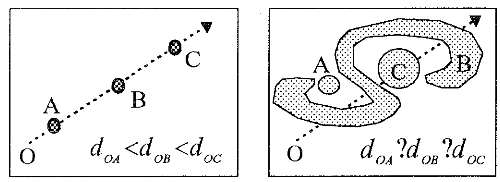

Fig.2(c). Ordering between points and objects

2.2. Object Feature Uncertainties

Object feature uncertainties of spatial relations in GIS are mainly caused by the ambiguous variables of object shapes, sizes and distributions. A metric relation between two arbitrarily-shaped objects is a fuzzy concept and is thus often dependent on human interpretation [**Fig.2(a)-(b)**].The simplest way to calculate the distance or direction between two objects is to convert the object calculations to the represented point calculations, such as using the distance between two capitals to represent the distance between two countries. But this method will cause many problems when the country is big and its shape is complicated. As the descriptions in Chen and et. al [1995, 1996], the rigorous method to calculate metric relations between two arbitrarily-shaped objects is to calculate the Hausdorff distances and directions between their sub-sets of objects and their fuzzy memberships. The general Hausdorff metric between two objects is just the special case of its fuzzy membership value equals to 1. Similarly, the object shape and distribution may cause the uncertainties of ordering relations [**Fig.2(c)**], but don't influence topologic relations. The ordering relations between objects can be derived by the method of partial-ordered segmentation of objects [Chen and et. at., 1995, 1996].

2.3. Data Uncertainties

Data uncertainties of spatial relations in GIS are mainly caused by imprecise measurements of spatial data. Generally, the locations of objects in spatial databases are not error-free, they may contain many kinds of errors, such as the errors of scanning, digitizing, selecting, projection, overlaying, and et.al. [Goodchild and et.al., 1989]. For describing the uncertainty in the positions of spatial objects (such as lines and areas), we can use the error model of ε -band developed by Chrisman [1982], in which the positional uncertainty of a spatial object K_i can

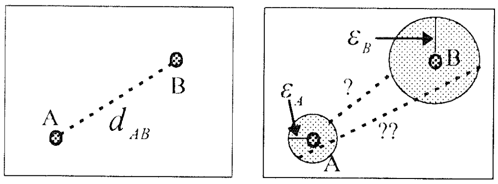

Fig.3(a). Positional uncertainties of distance relations

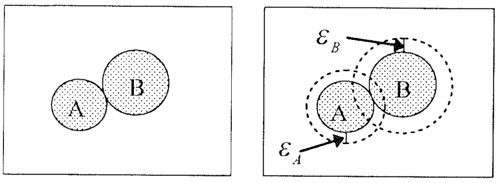

Fig.3(b). Positional uncertainties of topologic relations

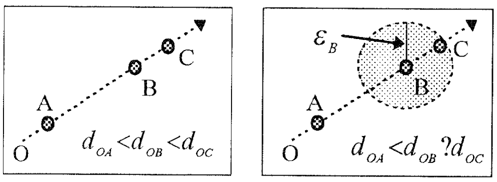

Fig.3(c). Positional uncertainties of ordering relations

be represented as $\tilde{K}_i = K_i \oplus \mu(\varepsilon)$, where ε is the buffer distance of error distribution and $\mu(\varepsilon)$ is the fuzzy membership function derived by ε. As shown in **Fig3(a)-(c)**, spatial metric, topologic and ordering relations between error ε -band generated objects will cause different kinds of uncertainties.

3. FUZZY 9-INTERSECTION MODEL

3.1. 9-Intersection

For driving binary topological relations between sets, Egenhofer et al., (1994) developed the 9-intersection model based on the usual concepts of point-set topology with open and closed sets, in which the binary topological relations between two objects, K_1 and K_2, in IR^2 is based upon the intersection of K_1's interior (K_1^o), boundary (∂K_1),and exterior (K_1^-) with K_2's interior (K_2^o), boundary (∂K_2), and exterior (K_2^-). A 3×3 matrix $\Im_9$, called the 9-intersection as follows:

$$\Im^9 = \begin{bmatrix} K_1^o \cap K_2^o & K_1^o \cap \partial K_2 & K_1^o \cap K_2^- \\ \partial K_1 \cap K_2^o & \partial K_1 \cap \partial K_2 & \partial K_1 \cap K_2^- \\ K_1^- \cap K_2^o & K_1^- \cap \partial K_2 & K_1^- \cap K_2^- \end{bmatrix} \quad [1]$$

By considering the values empty (0) and non-empty (1) in equation [3], one can distinguish between 2^9=512 binary topological relations in which only a small subset can be realized when the objects of concern are embedded in IR^2 [Egenhofer and Franzosa, 1991; Mark and et. al., 1995]. The beauty and simplicity of 9-intersection model come from the fact that it can solve the topologic and geometric problems by using the formal logic and algebraic methods. Since present digital computers are very strong for logic and arithmetic calculations, but they are poor for high level geometric and topologic reasoning. So the 9-intersection model has the potential abilities for automatically spatial and temporal reasoning.

3.2. Dynamic 9-Intersection

For integrally deriving different kinds of spatial relations between sets, Chen and et al. (1995, 1996) developed the dynamic 9-intersection model based on the concepts of the metric topology with open and closed sets and the morphological dilation, in which the general 9-intersection of equation [1] is extended as follows:

$$\Im^9_{(1,2)}(\varepsilon_i) = \begin{bmatrix} [K_1 \oplus B(\varepsilon_i)]^o \cap K_2^o & [K_1 \oplus B(\varepsilon_i)]^o \cap \partial K_2 & [K_1 \oplus B(\varepsilon_i)]^o \cap K_2^- \\ \partial[K_1 \oplus B(\varepsilon_i)] \cap K_2^o & \partial[K_1 \oplus B(\varepsilon_i)] \cap \partial K_2 & \partial[K_1 \oplus B(\varepsilon_i)] \cap K_2^- \\ [K_1 \oplus B(\varepsilon_i)]^- \cap K_2^o & [K_1 \oplus B(\varepsilon_i)]^- \cap \partial K_2 & [K_1 \oplus B(\varepsilon_i)]^- \cap K_2^- \end{bmatrix}$$

[2]

where the K_1 and K_2 are given two closed sets, the $K_i \oplus B(\varepsilon_t)$ means relevant morphological dilation by the closed ball B with radius ε_t, and the $\Im^9_{(i,j)}(\varepsilon_t)$ means dynamic 9-intersection with parameter ε_t from K_i to K_j. Based on the equation [2], we can derive dynamic topological relations by using the different parameter ε_t. In particular case, when $\varepsilon_t = 0$, we have $K_i \oplus B(\varepsilon_t) = K_i \oplus \{o\} = K_i$, then the dynamic 9-intersections $\Im^9_{(i,j)}(\varepsilon_t)$ coincide with the general 9-intersection $\Im_9$ [Chen and et. al., 1995, 1996].

3.3. Fuzzy 9-Intersection

For deriving different kinds of spatial relations between uncertain sets, we can extend the 9-intersection model to the fuzzy 9-intersection model as follows:

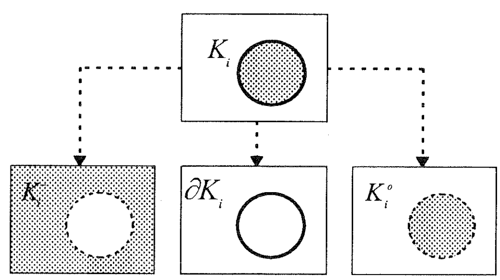

Fig.4. Space segmentation of 9-intersection model

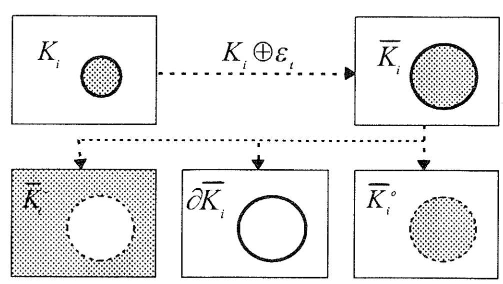

Fig.5. Space segmentation of dynamic 9-intersection model

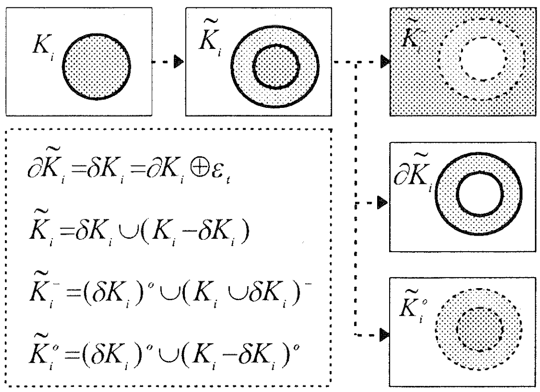

Fig.6. Space segmentation of fuzzy 9-intersection model

$$\tilde{\Im}^9_{(1,2)} = \begin{bmatrix} \mu(\tilde{K}_1^\circ \cap \tilde{K}_2^\circ) & \mu(\tilde{K}_1^\circ \cap \partial\tilde{K}_2) & \mu(\tilde{K}_1^\circ \cap \tilde{K}_2^-) \\ \mu(\partial\tilde{K}_1 \cap \tilde{K}_2^\circ) & \mu(\partial\tilde{K}_1 \cap \partial\tilde{K}_2) & \mu(\partial\tilde{K}_1 \cap \tilde{K}_2^-) \\ \mu(\tilde{K}_1^- \cap \tilde{K}_2^\circ) & \mu(\tilde{K}_1^- \cap \partial\tilde{K}_2) & \mu(\tilde{K}_1^- \cap \tilde{K}_2^-) \end{bmatrix} \quad [3]$$

where the K_1 and K_2 are given two closed sets; the generated fuzzy sets in consideration of data positional uncertainties are defined as $\partial\tilde{K}_i = \delta K_i = \partial K_i \oplus \varepsilon_i$, $\tilde{K}_i = \delta K_i \cup (K_i - \delta K_i)$, $\tilde{K}_i^- = (\delta K_i)^\circ \cup (K_i \cup \delta K_i)^-$ and $\tilde{K}_i^\circ = (\delta K_i)^\circ \cup (K_i - \delta K_i)^\circ$; and $\mu(*)$ is a kind of metric functions which are used for deriving the fuzzy memberships based on the generated sets by logic intersections. For different purposes, the function $\mu(*)$ may take the different forms as used below.

4. SPATIAL RELATIONS BETWEEN UNCERTAIN SETS

Since the object feature caused uncertainties of spatial relations can solved by using Hausdorff metrics between sub-sets [Chen and et. al., 1995, 1996], we only discuss the problems of conceptual and positional uncertainties of spatial relations in this paper. For reasons of simplicity the spatial relations between closed regions discussed in this paper only, related models for estimation of conceptual and positional fuzzy membership functions are defined as following sections.

4.1. Conceptual Uncertain Relations

For deriving conceptual fuzzy topologic relations, such as weak meet and strong meet which were discussed in the section 2.1, we can use the fuzzy 9-intersection model by selecting $\varepsilon_i \equiv 0$ and $\tilde{K}_i \equiv K_i$, then choice the following $\mu(*)$ to calculate related fuzzy memberships from object A to B:

<u>Line length</u>: $$\mu(\partial A \cap \partial B) = \frac{\ell(\partial A \cap \partial B)}{\ell(\partial A)} \quad [4]$$

<u>Area size</u>: $$\mu(A^\circ \cap B^\circ) = \frac{\Lambda(A^\circ \cap B^\circ)}{\Lambda(A^\circ)} \quad [5]$$

<u>Others</u>: $$\mu(*) = \begin{cases} 0, & \text{when } * = \phi; \\ 1, & \text{when } * \neq \phi; \end{cases} \quad [6]$$

where " * " means logically intersected sets, $\ell(*)$ and $\Lambda(*)$ are the line lengths and area sizes. Generally the binary topologic relations between objects A and B derived by equation [3] are not symmetry , i.e. $\tilde{\Im}^9_{(1,2)}$ usually is not equals to $\tilde{\Im}^9_{(2,1)}$. According to the fuzzy set theoretic operations [Zadeh, 1965], we cans either use union $Max[\mu(A,B),\mu(B,A)]$ or intersection $Min[\mu(A,B),\mu(B,A)]$ of two fuzzy memberships to integrated deriving fuzzy topological relations. For example, to calculate the fuzzy topologic relations of **Fig.1(b)** by using the union fuzzy memberships, we can get following results for left and right figures respectively:

<u>the left figure in Fig.1(b)</u>:

$$\tilde{\Im}^9_{(1,2)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.03 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \tilde{\Im}^9_{(2,1)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.08 & 1 \\ 1 & 1 & 1 \end{bmatrix};$$

$$Max(\tilde{\Im}^9_{(1,2)}, \tilde{\Im}^9_{(2,1)}) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.08 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

<u>the right figure in Fig.1(b)</u>:

$$\tilde{\Im}^9_{(1,2)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.23 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \tilde{\Im}^9_{(2,1)} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.79 & 1 \\ 1 & 1 & 1 \end{bmatrix};$$

$$Max(\tilde{\Im}^9_{(1,2)}, \tilde{\Im}^9_{(2,1)}) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.79 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

For deriving conceptual fuzzy metric relations, such as near and far, we can firstly use the fuzzy 9-intersection model to generate uncertain set $\tilde{K}_{ii}$ based on given fuzzy membership functions, then use the uncertain object $\tilde{K}_i$ as instead of the general object K_i to calculate Hausdorff metrics between uncertain sets or sub-sets [Chen and et. al., 1996]; After that we quantitatively estimate the conceptual fuzzy membership functions between uncertain subsets by using the $\mu(*)$ in equation [3] as follows:

<u>Solid volume</u>: $$\mu(A^\circ \cap B^\circ) = \frac{\Lambda(A^\circ \cap B^\circ) * \mu_A(x,y)}{\Lambda(A^\circ) * \mu_A(x,y)} \quad [7]$$

<u>Others</u>: $$\mu(*) = \begin{cases} 0, & \text{when } * = \phi; \\ 1, & \text{when } * \neq \phi; \end{cases} \quad [8]$$

where " * " means logically intersected sets, $\mu_A(x,y)$ is the given fuzzy membership function for description of uncertain metric concepts, $\Lambda(*)$ is area size of logically intersected two objects, and $\mu(A^\circ \cap B^\circ)$ is calculated fuzzy memberships of derived uncertain spatial metric relations.

4.2. Positional Uncertain Relations

For deriving positional fuzzy topologic relations, we can use the fuzzy 9-intersection model to generate uncertain set $\tilde{K}_{ii}$ based on uncertain ε-band, then choice the following $\mu(*)$ to calculate related fuzzy memberships from object A to B **[Fig.7]**:

$$\mu((\widetilde{A}^{*}\cap B^{*})=\frac{1-d_{min}\{[(\widetilde{A}^{*}\cap B^{*})\cap\partial\widetilde{A}],\widetilde{A}^{\circ}\}}{2\varepsilon_{A}} \qquad [9]$$

where " * " means either interior (o) or exterior (-), $d_{min}(*)$ is the minimum distance between two sets, and the ε_A is the positional uncertain ε -band of object A. Generally the binary topologic relations between objects A and B derived by equation [3] are not symmetry , i.e. $\widetilde{\Im}^{9}_{(1,2)}$ usually is not equals to $\widetilde{\Im}^{9}_{(2,1)}$. According to the fuzzy set theoretic operations [Zadeh, 1965], we cans either use union or intersection of two fuzzy memberships to integrated deriving fuzzy topological relations.

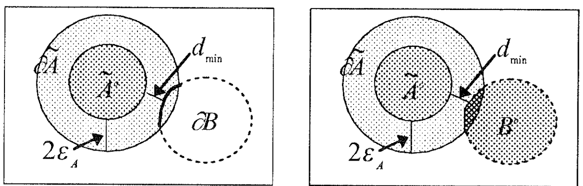

Fig.7. Fuzzy memberships for deriving topologic relations

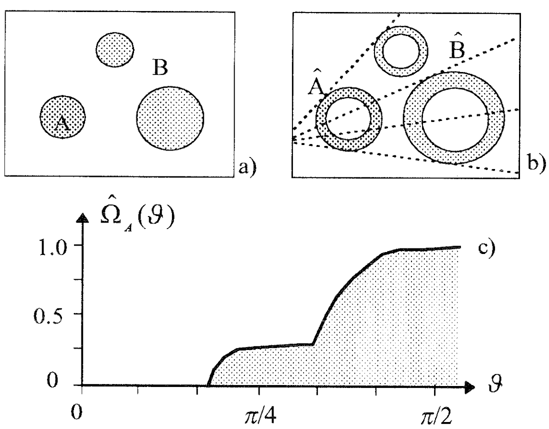

Fig.8. Fuzzy memberships for deriving directional relations

For describing the uncertainty in the positions of spatial objects (such as lines and areas) and deriving positional fuzzy metric relations, we also use the error model of ε -band in which the positional uncertainty of a spatial object K_i can be represented as $\widetilde{K}_i=K_i\oplus T(\varepsilon)$, where ε is the buffer distance of error distribution . According to Chen and et. at. [1995,1996], we can separate the processing of deriving spatial metric relations between uncertain sets to two steps. Firstly we use the uncertain object $\widetilde{K}_i$ as instead of the general object K_i to calculate Hausdorff metrics between uncertain sets or sub-sets ; Secondly we quantitatively estimate the positional fuzzy membership functions between uncertain subsets by using the measurement of dynamic covering uncertain areas as follows:

$$\left.\begin{aligned}\hat{\Phi}_A(\lambda)&=\frac{A\{[\widetilde{K}_1\oplus B(\lambda)]\cap\hat{K}_2\}+A\{\hat{K}_1\cap[\widetilde{K}_2\oplus B(\lambda)]\}}{A(\hat{K}_1)+A(\hat{K}_2)}\\ \hat{\Omega}_A(\theta)&=\frac{A\{[\widetilde{K}_1\oplus R(\theta)]\cap\hat{K}_2\}+A\{\hat{K}_1\cap[\widetilde{K}_2\oplus R(\theta\pm\pi)]\}}{A(\hat{K}_1)+A(\hat{K}_2)}\end{aligned}\right\} \qquad [10]$$

where $\hat{K}_i=\widetilde{K}_i-K_i=[K_i\oplus T(\varepsilon)]-K_i$ means the uncertain area generated by the related ε -band, $A\{*\}$ means the covered area sizes, and $\hat{\Phi}_A(\lambda)$ and $\hat{\Omega}_A(\theta)$ are distance and directional fuzzy membership functions separately. An example of positional uncertainty of spatial directional relations between two spatial regions is shown in **Fig. 8**.

6. CONCLUSIONS AND OUTLOOKS

9-intersection model as powerful tool for formally deriving topological relations. The beauty and simplicity of 9-intersection model come from the fact that it can solve the topologic and geometric problems by using the formal logic and algebraic methods. Since present digital computers are not very strong for high level geometric and topologic reasoning, the 9-intersection model has the potential abilities for automatically spatial and temporal reasoning. As the natural extension of general 9-intersection model, a fuzzy 9-intersection model is developed for studying different kinds of spatial relations between uncertain sets. Even though the presented approach is only focus on the applications in GIS field, the related results for deriving spatial relations between sets can be also used for many other fields, such as CAD, computer vision, pattern recognition, robot space searching and so on. However, only the theoretical models and algorithms have be presented in this paper, a wide field of practical application for data management and spatial data analysis in 2-D and 3-D GIS environments has not been touched. Therefore, the reported results must be verified and extended in order to be used in different practical environments.

Our further research will be concentrated on two main directions, one is the applications of the presented theoretical models and algorithms in 2-D and 3-D GIS environments for developing the new tools of spatial and temporal reasoning; another one is the extensions of presented theories and models for formally deriving complex spatial relations among spatial objects with multiple representations.

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