PROMPT - A NEW BUNDLE ADJUSTMENT PROGRAM USING COMBINED PARAMETER ESTIMATION

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ABSTRACT:

In the past years, multi image triangulation through bundle adjustment has become an increasingly important issue in close range photogrammetry. The new Rollei Fototechnic software PROMPT (**PR**ofessional **O**rientation applying **M**odern **P**arameter adjustment and analysis **T**ools) takes advantage of the WINDOWS environment. It has been designed to operate large amounts of data and to ensure most efficient and reliable results by introducing semi automatic analysis and adjustment tools.

1. INTRODUCTION

The functional relation used in bundle adjustment programs is the central projection [WESTERBBINGHAUS 1985] where the parameters of the interior orientation, the parameters of the exterior orientation and the three co-ordinates of the object points are included (ref. equations 1, 2 and figure 1a, 1b).

$$\begin{bmatrix} x_{ij} - x_H - dx \\ y_{ij} - y_H - dy \end{bmatrix} = \frac{-c}{Z_{ij}^*} \cdot \begin{bmatrix} X_{ij}^* \\ Y_{ij}^* \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} X_{ij}^* \\ Y_{ij}^* \\ Z_{ij}^* \end{bmatrix} = D(\omega_j, \varphi_j, \kappa_j) \begin{bmatrix} X_i - X_{oj} \\ Y_i - Y_{oj} \\ Z_i - Z_{oj} \end{bmatrix} \quad (2)$$

With x_{ij}, y_{ij} being co-ordinates of the image point P_{ij} in the image co-ordinate system; c being the calibrated focal length; x_H, y_H being co-ordinates of the principal point in the image co-ordinate system; dx, dy being systematic image errors in the image co-ordinate system; X^*_{ij}, Y^*_{ij}, Z^*_{ij} being co-ordinates of the point P_i in the auxiliary co-ordinate system parallel to the image co-ordinate system; X_{oj}, Y_{oj}, Z_{oj} being co-ordinates of the projection center O_j in the object co-ordinate system; $D(\omega_j,\varphi_j,\kappa_j)$ being the rotation matrix to transfer the object co-ordinate system into a position parallel to the auxiliary co-ordinate system; X_i, Y_i, Z_i being co-ordinates of the point P_i in the object co-ordinate system.

The geometric model of central projection is extended by parameters for systematic image errors (e.g. distortion parameters) for applications which require high accuracy. Besides photogrammetric image co-ordinate measurements the introduction of additional (geodetic) observations is possible as well.

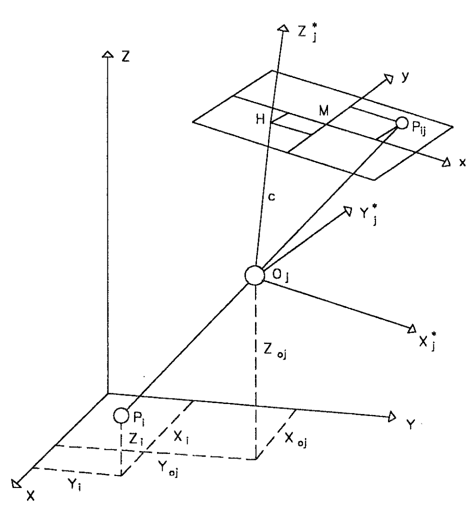

Figure. 1a: Geometry of the central projection

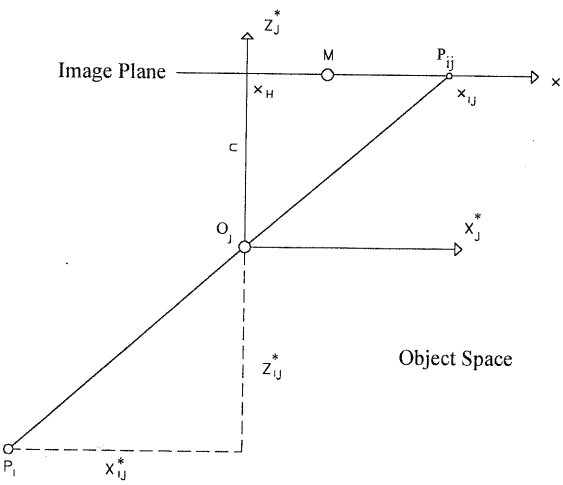

Figure. 1b: Geometry of the central projection

2. ADJUSTMENT TECHNIQUES WITHIN *PROMPT*

Within photogrammetric adjustment and analysis of data there are different tasks the numerical procedures have to serve. Besides the necessary analysis of the data (amount of redundancy, rank deficiency of the design matrix within the adjustment of indirect observations, detection of multiple observations, number of independent normal equations within the observations and more) the determination of the parameters from the redundant observations is most important.

Within the mathematical idea of the Lp-Norm estimators the method of least squares is central for the determination of the parameters and their related variances.

Let there be an (n,1) size vector $\underline{v}$ of residuals then the norm of this vector

$$N_p(\underline{v}) = \|\underline{v}\|_p = \left(\sum_{i=1}^{n} |v_i|^p \right)^{1/p} \qquad (3)$$

can be minimized due to its value p. For p=1 the least absolute value estimation (LAVE) is obtained, for p=2 the method of least squares (LS) and for p=infinite the so-called MINIMAX method (sometimes called Tschebyscheff-norm). From the statistical point of view the LAVE is a maximum likelihood estimate for random errors following a Laplacian distribution, corresponding the method of least squares for random errors following a normal distribution and the MINIMAX method for random errors following a rectangular distribution.

Within the idea of norm estimation techniques the LAVE technique may be regarded as a so called robust adjustment technique. These numerical procedures are introduced to adjustment and analysis tools to ensure blundered data (so-called outliers) to be detected by the size of their corresponding residual. LS is well known to obtain statistically best results, this means the corresponding parameter's variances become minimal within all Lp-norm estimators. MINIMAX might be applied to reduce the absolute size of the maximum residual to its minimum (among all Lp-Norm estimators). Obviously different adjustment techniques might obtain different results for the parameters. From the statistical point of view these techniques obtain almost the same numerical results for a large amount of data with the random errors following a normal distribution [KAMPMANN, KRAUSE, 1995].

PROMPT applies different numerical strategies for the computations of the parameters and carries out some important numerical checks. Besides the well known normal equation formulations from least squares technique, additional techniques from Linear Programming are introduced. To reduce the amount of computation so called sparse techniques are applied for several purposes. Especially when operating nonregular normal equations within the sparse technology, the so-called datum transformation (S-transformation) of the parameters had to be applied [GREPEL, 1987]. Numerical checks for the accuracy of the computation are carried out.

PROMPT was designed to operate different adjustment models other then the adjustment of indirect observations. The introduction of additional equality constraints for the parameters $\underline{x}$ may be introduced for several Lp-Norm estimators applying the transformation of them into the adjustment model of indirect observations [KAMPMANN, 1992].

3. ADJUSTMENT WITH RANK DEFICIENCY DESIGN MATRICES

Consider the (n,u) size design matrix $\underline{\mathbf{A}}$ with n denoting the number of observations and u denoting the number of parameters to determine and rank $\underline{\mathbf{A}}$ = q = u-r. The integer value r denotes the rank deficiency of the design matrix. If r>0 the matrix of normal equations $(\underline{\mathbf{A}}^T\underline{\mathbf{A}})^{-1}$ is a nonregular (u,u) size matrix with rank $(\underline{\mathbf{A}}^T\underline{\mathbf{A}})^{-1}$ = u-r.

To obtain a unique determination of the (u,1) size vector $\underline{\mathbf{x}}$ of the parameters r equality constraints may be introduced to the least squares adjustment. These constraints are formulated within the (r,u) size matrix $\underline{\mathbf{E}}$ and may be numerically derived from the necessary condition $\underline{\mathbf{A}}\,\underline{\mathbf{E}}^T = \underline{\mathbf{0}}$ where $\underline{\mathbf{0}}$ denotes a (n,r) size zero matrix.

It is well known that the parameters $\underline{\mathbf{x}}$ are calculated that $\underline{\mathbf{E}}$ $\underline{\mathbf{0}} = \underline{\mathbf{o}}$ is a necessary result, where $\underline{\mathbf{o}}$ denotes the (r,1) size zero vector and additionally $\underline{\mathbf{x}}^T \underline{\mathbf{x}} = \min$.

To obtain the matrix $\underline{\mathbf{E}}$ from the sparse matrix $\underline{\mathbf{A}}$ the numerical condition $\underline{\mathbf{A}}\,\underline{\mathbf{E}}^T = \underline{\mathbf{0}}$ is considered for the computation of $\underline{\mathbf{E}}$. To minimize rounding errors and numerical inaccuracies arising from the large amount of data within the photogrammetric application iterations are carried out.

4. COMPUTATIONAL CHECKS WITHIN PROMPT

When dealing with a large amount of data, it is very important to check the adjustment and analysis results with an independent computation. PROMPT was designed to provide high quality results and to insure their accuracy. Some important numerical checks are displayed, such as the well known ANSERMET-Check. Consider the (n,n) size diagonal matrix of weights $\underline{\mathbf{P}}$ (with $p_i > 0.0$) and the (n,n) size unit matrix $\underline{\mathbf{I}}$ then the (n,n) size matrix $(\underline{\mathbf{I}}\text{-}\underline{\mathbf{C}})$ contains the so called observations' redundancies r_i on their diagonal elements

$$(\underline{I} - \underline{C})_{ii} = \left(\underline{I} - \underline{A}\left(\underline{A}^T \underline{P}\underline{A}\right)^{-1} \underline{A}^T \underline{P}\right)_{ii} = r_i \quad (4)$$

Those observations' redundancies may be taken for the determination of the so called inner reliability parameters and for statistical testing like the standardized or studentized residuals.

The ANSERMET check displays the sum of the diagonal elements (trace) of the (n,n) size matrix $\underline{\mathbf{A}}\,(\underline{\mathbf{A}}^T\underline{\mathbf{PA}})^{-1}\,\underline{\mathbf{A}}^T\,\underline{\mathbf{P}}$ that has to equal the rank q of the design matrix $\underline{\mathbf{A}}$. Remember that q is an integer value. For an adjustment with rank deficiency design matrices the numerical checks $\underline{\mathbf{A}}$ $\underline{\mathbf{E}}^T = \underline{\mathbf{0}}$ and $\underline{\mathbf{E}}\,\underline{\mathbf{x}} = \underline{\mathbf{o}}$ are displayed by summing up the absolute sum of the elements of the (n,r) size matrix $\underline{\mathbf{0}}$ and the (r,1) size vector $\underline{\mathbf{o}}$.

These numerical checks are important to control the proper working of the computer's processor and to decide whether to add some iterations to the computations and increase the accuracy.

When calculating the error ellipsis for the object points the following equation may be taken for a numerical check

$$s_x^2 + s_y^2 + s_z^2 = A^2 + B^2 + C^2 \quad (5)$$

where s denotes the standard deviation of the points' coordinate and A,B,C denote the axis of the corresponding error ellipse.

5. PERFORMANCE OF PROMPT

It is the target of PROMPT within the CDW (Close Range Photogrammetric Workstation) of Rollei Fototechnic to provide a most convenient and easy-to-use environment. Using the WINDOWS platform for PC applications the number of observations and parameters may be extended to several thousand.

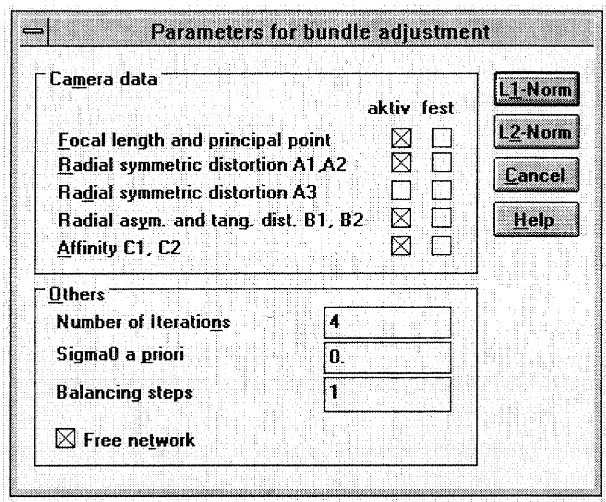

Figure. 2a: Initial PROMPT screens

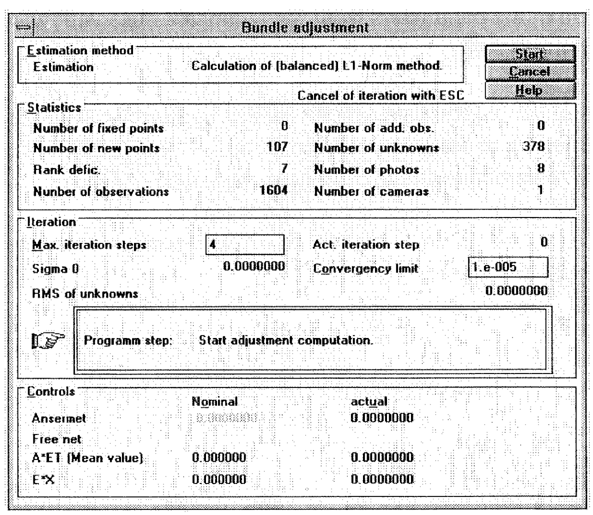

Figure. 2b: Initial PROMPT screens

Graphical tools for the judgment of the results are also integrated (ref. figure 3).

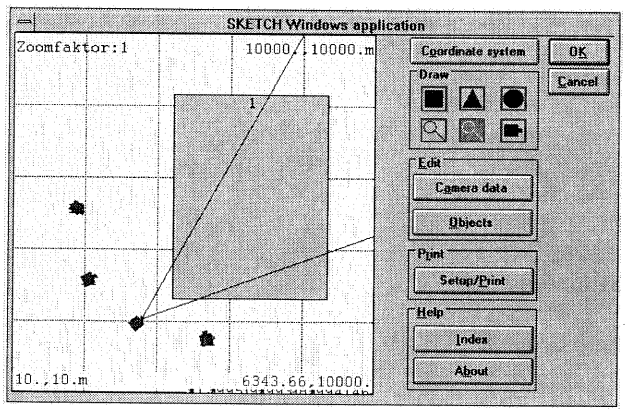

Figure. 3: Graphical representation of camera stations

An extensive analysis of the data is carried out to provide all necessary information. This includes the number of (non-multiple) observations being introduced to the adjustment and error detection using robust parameter estimation techniques as well as initial information. In addition with the implemented above mentioned independent control algorithms a most reliable result is ensured. The userfriendlyness of PROMPT is furthermore increased through the automatic determination of the E matrix for the free network calculation. With the large variety of different models for the calculation of the interior orientation a wide range of applications from low to high accuracy and analogue to digital imagery is covered (ref. figure 2a, 2b).

6. CONCLUSION

After introducing modern parameter estimation techniques in the program NAWE_OPT [ref. FELLBAUM 1994] with PROMPT the complete orientation computation of Rollei Fototechnic are now equipped with these sophisticated algorithms. PROMPT has been designed to operate semi automatically for the close range photogrammetry application. It has been the target to provide the user with any necessary information on the one hand, and on the other hand to avoid any unnecessary or sophisticated operation if it is not necessary. Robust adjustment techniques taking into account initial information ensure against blundered observations. To judge the accuracy of the computations and the results a variety of checks is provided.

With all the mentioned features in connection with the modern WINDOWS environment a powerful and easy to handle bundle adjustment program has been created which will give a wider range of users access to multi image triangulation through bundle adjustment.

7. LITERATURE

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