AIRBORNE GPS

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ABSTRACT:

The paper gives the theory of airborne GPS related to Photogrammetry and the results of a self calibration used to validate the theory. Accordingly, no ground control points are required for mapping using a strip or block of photographs provided the site is within 10 Km of the calibration site.

INTRODUCTION

Global Positioning System (GPS) technology has been developed in recent years such that the position of moving objects can be determined to about $\pm$0.2mm relative accuracy and about $\pm$ 2cm absolute accuracy. The airborne GPS research was conducted at Iowa State University from April 1993 to May 1995. A series of four tests were carried out in St. Louis, Missouri and in Ames, Iowa. All tests, except one, were done in cooperation with Ashtech and Surdex Inc., using Cessna aircraft, LMK 2000 camera and Ashtech receivers. The objective of this research was to use airborne GPS to take aerial photographs at predetermined locations, to determine the best aerial camera location and orientation for mapping.

The research showed that Airborne GPS is feasible. In block triangulation no ground controls are required. In a strip, no ground controls are required provided either the omega angle (rotation about the y axis) of the camera is known or the height difference between 2 or more points in the y direction are known. The research showed that the omega angle of the camera can be determined to an accuracy of 0.0001 radians by a self calibration from the omega angle of the aircraft determined by airborne GPS, provided the calibration site is within 10km of the photographic site.

The objectives of this paper are to describe briefly photogrammetry and kinematic GPS as they relate to airborne GPS, give the summary of the self calibration used to test airborne GPS, and the conclusions and recommendation.

PHOTOGRAMMETRY

In photogrammetry the photo coordinates (x, y) are related to the ground coordinates (x_G,y_G,z_G) by the following equation:

$$x - x_o = f(a11(x_G - x_o) + a12\,(y_G-y_o)+a13(z_G-z_o))/(a31(x_G - x_o) + a32\,(y_G-y_o)+a33(z_G-z_o)) + \text{radial distortion} + \text{decentering distortion} + \text{refraction}$$

$$y-y_o = f(a21(x_G - x_o) + a22\,(y_G-y_o)+a23(z_G-z_o))/(a31(x_G - x_o) + a32\,(y_G-y_o)+a33(z_G-z_o)) + \text{radial distortion} + \text{decentering distortion} + \text{refraction} \quad (1)$$

where x_o, y_o, f are interior orientation elements, (x_o , y_o , z_o) are the nodal point coordinates in the ground coordinates system.

$$\text{and } A = R_k R\phi R\omega = \begin{matrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{matrix}$$

Where $R_K R\phi, R\omega$ are the rotation matrix required to make the photo coordinates axes (x,y,z) parallel to the ground coordinate axes by rotating first about x axis by ω , then about y axis by ϕ and finally about z axis by K. The K,ϕ,ω are known as the orientation angles. The x_o, y_o, z_o, K,ϕ,ω are known as the exterior orientation elements.

The objective of photogrammetry is to determine (x_G, y_G, z_G) of a point from the photo coordinates of two or more photographs. This is done by three methods: Analog, analytical and self calibration.

In the analog method, the interior orientation, radial and decentering distortions are assumed small. The projectors are used to project the images and produce the stereo models. When producing the stereo model, five of the twelve exterior orientation elements are determined by relative orientation. The stereo model is scaled and leveled using external ground control points, thus determining the other seven exterior orientation elements. Special instruments such as Zeiss Z8 are designed to produce the stereo model and then plot the map.

In the analytical method, the photo coordinates are corrected for interior orientation, radial and decentering distortions given by the calibration of the camera. The photo coordinates of two or more photos, together with three or more known ground control are simultaneously adjusted to give the ground coordinates. Software such as "Albany" is capable of such adjustment. Some stereo plotters which are connected to computers for doing these computations in real time and which assist in driving the plotters are known as analytical plotters.

In self calibration, the interior orientation elements, the radial and decentering lens distortion elements, and the exterior orientation elements are simultaneously determined with unknown ground control points using the photo coordinates of two or more photos and a number of ground control points. The method used is normally the least squares constraint method in which any of the parameters are constrained to its known accuracy. The program such as "Calib" is capable of this adjustment.

KINEMATIC GPS

The GPS consists of 24 satellites orbiting about 20,000 kilometers above the earth. The satellites transmit information in two carrier frequencies L_1 and L_2 and modulated by two codes P and C/A code.

Differential GPS tracks the same satellites from two stations. Using the carrier phase frequency, the base line vector can be computed accurately. The accuracy depends on the accuracy of the phase measurement, error due to multipath and the ionospheric error depending on the distance between the two stations. The use of P and C/A code may eliminate the multipath and use of L_1 and L_2 may eliminate the ionospheric error. The receivers, such as the Z12 Ashtech receiver, measures the phase to an accuracy of 0.2 millimeters or better and has the capability of tracking L_1 and L_2 frequencies.

In Kinematic GPS one of the receivers is fixed at the base station and the other is free to move. The phase angle from each satellite is measured continuously. However, only portions of the phase angle less than 2π are measured at one time; hence the receiver has to keep track of the total phase angle, and the integer number of 2π. When a receiver moves, there is a possibility that it may loose track of a satellite and loose the integer number of 2π. Knowing the position of the base receiver and the position of the rover, using the other satellites, it is possible to calculate the lost integer count. The PNAV software is capable of resolving the integer ambiguity on the fly, provided there are more than 7 satellites at a time.

APPLICATION OF KINEMATIC GPS IN PHOTOGRAMMETRY

If a GPS antenna is fixed above the camera nodal point in an aircraft (camera antenna), then its position, (see Fig. 1) determined in real time by kinematic mode, can be used to take aerial photos at predetermined locations. Thus Kinematic GPS is used in pin-point navigation for photogrammetric mapping.

Using differential Kinematic GPS, the camera's location (x_o, y_o, z_o) can be determined precisely. Thus, in a stereo pair, of the 12 exterior orientation elements, six can be determined by Kinematic GPS methods. Five of the exterior elements can be determined by relative orientation and 12th element, $\acute{\omega}$, has to be determined by external ground control.

In a triplet with two photos in the y direction and two photos in the x direction (see Fig. 2), the kinematic GPS can be used to determine 9 exterior orientation elements and the relative orientation to determine the other nine exterior orientation elements.

In an aircraft, if 4 antennas are mounted as shown in Fig. 1 such that the left wing antenna and the right wing antenna is along the y axis of the aircraft, the camera antenna C and the forward antenna F is along the x axis, then the Kinematic GPS can be used to determine the locations of these antennas at the time of the exposure. From the location of the antennas, the rotation angles of the aircraft with respect to the ground system (x_G,y_G,z_G) can be obtained from:

$$\begin{aligned} \sin \acute{\omega}_G &= (Z_r - Z_l)/LR \\ \sin \phi_G &= (Z_f - Z_c)/ FC \\ \sin K_G &= (Y_f - Y_c)/ FC \end{aligned} \tag{2}$$

If R is the rotation matrix which makes the camera axis (x_c,y_c,z_c) parallel to the aircraft axis (x_A,y_A,z_A), then the rotation angles of the camera is given by:

$$A = R\,A'\,R^T \tag{3}$$

where

$$A' = R\,K_G * R\phi_G * R\acute{\omega}_G \text{ and } A = R\,K_c * R\phi_c * R\acute{\omega}_c$$

A' = rotation of the aircraft obtained by GPS
A = rotation of the camera
R( ) = Rotation matrix about z, y or x axis

Thus in an aerial photo all the exterior orientation can be determined by kinematic GPS provided the parameters of the matrix R are determined by calibration. This means that no ground control is required for rectification, stereo plotting, and orthophoto production.

RESULTS OF SELF CALIBRATION

On June 20, 1994, the Cessna aircraft fitted with four L_1/L_2 antennas and a L_1 antenna for navigation, was used to test the airborne GPS concepts (see Fig. 3).

The aircraft was taxied over the Taxi point; the four GPS Z12 receivers were connected to the L_1/L_2 antennas and arranged to collect the data on flight. Two Z12 GPS receivers were set on the nearby reference points Base1 and Base 2.

The flight plan consists of one flight in the East - West direction at a flying height of 3000 feet over the ISU campus, and another over the ISU campus and continuing over the Highway 30 test site at a flying height of 1500 feet (see Fig. 4). The campus site is 3 to 5 kilometers from the airport and the Highway 30 site is about 17 to 30 kilometers from the airport.

The results were smooth and the positions of the antennas with respect to all three references agreed within acceptable limits. Fig.3 shows the location of the left wing, right wing and camera antennas with respect to the Taxi point. The difference between the camera antenna coordinates determined by PNAV when the aircraft is over the Taxi point and the coordinates from control survey is 0.06 meters in x and 0.13 meters in y indicating that the PNAV position determination is accurate and the small difference shows the ability of the pilot to taxi the plane exactly over the Taxi point. The height of the camera antenna above the camera's nodal point given by PNAV and the tape measurement is 1.541 meters which compares with the previous calibrated value of 1.464 meters; the difference is due to the use of a cloth tape for measurement and the lack of knowledge of the exact location of the nodal point.

Using the time, antenna locations, and angles at all times of flight; the angles at camera exposure times are prepared by utilizing a spreadsheet.

For this study, it was sufficient to accept the data with Base 2 as a reference and the interpolated antenna positions given by the PNAV software. Photos 1-3 from flight 1; and photos 4,5,6, and 7 are from flight 2 campus site and photos 8 & 9 are from flight 2 Highway 30 site are used in the analysis .

Table 5 shows that the difference in orientation angles between the photogrammetry and GPS methods were consistent for the campus site in flight 1 and flight 2 and not consistent between campus site and the Highway 30 site in flight 2. This is due to the highway 30 site is about 25 kilometers from Base 2, and lack of good targeted pass and control points.

As discussed earlier, in order to do strip adjustment using airborne GPS without any ground control we need to determine the omega orientation angle by airborne GPS. Previous tests have shown that:

$$\acute{\omega}_p = \acute{\omega}_o + (a \cos K_G + b \sin K_G)\,\acute{\omega}_G + c\,\Phi_G \qquad (4)$$

where $\acute{\omega}_p$ = omega by photogrammetry
$\acute{\omega}_G$ = omega by GPS
$\acute{\omega}_o$ = a constant parameter

The variable Φ_G was added as the camera's swing motion was locked.

Table 6 shows the results of the least squares fit of the above equation using the campus site flight 1 and flight 2 data. The standard error of 0.0005 indicates that the accuracy of $\acute{\omega}$ is better than or equal to 0.0005 radians and is acceptable for highway application using 1500 feet or 500 meters in flying height photos.

Table 6 shows the transformation parameters for transferring $\acute{\omega}_G$ to $\acute{\omega}_p$ obtained from campus site data. They are not suitable for the Highway 30 site.

In order to test the feasibility of using the transformation parameters from the campus site to the Highway 30 site, a combined adjustment of flights 1 and 2 was done using the "Calib" software. By trial and error, a satisfactory solution was found by assigning different weights to interior orientation elements (x_o, y_o, f) (see Table 7), Airborne GPS coordinates (X_c, Y_c, Z_c) and ground control. The parameters from the campus site were used to obtain $\acute{\omega}_p$ from $\acute{\omega}_G$ in the Highway 30 site strip. When these values were used in the Highway 30 site strip adjustment, even without ground control, they gave satisfactory pass point coordinates. This suggests a self calibration for a site (eg. Highway 30) can be used to convert $\acute{\omega}_G$ to $\acute{\omega}_p$.

Table 5 shows the error of (K_p - K_G) is about 0.0005 radians and (ϕ_p - ϕ_G) is about 0.001 radians even though the distance between the camera antenna and the forward antenna is only 1 meter. This suggests that the relative error of GPS coordinates is better than 1 millimeter and that Φ_G and K_G can be used to rectify aerial photos and also produce orthophoto. The error in (K_p - K_G) is better than (ϕ_p - ϕ_G) because the determination of K_p by photogrammetry is more accurate than ϕ_p.

Because of the possibility of small errors in the initial data, steps were taken to refine the ground control, the photogrammetric coordinates, and the GPS data.

The refined data for the nine photos were then adjusted by "Calib". The difference in camera coordinates for the campus site (photos 1-7), see Table 8, clearly show that the airborne GPS coordinates are better than 10 centimeters irrespective of the flight altitude and flights. The error in the z direction of 0.7 meters for the Highway 30 site is probably due to integer ambiguity resolution by the PNAV software because the Highway 30 site is more than 10 kilometers from the reference station Base 2.

Table 5 shows that the difference in orientation angles between GPS and photogrammetry are constant for flight 1 and flight 2 on the campus site. However, the orientation angles from GPS for the Highway 30 and campus site appear to be different. Again, this is because the Highway 30 site is more than 10 kilometers away from the reference station Base 2. This suggests the importance of having the reference station within 10 kilometers of the site or of knowing the elevation difference for two or more points in the y direction perpendicular to the flight to determine the transformation parameter when obtaining $\acute{\omega}_p$ from $\acute{\omega}_G$.

Table 9 shows that the standard error of the fit between $\acute{\omega}_p$, from refined data and $\acute{\omega}_G$ is 0.00008 radians. The accuracy of 0.0001 radians in $\acute{\omega}$ is sufficient for drawing 2 foot contours either from 1500 or 3000 feet flying height photos.

Table 10 shows the difference between $\Delta\acute{\omega}_1 = \acute{\omega}_G - \acute{\omega}_p$ of flight 1 and $\Delta\acute{\omega}_2 = \acute{\omega}_G - \acute{\omega}_p$ of the flight 2. The table also shows the second difference, $\Delta\acute{\omega}_{12} = \Delta\acute{\omega}_1 - \Delta\acute{\omega}_2$. The standard error of $_1\Delta\acute{\omega}$, is 0.00003 radians which agrees with the expected error of 0.00002 for a height difference of 0.2 millimeters at 10 meters apart.

CONCLUSION AND RECOMMENDATION

The airborne GPS is feasible. The coordinates of the camera antenna can be determined with an accuracy better than ± 10 centimeters or better provided the base reference station is within 10 kilometers of the photographic site. This is acceptable for mapping at all scales.

The PNAV software resolves the integer ambiguity satisfactorily for fast static computation provided the rover receiver is within 10 kilometers of the base station.

Camera, wings and foresight are suitable for antenna location. However the tail is not. The motion of the left and wing antennas are symmetrical and can be used for computing the angle of rotation.

The accuracy of the Z12 GPS receiver is 0.2 millimeters and the noise due to multipath at the camera, foresight and wing locations is negligible. The accuracy of the ω obtained from left and right wing antennas at a separation of 10 meters is better than ±0.0001 radians. This is acceptable for 2 foot contours using 3000 feet or lower flying heights.

For a block with more than one strip, no ground control is required. The base station has to be within 10 kilometers of the block and local geoid undulation must be applied to the elevations.

For a strip, self calibration is required for transferring ω_G to ω_p. This calibration is valid for projects within 10 kilometers. In order to determine the true parameters which will give true ground values, the self calibration has to be designed to eliminate linear dependency between interior orientation elements and exterior orientation elements, as well as within the interior orientation elements. This can be accomplished by a self calibration using Airborne GPS with a minimum quadruplet of photos (see Fig. 11); two in the direction of flight, one perpendicular to the flight and another at low altitude flying height. This self calibration can be done every 10 Km along the strip. Since no additional targeting is required and only observation in additional two photos for every 10 Km is required the method is economically feasible.

Further research is required to obtain ω_p from ω_G with accuracy of ±0.00002 radians. GPS is capable of providing this accuracy.

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References:

1. Ashtech Inc. Prism II Advanced Software Modules, Sunnyvale, California, September, 1994.
2. Jeyapalan, k., Photogrammetry, Encyclopedia of the Earth Science, volume 3, Academic Press Inc., 1992.
3. Jeyapalan, K., et al., Use of GPS for Photogrammetry, Iowa State University, Ames, Iowa, 1992.
4. Jeyapalan, k., et al., Airborne GPS, Iowa State University, Ames, Iowa, 1995.
5. Lapine, L.A., NOAA Tests Kinematic GPS, ACSM Bulletin 12-14, August, 1990.
6. Merchant, Dean C, Airborne GPS-Photogrammetry for Transportation Systems, ACSM-ASPRS, Annual Convention, Reno, Nevada, 1994.
7. Thompson, E.H., Introduction to the Algebra of Matrices with some applications, The University of Toronto Press, 1969.

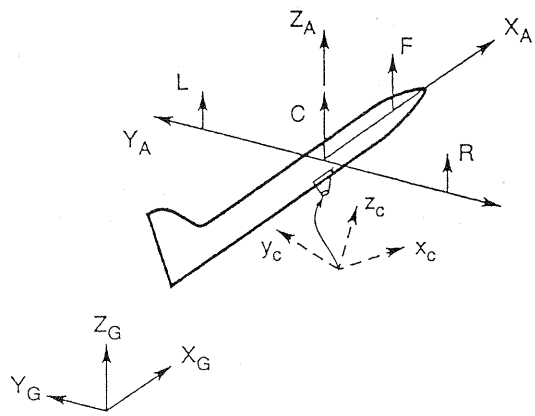

Figure 1. Multiantenna locations.

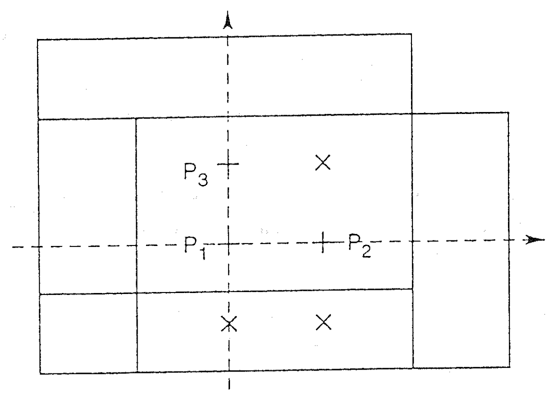

Figure 2. Triplet

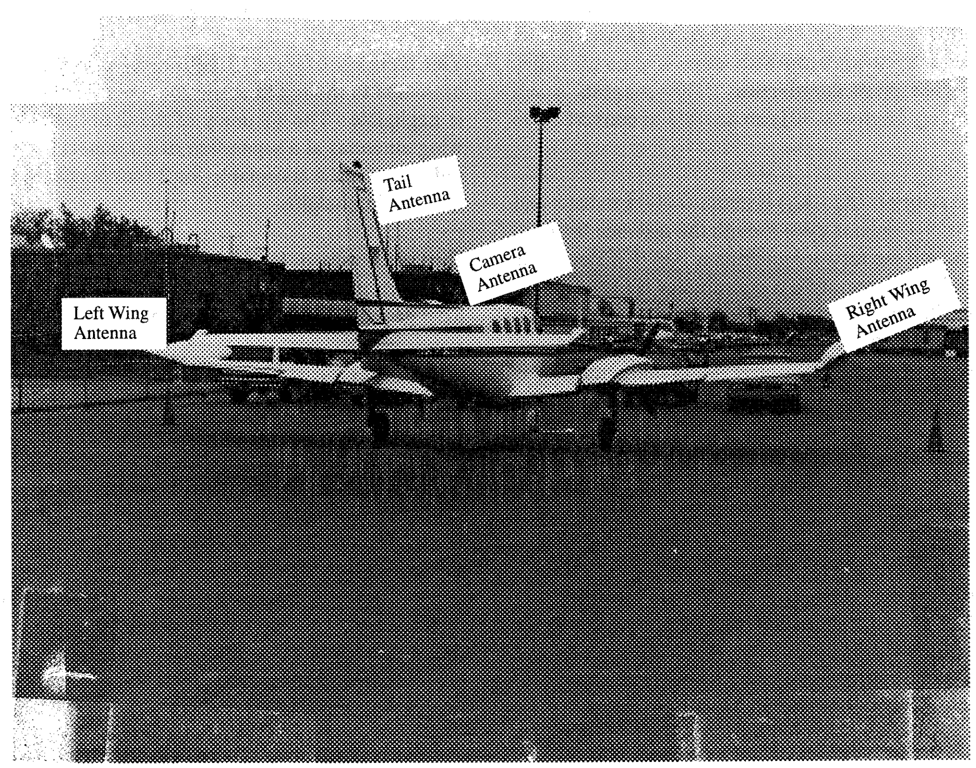

Figure 3. Aircraft with four antennas.

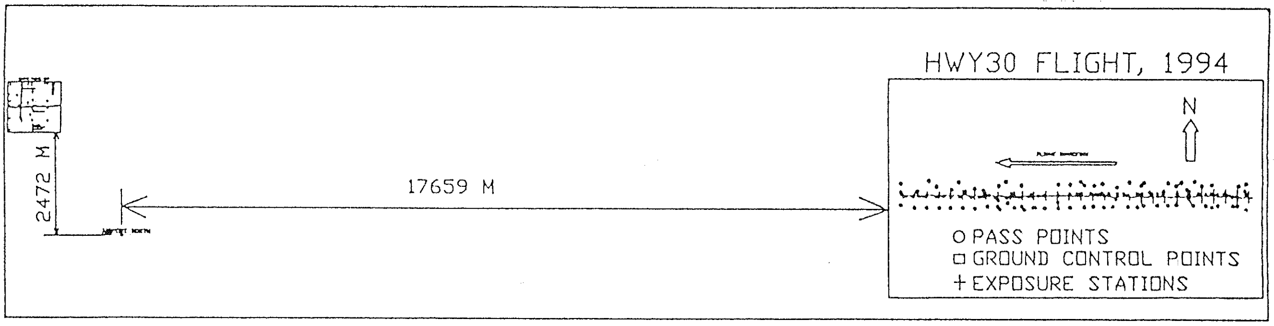

Figure 4. Project 1994.

	ω	φ	κ
1	0.0045	0.0151	0.0032
2	0.0053	0.0130	0.0024
3	0.0037	0.0117	0.0019
Std. mean	0.0045	0.0133	0.0025
Std. error	0.0006	0.0014	0.0005
4	0.0055	0.0168	0.0076
5	0.0071	0.0255	-0.0117
6	0.0054	0.0177	-0.0010
7	0.0036	0.0188	+0.0089
Std. mean	0.0055	0.0197	0.00094
Std. error	0.0012	0.0034	0.0082
8	0.0116	0.0027	-0.0061
9	0.0118	0.0029	-0.0057
Std. mean	0.0117	0.0028	-0.0059
Std. error	0.0001	0.0001	0.0003

Table 5. GPS-Photo orientation.

```
INPUT DATA ARE: (OMEGAp,OMEGAg,KAPPAg, PHIg)

   OMEGAp        OMEGAg        KAPPAg         PHIg

 0.043108000   0.047640007   0.075824965   -0.061304218
 0.042177000   0.047439621   0.044556999   -0.060995646
 0.015774000   0.019469214   0.041470049   -0.057373606
 0.036485000   0.042290545   0.045362608   -0.030498599
 0.047588000   0.054703783   0.023387758   -0.023387548
 0.011352000   0.016753616   0.052766161   -0.040786984
 0.007231000   0.010923818   0.067033636   -0.033132291

FORMULA IS OMEGAp-OMEGAg=OMEGAo+OMEGAg(a*COS(KAPPAg)+b*SIN(KAPPAg))+d*PHIg

-0.0048850666  -0.0641132373   0.4197135928  -0.0300743669

THESE ARE THE ERRORS, (COMPUTED - REAL)

  -0.000040      0.000060     -0.000373     -0.000066     -0.000035
   0.001041     -0.000587

THE STANDARD DEVIATION IS
   0.000513
```

Table 6. Results of combination of high and low flights.

Weight on photo coordinates = 5000

Standard error on ground contact = 0.01m

Standard error on Airborne GPS (low flight)
X_G, Y_G = 0.01m
Z_G = 0.001m

Standard error on Airborne GPS (high flight)
X_G, Y_G, Z_G = 0.01m

```
FORMULA IS OMEGAp=OMEGAo+OMEGAg(a*COS(KAPPAg)+b*SIN(KAPPAg))+d*phi

 -0.0107795884   0.8991249682   0.7631507940  -0.0508904326

 THESE ARE THE OMEGAp:

 -0.0128428  -0.0205923  -0.0156900  -0.0372186  -0.0150666  -0.0230069
 -0.0091735  -0.0128985   0.0087594  -0.0011218  -0.0134105  -0.0244845
 -0.0076987  -0.0044199  -0.0056232  -0.0213606  -0.0143648  -0.0108987
  0.0026834  -0.0073671  -0.0023437  -0.0119827  -0.0178356  -0.0114358
 -0.0118249  -0.0080444  -0.0273579  -0.0311399  -0.0261177  -0.0109928
```

Table 7. ω_G to w_p using different weights.

	X	Y	Z
1	-0.081	0.277	0.008
2	0.305	0.205	-0.148
3	0.005	0.491	-0.167
Mean	0.076	0.324	-0.102
Std. Error	0.165	0.120	0.078
4	0.048	0.343	0.258
5	-0.004	0.1	0.081
6	-0.099	-0.204	0.046
7	0.059	-0.359	0.131
Mean	0.001	-0.03	0.124
Std. Error	0.062	0.27	0.080
8	-0.243	0.286	-0.916
9	0.467	0.249	-0.865
Mean	0.112	0.267	-0.891
Std. Error	0.335	0.018	0.025

Table 8. GPS-Photo locations using refined data.

INPUT DATA ARE: (OMEGAp,OMEGAg,KAPPAg, PHIg,TIME)

OMEGAp	OMEGAg	KAPPAg	PHIg	SECONDS
0.042392	0.047640	0.075825	-0.061304	0.570116
0.041186	0.047440	0.044557	-0.060996	6.775577
0.014783	0.019469	0.041470	-0.057374	12.882438
0.036995	0.042291	0.045363	-0.030499	1457.599484
0.048136	0.054704	0.023388	-0.023388	1460.871496
0.012002	0.016754	0.052766	-0.040787	1464.134522
0.007390	0.010924	0.067034	-0.033132	1467.263800

FORMULA: OMEGAp-OMEGAg=OMEGAo+OMEGAg(a*COS(KAPPAg)+b*SIN(KAPPAg))+d*phi+E*Kp+F*T

0.0000550947 -0.0763984723 0.5237227962 0.0729936964
0.0129816652 -0.0000011097

THESE ARE THE ERRORS, (COMPUTED - REAL)

0.0000727 -0.0000867 0.0000140 -0.0001274 0.0000901 0.0000744
-0.0000370

THE STANDARD ERROR IS
0.0000859

Table 9. ω_G to ω_p using refined data.

Flight	Photo	Omega GPS-Photo (radians)	Average	Difference 1st & 2nd	Average
	1	0.005242			
1	2	0.006254	0.005396		
	3	0.004686			
	4	0.005296		0.000048	
2	5	0.006568	0.0050375	0.000314	0.000142
	6	0.004752		0.000066	
	7	0.003535			

Table 10. First and second difference in $\omega_p - \omega_G$

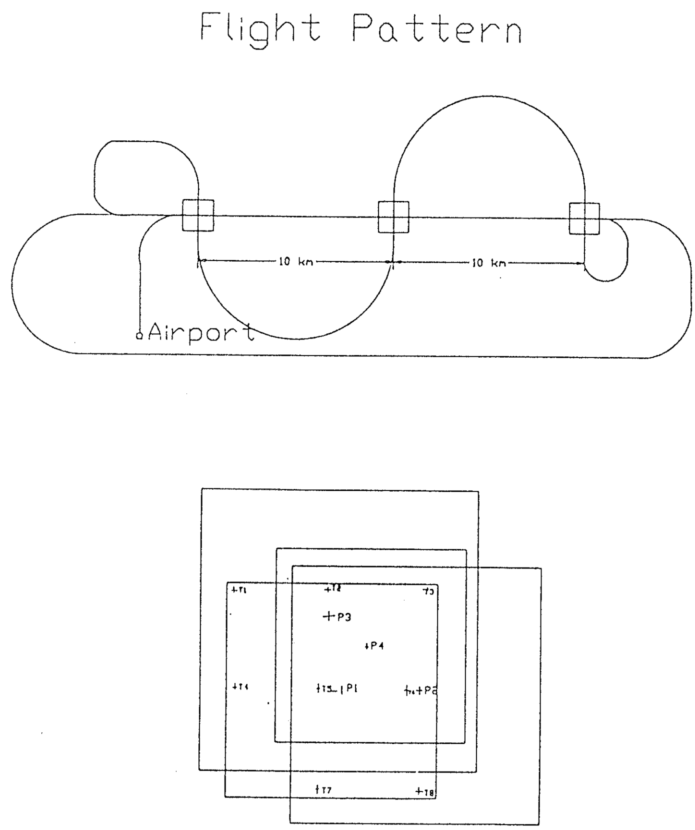

Figure 11. Flight Pattern