# FEATURE-BASED PHOTOGRAMMETRIC AND INVARIANCE TECHNIQUES FOR OBJECT RECONSTRUCTION

H. F. Barakat, K. Weerawong, and Edward M. Mikhail, Purdue University

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## ABSTRACT

Linear features, with independent descriptors in the object space, are affectively used in photogrammetric restitution and object reconstruction. Point-based invariance is discussed and its applications in Image Understanding/Computer Vision are contrasted with photogrammetry. Object reconstruction, being a common invariance and photogrammetric task is evaluated by both techniques using synthetic and real image data. Research is continuing on multi-image invariance, multi-feature construction, and combined invariance/photogrammetry techniques.

## 1. INTRODUCTION

Imagery used to reconstruct the objects recorded is in general two-dimensional representation of usually three-dimensional objects. Image features are of three types: points, lines and areas. Until recently, photogrammetric methodology has been based primarily on point features, particularly because of extensive use of hard-copy image input. The increased use of digital imagery has opened up opportunities for exploiting linear features since they are both abundant in imagery of human infrastructure, and amenable to extraction by automated algorithms. The inclusion of linear features, alone or in combination with point features, into photogrammetric reduction algorithms requires careful development and analysis.

Image invariance refers to the existence of properties, derived from images, which are invariant under specific imaging geometry, the most common of which is central or perspective projection. One very early property used in graphical rectification is the anharmonic or cross-ratio. In recent years, activities in Image Understanding (IU) and computer Vision (CV) has resulted in significant development in image invariance. As in photogrammetric research, point-based development preceded line-based invariance. Although IU/CV applications of invariance encompass different tasks, object reconstruction from overlapping imagery is an application which is also common to photogrammetry.

Feature-based, particularly linear features, photogrammetric techniques for the reconstruction of imaged three-dimensional objects is discussed in section 2. A brief introduction to the invariance concept and its uses is given in section 3. Results from experiments using both simulated and real imagery are provided for both techniques. Conclusions and continuing research are in the fourth and last section.

## 2. LINE-BASED PHOTOGRAMMETRIC RESTITUTION

### 2.1 Linear Feature Description

A point feature is represented by two coordinates in 2D space and three coordinates in 3D space. Linear features can be similarly described. Considering straight lines, they are defined by two parameters in 2D space, and four independent parameters in 3D space, expressed by equations (2.1) and (2.2), where $p$ (the distance from the origin to the line) and $\alpha$ (its angle with the $x$-axis) are the 2 parameters in 2D; $q$ (the distance from the origin to the line), and $\beta_1, \beta_2, \beta_3$ (angles effecting rotation such that the line is along one coordinate axis) are the 4 parameters in 3D. A circle is defined by 3 parameters in 2D, and 6 independent parameters in 3D, and represented by equations (2.3) and (2.4) in which $x_c, y_c$ are coordinates of its center in the plane, and $r$ its radius; $X_c, Y_c, Z_c$ are coordinates of the center; $R$ its radius, and $\alpha_1, \alpha_2$ are the angles defining the unit vector $\vec{p}$ perpendicular to the plane of the circle in 3D.

$$x_i \cos\alpha + y_i \sin\alpha = p \tag{2.1}$$

$$\begin{bmatrix}X_i\\Y_i\\Z_i\end{bmatrix}=\lambda_i\begin{bmatrix}\sin\beta_1\sin\beta_3-\cos\beta_1\sin\beta_2\cos\beta_3\\\cos\beta_1\sin\beta_3+\sin\beta_1\sin\beta_2\cos\beta_3\\\cos\beta_2\cos\beta_3\end{bmatrix}+q\begin{bmatrix}\cos\beta_1\cos\beta_2\\-\sin\beta_1\cos\beta_2\\\sin\beta_2\end{bmatrix} \tag{2.2}$$

$$(x_i-x_c)^2+(y_i-y_c)^2=r^2 \tag{2.3}$$

$$\vec{V}\cdot\vec{V}=R^2 \tag{2.4}$$

$$\vec{V}\cdot\vec{p}=0 \tag{2.5}$$

$$\vec{V}=[(X_i-X_c)\ \ (Y_i-Y_c)\ \ (Z_i-Z_c)] \tag{2.6}$$

$$\vec{p}=[\cos\alpha_1\cos\alpha_2\ \ \sin\alpha_1\cos\alpha_2\ \ \sin\alpha_2]^T \tag{2.7}$$

## 2.2 Geometric Constraints

One of the rich sources of information when dealing with linear features is the existence of various types of geometric constraints in the object space among such features. These are of two types: one providing *relative information*, such as parallel, perpendicular, coplanar, etc., and the other *partial absolute information* with respect to the reference coordinate system, such as horizontal, vertical, etc., features. Constraints **among straight lines** include: *relative*: 2 parallel lines (2 Equations); 2 perpendicular lines (1 Eq.) 2 coplanar lines (1 Eq.); *partial absolute*: line parallel to $X$-, $Y$-, or $Z$-axis (vertical) each provides 2 Eq., horizontal line (1 Eq.). Constraints **among circular features** include: *relative*: 2 parallel circles (2 Eq.); 2 coplanar circles (3 Eq.); 2 circles in perpendicular planes (1 Eq.); *partial absolute*: circle in ***XY*** (horizontal), ***YZ***, or ***ZX*** planes, each provides 3 constraint equations. Constraints **between straight lines and circles** include: 1 line coplanar with 1 circle (2 Eq.); 1 line perpendicular to circle plane (2 Eq.), 1 line passing through circle center (2 Eq.), all of these provide relative information.

## 2.3 Photogrammetric Conditions

Classical photogrammetric condition equations were all derived on the basis of point features and therefore need to be re-developed for linear features. Each type of linear features requires a suitable form. For a straight line feature, an equivalent *pair* of collinearity equations relating the line image parameters, $p,\alpha$ to its object descriptors, $q,\beta_1,\beta_2,\beta_3$:

$$\begin{aligned}&(Dm_{31}+f\cos\alpha m_{11}+f\sin\alpha m_{21})(m_{\beta 11}q-X_L)\\&+(Dm_{32}+f\cos\alpha m_{12}+f\sin\alpha m_{22})(m_{\beta 21}q-Y_L)\\&+(Dm_{33}+f\cos\alpha m_{13}+f\sin\alpha m_{23})(m_{\beta 31}q-Z_L)=0\end{aligned} \tag{2.8}$$

$$\begin{aligned}&(Dm_{31}+f\cos\alpha m_{11}+f\sin\alpha m_{21})\\&+(Dm_{32}+f\cos\alpha m_{12}+f\sin\alpha m_{22})\\&+(Dm_{33}+f\cos\alpha m_{13}+f\sin\alpha m_{23})=0\end{aligned} \tag{2.9}$$

$$D=p-x_0\cos\alpha-y_0\sin\alpha \tag{2.10}$$

in which $m_{ij}$ are elements of the image orientation matrix, $m\beta_{ij}$ are elements of the line rotation matrix. For a circular feature in the object space, the collinearity condition reduces to a single equation for each image

points, $i,j$, on its image. The image vector is

$$\begin{bmatrix}\rho_x\\\rho_y\\\rho_z\end{bmatrix}=M^T\begin{bmatrix}x_i-x_0\\y_i-y_0\\-f\end{bmatrix} \tag{2.11}$$

and the condition equation is

$$\vec{U}\cdot\vec{U}=R^2 \tag{2.12}$$

$$\vec{U}=\begin{bmatrix}X_L-X_c\\Y_L-Y_c\\Z_L-Z_c\end{bmatrix}-\frac{p_x(X_L-X_c)+p_y(Y_L-Y_c)+p_z(Z_L-Z_c)}{p_x\rho_x+p_y\rho_y+p_z\rho_z}\begin{bmatrix}\rho_x\\\rho_y\\\rho_z\end{bmatrix} \tag{2.13}$$

in which $x_0,y_0,f,X_L,Y_L,Z_L,M$ represent the interior (IO) and exterior (EO) image parameters, $X_c,Y_c,Z_c,R,p_x,p_y,p_z$ (elements of $\vec{p}$, see Eq. (2.7)) the circle descriptors in the object space.

## 2.4 Line-Based Photogrammetric Operations

Line features, like point features, may be used as *pass* and *control* features. Therefore, all photogrammetric operations executed with point features can similarly be performed on the basis of linear features. Here are examples:

*Resection*: 3 control straight lines **or** 2 control circles are the minimum required to estimate the six exterior orientation elements of a single photograph. If the interior orientation elements are to be also recovered, two additional control straight lines would be required for a minimum solution. Combination of features and more than the minimum control may be used.

*Relative Orientation (RO)*: Pass straight lines **do not** contribute to RO of a pair. For a triplet, however, a pass line in 3 images contributes 2 equations to RO. A pass circle in 2-image overlap contributes 4 equations, and in 3-image adds 9 equations.

*Extended Relative Orientation (ERO)*: The coplanarity condition of the base vector $\vec{B}$, and two image vectors $\vec{\rho}_L, \vec{\rho}_R$ is given by:

$$\vec{B}\cdot(\vec{\rho}_L \times \vec{\rho}_R) = 0 \qquad (2.14)$$

Alternatively, it is given by

$$[x\ y\ 1]_L\ I_L^T M_L K M_R^T I_R \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_R = 0 \qquad (2.15)$$

in which

$$K = \begin{bmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{bmatrix} \qquad (2.16)$$

$$I_{L,R} = \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & -f \end{bmatrix}_{L,R} \qquad (2.17)$$

$E = M_L K M_R^T$ is called the *essential matrix* and is used for calibrated cameras when $x_0, y_0, f$ are known, while $F = I_L^T M_L K M_R^T I_R$ is called the *fundamental matrix* and is used for uncalibrated cameras. Since the rank of $K$ is 2, $|F| = 0$ (Barakat, 1994). Further, the 9 elements of $F$ are recoverable to a scalar multiple, hence the maximum number of independent parameters in $F$ is 7. Consequently, ERO of a stereopair can only recover 2 IO elements in addition to the classical 5 EO elements.

*Partial Absolute Orientation* For complete absolute orientation (AO) of a relatively oriented stereomodel, control linear features are needed. Each control straight line contributes four independent equations to the recovery of the 7 parameters of AO. Therefore, a minimum of 2 non-coplanar such lines is required. Frequently, no "control" lines may be available, and instead geometric constraints which yield partial absolute information exist. These may then be used to recover additional rotational elements depending upon the available constraints (horizontal or vertical lines, etc.).

*Block Adjustment*: This is the general method which when based on unified least squares and carries all the parameters and constraints as a priori weighted information, can be used to perform any of the operations discussed separately above.

### 2.5 Experiments and Results

A large number of experiments, with both simulated and real data, were conducted to test the developed mathematical models and study the effectiveness of the exploitation of linear features in photogrammetric applications. The results are:

**Case A - Simulated Data**: A pair of photographs with strong convergent geometry were simulated such that the set of perfect image coordinates were perturbed with errors having $\sigma_x = \sigma_y = 0.01\,mm$. Two experiments were conducted using the two photo block triangulation. Experiment #S1 is a regular two photo block adjustment, which recovers 12 exterior orientation parameters, using 10 control lines and 10 pass lines. Experiment #S2 attempts to recover both interior and exterior, 18, orientation parameters of the two photos using 10 control and 10 pass lines. Tables 1 and 2 list the RMS for $dX, dY, dZ$ computed at 5 points on each pass line for experiments S1 and S2 respectively. For each point; $dX, dY, dZ$ are the differences between $X, Y, Z$ computed using the a priori known line descriptors $(q, \beta_1, \beta_2, \beta_3)$ and $X, Y, Z$ computed using their estimated values after the block adjustment.

**Case B - Real Data (Bangor Imagery)**: The data set consists of two nearly vertical aerial photographs flown over an urban area in Bangor, Maine, at a scale of about 1:8660. Regular two photo block triangulation (i.e. solving for 12 parameters) was performed using 6 control lines and 9 pass lines. Table 3 lists the differences in the camera parameters between the original and the recovered parameters while table 4 lists the RMS for $dX, dY, dZ$ computed at 5 points on each pass line. The results show the applicability of using lines in the two photo block to recover both the camera and pass feature parameters.

## 3. INVARIANCE-BASED OBJECT RECONSTRUCTION

### 3.1 Invariance Versus Photogrammetry

Image invariance theory is based on a premise which is fundamentally different from photogrammetric theory. Image invariance deals with invariant quantities under perspective projection (transformation). The cross-ratio is the classic invariant of the projective line. For four points on a line, under projective transformation, the ratio of ratios of distances is invariant. In most photogrammetric activities, very careful modeling of the sensor elements as well as imagery acquisition parameters is central to the techniques used. By contrast, image invariance is almost totally built on the opposite thesis, it does not require knowledge about such parameters and relies instead on invariant properties derived directly from the overlapping imagery. Potential gain may be expected from analyzing these two different theories, establishing their relationships, and seeking a hybrid approach which maximizes the contribution of each. A hybrid approach may lead to improved techniques for object reconstruction with rigorous propagation of quality measures for a variety of imaging systems.

### 3.2 Invariance Applications in IU/CV

The central theme of CV is to achieve human level capability in the extraction of information from imagery for such applications as object recognition, navigation, and object modeling (Hartley, 1993). By contrast, the primary goal of photogrammetry is accurate reconstruction of 3D object from overlapping imagery. Thus, object model construction is a common goal of both IU/CV and photogrammetry, in which invariance plays a role. Other IU/CV applications of invariance include (Zisserman, 1995); (1) Image and object feature transfer for 2D objects; (2) Model based object recognition: given a perspective image of a scene, the task of model based vision is to identify which objects if any, from the model library, are in the scene; (3) Epipolar Geometry: a point in one image determines a line in the other on which the corresponding point must lie. This reduces the correspondence (matching) problem to 1D, rather than 2D search. (Used also extensively in photogrammetry); (4) Transfer (image transfer for 3D objects): given two images of a 3D structure, points in a new image are determined, given only a small number of point correspondences. This is accomplished without reconstructing the 3D structure, nor knowing the camera parameters or motion; (5) 3D structure recovery (3D object reconstruction): recovering non-euclidean 3D structure given only corresponding image points in a stereo pair of views. Using control points, the object is reconstructed in 3D euclidean space. (Main application in photogrammetry).

### 3.3 Photogrammetric Analysis of Invariance

Invariance is based on the same mathematical principles as photogrammetric theory. Therefore, one would expect that invariance techniques would have equivalents in photogrammetry. Such techniques, which we analyzed, include point- and line-based image and object transfer for 2D planar objects (Barakat, 1994). Invariance yields equations of straight lines the intersections of which give the positions of the points to be transferred. For non-redundant 4-point invariance, the *sequence* of points used yields line pairs of different geometric strengths. In redundant cases, using different point sequences to form *linear* condition equations results in least squares estimates which are different for *both the positions and their quality.* Corresponding photogrammetric techniques (which implement projective transformation between planes) based on point and line features, on the other hand, provide unique estimates and covariances for both non-redundant and redundant cases. A refined least squares approach, for which the linear invariance equations become non-linear, appears to alleviate the non-uniqueness problem.

Next, point-based image invariance is investigated for three-dimensional objects in multiple images; in particular the use of the *fundamental matrix* to transfer images from two photographs to a third. Introducing the constraint of zero determinant on the fundamental matrix stabilizes the solution, which otherwise leads to widely varying results. Accurate recovery of $\boldsymbol{F}$ is quite critical as will be discussed also in object reconstruction in the following section.

### 3.4 Object Reconstruction By Invariance

In the derivation of invariance relationships for image transfer, object coordinates are eliminated and the image acquisition parameters are usually lumped together and replaced by other nonphysically significant parameters such as the fundamental matrix. In an alternative derivation, algebraic elimination of the camera orientation parameters from the equations results in invariant coordinates of the object points. These coordinates are identical from any two images of the object, provided that 5 control points, not any four of which lie in a plane, are identified in both images. The 3-D object is, then, reconstructed from the invariant coordinates using a cross-ratio of determinants in a similar approach to the 2-D (planar object) case.

According to Barrett (Barrett, 1994) the method is explained as follows. Two points are selected, e.g. $P_1$, and $P_2$, and the line passing through them becomes the "spine" of a "pencil" of three planes; $P_1P_2P_3$, $P_1P_2P_4$, and $P_1P_2P_5$, as shown in the figure. For any other general object point, $P$, a fourth plane in this pencil is constructed, $P_1P_2P$. Then, the cross-ratio of these four planes is computed as the first invariant coordinate of $P$; $C_1(P)$. The procedure is repeated for two other choices of the "spine" of the pencil, e.g., $P_2P_3$ and $P_1P_3$. The resulting set of cross-ratios of planes; $C_1(P), C_2(P), C_3(P)$ provides invariant properties of the three planes in space hinged on the spines $P_1P_2, P_2P_3$ and $P_1P_3$. These three planes intersect at the general point $P$, whose object coordinates are thus calculated.

It is clear that in this case we also have many possible combinations and sequences. Results show that the uniqueness problem exists here as it does with the 2D case.

The original method by Barrett utilizes linear equations, avoiding the need for initial approximations, and uses the minimum (non-redundant) number of 5 control (Basis) points. As in the case of 2D invariance, discussed in our previous work (Barakat, 1995), the uniqueness problem exists. Different combinations/sequences of control (Basis) points lead to different results.

In our modification, a refined least squares technique, which allows for iteration on the observables as well as on the unknown parameters, was applied to alleviate the sequence problem. In addition, the use of redundant number of control points is introduced which significantly improves the results. Also, in view of our previous work the constrained least squares technique for the estimation of the fundamental matrix was implemented to get more accurate results than the original linear estimation of $\boldsymbol{F}$.

The improvements in the results due to the modification of the original method, are presented and discussed in the following section. Because object coordinates are involved together with image coordinates, the photogrammetric equivalent to this invariance task is in general two-photo block triangulation. Since in invariance no information is assumed with regard to the sensor, all 18 I.O. and E.O. acquisition parameters must be assumed to be unknown. Five control points yield 20 collinearity equations, and 8 pass points yield 8 coplanarity equations, thus a redundancy of 10 will exist for the equivalent invariance unique case. If the 5 control points are taken as a subset of the 8 pass points a redundancy of 5 still remains. The following sections presents comparative results of both approaches.

### 3.5 Experimentation and Results

Extensive experimentation has been performed employing the procedure described above for object reconstruction using invariance, and comparisons were made with the equivalent photogrammetric technique. The results of this experimentation are summarized in the following cases.

**Case A - Simulated Data:** Two pairs of photographs, one with convergent geometry and the other with normal vertical geometry, were simulated such that the set of perfect image coordinates were perturbed with errors having $\sigma_x = \sigma_y = 0.01\,mm$. Six well distributed control points and 16 object check points were used for object reconstruction. Table 5 summarizes the rms of $dX, dY, dZ$ for the original and the modified invariance and the 2 photo block. It is very clear that the modified method results are superior to those of the original method especially for the convergent (C) case. In the convergent geometry case, the 11 extended relative orientation parameters for the two photos (18-7=11) are distinct and have significant values. Therefore, lumping those 11 parameters into 7 recoverable elements of the fundamental matrix affects the solution and requires more accurate estimation of the fundamental matrix as implemented in the modified method. For the normal vertical geometry case, the number of well defined camera parameters is smaller than that of the convergent case. The 7 independent elements of the fundamental matrix can more easily recover those camera parameters for this geometry, as can be seen in the small amount of improvement between the original and the modified methods. It is important to note that, as in the case of 2D invariance, the control points configuration and the location of the check points have significant influence on the quality of the results. All subsets of 4 points, out of the total 7 points (6 control + 1 check), should be checked not being close to falling in a plane. The main advantage of the invariance technique, besides that no knowledge is required for the image acquisition parameters, is that no approximations for the ground coordinates of the check points are required.

**Case B - Real Data (Purdue Campus Imagery):** The modified method was applied on a pair of real vertical images flown over the Purdue campus, at a scale of 1:4000. The equivalent photogrammetric technique was performed using the same data set. Table 6 lists the rms of $dX, dY, dZ$ for 20 check points inside and around the border of the control points frame, where $dX, dY, dZ$ are the differences between the estimated coordinates and the known measured coordinates. Both invariance and photogrammetry worked equally well because of the well distributed control points and the location of the check points. The most significant conclusion from this experiment is the importance and sensitivity of the estimation of the fundamental, $\boldsymbol{F}$, matrix and its effect on the success of the invariance method. All subsets of 7 points out of the total number of points used to estimate the $\boldsymbol{F}$ matrix should have different $\boldsymbol{Z}$ values so that they are not close to being on a plane. This is even more important than having different $\boldsymbol{Z}$ values for the control points on the quality of the obtained results.

**Case C - Real Data (Bangor Imagery):** The data set is described in Case B in section 2.5. Table 7 lists the rms of $dX, dY, dZ$ of check points using both modified invariance and photogrammetric methods. Six well distributed control points were selected along the model perimeter (the overlap area of the two photos) with the 11 check points both inside and on the border defined by the control points. Improved results were obtained by photogrammetry, while invariance performed less accurately due to the location of the check points relative to the control points frame. It is clear that the invariance method works better if the check points are confined with in and closer to the control points frame.

## 4. CONCLUSIONS AND CONTINUING RESEARCH

1. Linear image features are significant source of information which when properly exploited facilitate three-dimensional object reconstruction, since they are abundant in human-made infrastructure, and are amenable to automated feature extraction.
2. Geometric constraints between various linear features provide substantial information in support of photogrammetric restitution and object reconstruction, both in absolute and partially absolute sense.
3. Feature recovery by photogrammetric techniques (triangulation or extended relative orientation) is accurate, even though the recovery of the interior orientation parameters may not be accurate, due to projective compensation.
4. Invariance provides a useful tool for object reconstruction, particularly since it does not require approximate values.
5. The point sequence used to construct the invariance equations can have a significant influence on the results, particularly for the redundant case where position estimates and their quality vary. A refined least squares approach, which requires linearization of the equations appears to alleviate this non-uniqueness problem.
6. It is crucial that the fundamental matrix, $\boldsymbol{F}$, be well recovered for the success of invariance technique, especially for the convergent geometry case. Furthermore, the configuration of control and check object points is rather critical to the quality of the results. Points used for both the estimation of $\boldsymbol{F}$ and as control points should not fall close to a plane.

Research is continuing on the following:

a. Experimentation to study the effects of various configuration of the ground points (both control and check) and the different camera geometry on the performance of the invariance technique.
b. Extension of the invariance technique to apply to multiple overlapping photos.
c. Investigate the line-based and combined point/line-based invariance techniques for object reconstruction.
d. Study the possibility of developing a hybrid approach combining invariance and photogrammetry for object reconstruction.

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## 6. REFERENCES AND BIBLIOGRAPHY

Barakat, H., Weerawong, K., and Mikhail, E.M., 1995. Comparison Between Invariance and Photogrammetry for Image and Object Transfer. SPIE, Orlando, FL, Vol. 2486.

Barakat, H., Doucette, P., and Mikhail, E.M., 1994. Photogrammetric Analysis of Image invariance. ISPRS Commission III, Munich, Germany, pp. 25-34.

Barrett, E., Gheen, G., and Payton, P., 1994. Algorithms for Invariant Model Transfer and Object Reconstruction. IU Workshop, Monterey, CA, Vol. II, pp 1429-1440.

Hartley, R., and Mundy. J., 1993. The Relationship Between Photogrammetry and Computer Vision. SPIE, Vol. 1944, pp. 92-105.

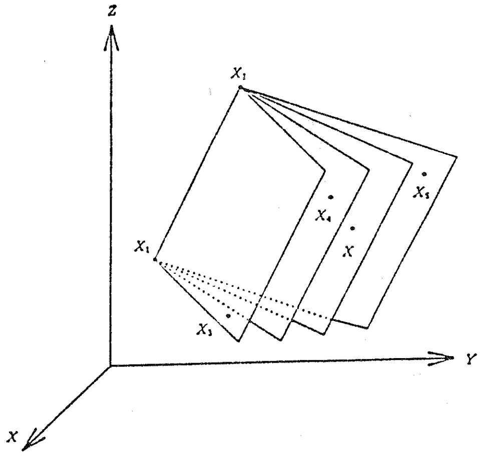
Mikhail, E.M., and Weerawong, K., 1994. Feature-Based Photogrammetric object construction. ASPRS, Reno, Nevada.

Mikhail, E.M., 1993. Linear Features for Photogrammetric restitution and Object completion, SPIE, Vol. 1944, pp. 16-30.

Mikhail, E.M., 1976. Observations and Least Squares. University Press of America, New York.

Mulawa, D.C., and Mikhail, E.M., 1988. Photogrammetric Treatment of Linear Features. ISPRS, Kyoto, Japan, Vol. 10, pp. 383-393.

Zisserman, A., 1995. Uncalibrated Vision. ISPRS, The Role of Models in Automated Scene Analysis, Stockholm, Sweden.




$C_1(X)$ = Cross Ratio of the 4 Planes:
$X_1X_2X_3, X_1X_2X_4, X_1X_2X_5, X_1X_2X$
(From Barrett 1994)

| Line # | dX (m) | dY (m) | dZ (m) |
|---|---|---|---|
| 1 | 0.023 | 0.077 | 0.013 |
| 2 | 0.041 | 0.033 | 0.070 |
| 3 | 0.069 | 0.008 | 0.041 |
| 4 | 0.031 | 0.042 | 0.004 |
| 5 | 0.454 | 0.512 | 0.123 |
| 6 | 0.207 | 0.118 | 0.288 |
| 7 | 0.019 | 0.066 | 0.045 |
| 8 | 0.648 | 0.245 | 0.411 |
| 9 | 0.042 | 0.056 | 0.019 |
| 10 | 0.018 | 0.042 | 0.177 |

Table 1 RMS at 5 Points on each Check Line Two Photo Block solving for E.O. Parameters Using 10 Control Lines + 10 Pass Lines (S1)

| Line # | dX (m) | dY (m) | dZ (m) |
|---|---|---|---|
| 1 | 0.045 | 0.085 | 0.017 |
| 2 | 0.008 | 0.060 | 0.013 |
| 3 | 0.075 | 0.044 | 0.052 |
| 4 | 0.094 | 0.085 | 0.015 |
| 5 | 0.323 | 0.395 | 0.108 |
| 6 | 0.310 | 0.184 | 0.505 |
| 7 | 0.041 | 0.099 | 0.063 |
| 8 | 0.612 | 0.216 | 0.403 |
| 9 | 0.106 | 0.080 | 0.027 |
| 10 | 0.170 | 0.043 | 0.131 |

Table 2 RMS at 5 Points on each Check Line Two Photo Block solving for I.O.+E.O. Parameters Using 10 Control Lines+10 Pass Lines (S2)

| | dX (m) | dY (m) | dZ (m) |
|---|---|---|---|
| Invariance | 0.004 | 0.005 | 0.002 |
| 2-Photo Block | 0.004 | 0.005 | 0.000 |

Table 6 RMS at 20 Check Points for Invariance and Photogrammetric Methods for Object Reconstruction Using 6 Control Points (Purdue Campus, Real Data, 1:4000)

| | dX (m) | dY (m) | dZ (m) |
|---|---|---|---|
| Invariance | 0.323 | 0.297 | 0.739 |
| 2-Photo Block | 0.059 | 0.037 | 0.689 |

Table 7 RMS at 11 Check Points for Invariance and Photogrammetric Methods for Object Reconstruction Using 6 Control Points (Bangor, 1:8660)

| Line # | dX (m) | dY (m) | dZ (m) |
|---|---|---|---|
| 1 | 0.187 | 0.051 | 1.246 |
| 2 | 0.139 | 0.515 | 0.168 |
| 3 | 0.005 | 0.020 | 0.441 |
| 4 | 0.005 | 0.049 | 0.408 |
| 5 | 0.060 | 0.109 | 0.461 |
| 6 | 0.218 | 0.124 | 0.670 |
| 7 | 1.611 | 0.286 | 0.240 |
| 8 | 0.003 | 0.035 | 0.119 |
| 9 | 1.375 | 0.981 | 7.148 |

Table 4 RMS at 5 Points on each Check Line Two Photo Block solving for E.O. Parameters Using 6 Control Lines + 9 Pass Lines (Bangor, Real Data, 1:8660)

| Photo # | $\Delta\omega\,(deg)$ | $\Delta\phi\,(deg)$ | $\Delta\kappa\,(deg)$ | $\Delta X_L\,(m)$ | $\Delta Y_L\,(m)$ | $\Delta Z_L\,(m)$ |
|---|---|---|---|---|---|---|
| 1 | 0.254 | 0.070 | -0.006 | -0.729 | -1.182 | 2.621 |
| 2 | -0.043 | -0.074 | 0.002 | -1.663 | 0.939 | 2.216 |

Table 3 Differences in Camera Parameters - Two Photo Block using 6 Control Lines and 9 Pass Lines - Real Data (Bangor (1:8660))

| | Convergent (C) Geometry | | | Normal (N) Geometry | | |
|---|---|---|---|---|---|---|
| | dX (m) | dY (m) | dZ (m) | dX (m) | dY (m) | dZ(m) |
| Original Invariance | 3.855 | 4.007 | 3.165 | 0.196 | 0.089 | 0.320 |
| Modified Invariance | 0.307 | 0.169 | 0.408 | 0.112 | 0.045 | 0.315 |
| 2-Photo Block | 0.033 | 0.029 | 0.061 | 0.034 | 0.037 | 0.136 |

Table 5 RMS at 16 Check Points for Invariance and Photogrammetric Methods for Object Reconstruction Using 6 Control Points - Simulated perturbed Data (0.01 mm)