# Orientation and Free Network Theory of Satellite CCD Line-Scanner Imagery

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#### ABSTRACT

In 1992 Okamoto et al. proposed an orientation theory of satellite CCD line-scanner imagery based on affine transformation. However, in order to employ this orientation approach, the central-perspective satellite CCD line-scanner images are required to be transformed into affine ones and this image transformation cannot be performed without errors due to height differences in the terrain. Therefore, the original orientation method could be applied only for the case where the terrain was hilly and the image transformation errors were negligibly small. In this paper, we will first present an iterative orientation method of satellite CCD line-scanner imagery where the image transformation errors are corrected by using the ground height information obtained in the previous iteration step. Tests with many simulation models clarified that two iteration steps are quite enough to remove the image transformation errors, even when height differences in the terrain exceed 4,000 meters. Next, the free network theory of satellite CCD line-scanner imagery is constructed, because this problem is becoming of greater importance for the precise analysis of satellite CCD line-scanner imagery having very high ground resolution (1m - 4m) which may appear in a few years.

## INTRODUCTION

The general orientation theory of satellite CCD line-scanner imagery such as SPOT imagery can rigorously be constructed based on projective transformation. However, this theory itself is of little practical use, because the attained accuracy is rather low due to very high correlations among the orientation parameters. In order to overcome this difficulty, an orientation theory of one-dimensional affine images was derived by Okamoto et al. in 1992, which can be applied for the analysis of satellite CCD line-scanner imagery by transforming the central-perspective images into affine ones. However, this image transformation cannot be performed without errors due to height differences in the terrain. Therefore, a correction technique of the image transformation errors is required to be developed to employ the orientation method using affine transformation for the analysis of satellite CCD line-scanner imagery of mountainous terrain where the image transformation errors are not negligibly small. Also, this problem can be overcome by introducing an iterative orientation approach using affine transformation, where the image transformation errors are corrected by using the ground height information obtained in the previous iteration step.

Very little has been written that provides a general and rigorous approach to the free network analysis of satellite CCD line-scanner imagery, though this

problem is becoming of greater importance for the precise analysis of forthcoming satellite CCD line-scanner imagery having very high ground resolution (1m - 4m). However, if the orientation problem of overlapped images is solved and the characteristics of the geometrical one-to-one correspondence relating the model and object spaces are clarified, we can easily find linearly independent free network vectors required for obtaining unbiased solutions with minimum variance. In the general orientation problem of one-dimensional affine images, the absolute orientation can be expressed by a two-dimensional affine transformation having six independent coefficients and the equation of the photographing plane in the object space coordinate system. Thus, in the free network analysis of satellite CCD line-scanner imagery, we need nine linearly independent vectors which can be given by linearizing the one-to-one correspondence between the model and object spaces.

In this paper, the general orientation problem of one-dimensional affine images is first briefly discussed. Then, the transformation of central-perspective line-scanner images into affine ones is described with correction of the image transformation errors, and simulation results of the proposed iterative orientation approach are discussed. Regarding the free network analysis of satellite CCD line-scanner imagery, the nine linearly independent vectors are given and the practical

characteristics are discussed which are obtained from many simulation calculations.

ORIENTATION PROBLEM OF AFFINE LINE-IMAGES

The basic equations for the three-dimensional analysis of affine line-scanner imagery are described in the form (Okamoto, et al. (1992))

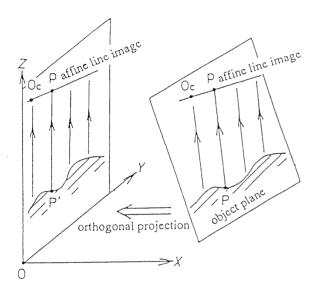


Figure-1: three-dimensional analysis of affine line image

$$\begin{array}{rcl}
0 &=& X + D_1 Y + D_2 Z + D_3 \\
y_c &=& D_4 Y + D_5 Z + D_6
\end{array} (1)$$

The first equation of Equation 1 denotes the equation of a photographing plane in the object space coordinate system, and the second equation expresses the relationship between the affine line image and an image of the object orthogonally transformed into the Y-Z plane of the reference coordinate system (X,Y,Z)(See Figure 1.). Also, we can see from Equation 1 that the three-dimensional analysis of an affine image can be separated into the following two processes: the determination of the plane including the object and the affine image with respect to the reference coordinate system and the orientation of the image in the Y-Z plane, because the first and second equations in Equation 1 have no common coefficients. orientation theory derived by Okamoto et al. in 1992 can rigorously be applied to the second phase of the three-dimensional analysis of overlapped affine images.

TRANSFORMATION OF CENTRAL-PERSPECTIVE LINE-IMAGES INTO AFFINE ONES

In reality, satellite CCD line-scanner imagery is taken central-perspectively. Thus, in a rigorous sense, we must analyze the imagery based on projective transformation. However, such analysis may not be effective, because the satellite CCD line-scanner conventionally has an extremely narrow field angle and thus very high correlations arise among the orientation parameters. This may especially be true when the photographed terrain is hilly. In order to overcome this difficulty, we will employ the orientation theory based on affine transformation by transforming the central-perspective images into affine ones. This transformation will be explained as follows.

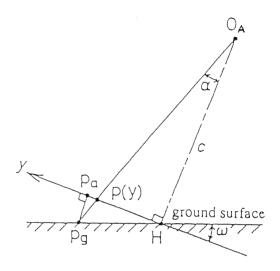


Figure-2: transformation of a centralperspective line image into an affine one

Let the ground surface be flat and a central-perspective line image be taken at a convergent angle  $_{\mathfrak{W}}$  (See Figure-2.). Further, the image is assumed to intersect the terrain at its principal point H. p(y) denotes a real image point and  $p_g$  is the intersecting point of the ray  $p_g$  and the ground surface. The corresponding affine image point  $p_g(y_a)$  can be found by drawing the normal to the central-perspective line image from  $p_g$ . The relationship between the central-perspective image point p(y) and the corresponding affine one  $p_g(y_a)$  is given in the form

$$y_a = \frac{y_c - y_H}{1 - (\tan \omega)(y_c - y_H) / c}$$
  
 $y = y_c - y_H$  (2)

in which c,  $y_c$ , and  $y_H$  denote the principal distance of the scanner, the measured image coordinate, and the principal point coordinate, respectively. The rotation angle  $\omega$  and the interior orientation parameters

 $(y_H,c)$  of the scanner are approximately known in the conventional analysis of satellite CCD line-scanner imagery. Thus, the image transformation errors due to the errors of the orientation elements are considered to be small, if the ground surface is flat. In addition, such errors can be corrected in the orientation calculation using Equation 1, because they may be modeled in a linear form. However, we must remove the image transformation errors due to height differences in the terrain. Let  $\Delta_Z$  indicate the scaled height difference of a ground point from the average height and  $\alpha_X$  denote the half of the field angle of the scanner. The image transformation error  $\Delta y$  due to neglecting the height difference  $\Delta_Z$  is shown as  $\Delta_X$  in Figure-3 and is given in the form

$$\Delta y = \Delta z (\tan(\omega + \alpha) - \tan \omega) \cos \omega$$
 (3)

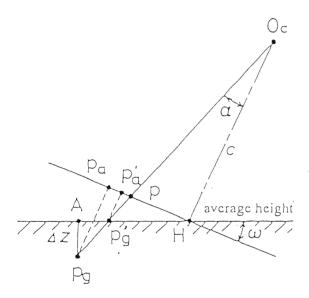


Figure-3: image transformation error due to height difference in the terrain

In the case where the field angle is 4 degrees, the rotation angle is 30 degrees, and the maximum height difference in the terrain is 500m, the maximum image transformation error amounts to 10.3m at the ground scale. Thus, the image transformation error due to the height difference in the terrain must be considered in the orientation calculation using affine transformation, and this correction can be performed iteratively as follows (See Figure-4).

 In the first step the orientation calculation is carried out under the assumption that the photographed terrain is flat. Then the approximate height

- information for each ground point is obtained. The obtained Z-coordinates of the total ground points yield the average ground height and the height difference  $\Delta_Z$  of each point with respect to the average height can be calculated.
- 2) The image transformation error due to the height difference ∆Z (scaled) can be removed by changing the principal distance of the scanner for each point in a following way.

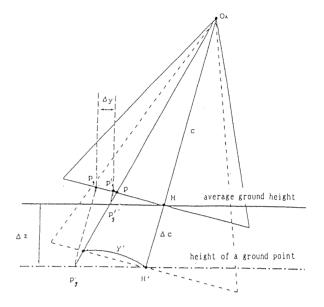


Figure-4: correction of image transformation error

$$\Delta c = \frac{\Delta z}{\cos \omega}, \quad c' = c + \Delta c, \quad y' = \frac{yc'}{c}$$

$$y_a' = \frac{y'}{1 - y' \tan \omega / c'}$$
(4)

 The orientation calculation using affine transformation is repeated with the corrected image coordinate ya'.

FREE NETWORK ANALYSIS OF SATELLITE CCD LINE-SCANNER IMAGERY

In the orientation problem of overlapped affine line-scanner images, the one-to-one correspondence relating the model and object spaces can be expressed by a two-dimensional affine transformation having six independent coefficients and the equation of the model plane in the object space coordinate system which has three independent coefficients. Therefore, in the free network analysis of satellite CCD line-scanner images

using affine transformation we need nine linearly independent vectors for obtaining the unbiased solutions with minimum variance. These vectors can be found as follows.

The basic equations (Equation 1) can be linearized in the form

$$0 = g_0 + Y \triangle D_1 + Z \triangle D_2 + 1 \bullet \triangle D_3 + 1 \bullet \triangle X$$

$$+ D_1 \triangle Y + D_2 \triangle Z$$

$$y_a = f_0 + Y \triangle D_4 + Z \triangle D_5 + 1 \bullet \triangle D_6 + D_4 \triangle Y$$

$$+ D_5 \triangle Z$$
(5)

Setting up Equation 5 for all affine line-scanner images under consideration simultaneously, we have a system of linear equations in a matrix form as

$$\mathbf{A} \, \Delta \mathbf{x} = \mathbf{c} \tag{6}$$

in which

A : a coefficient matrix of the system of linear equations

Ax: a vector of corrections to unknownsc: a vector of constants.

Without object space controls the matrix A is singular and its rank deficiency is nine. In the conventional orientation calculation object space controls are used to overcome the rank deficiency of the coefficient matrix. On the other hand, in the free network analysis of affine line-scanner images we employ nine linearly independent vectors satisfying the following relationship:

$$\mathbf{AG} = 0 \tag{7}$$

where G is a matrix constructed from the 9 linearly independent vectors, i.e.,

$$G = (g_1, g_2, \dots, g_9)$$
 (8)

Also, these 9 vectors can be found by linearizing the one-to-one correspondence between the model and object space. Unlike the conventional way of finding the free network vectors, however, the nine linearly independent vectors are searched in two different phases, because the absolute orientation of affine line-scanner images can be separated into two geometrical processes: transformation of the model into an image of the object orthogonally projected into the Y - Z plane of the reference system and transformation of the orthogonal image into the object. Three linearly independent vectors correspond to the first equation of Equation 5 and the remaining six vectors pertain to the second equation. These nine vectors are described as

$$\begin{aligned} \mathbf{g}_{1}^{T} &= (1, 0, 0, 0, 0, 0, -Y, 0, 0) \\ \mathbf{g}_{2}^{T} &= (0, 1, 0, 0, 0, 0, -Z, 0, 0) \\ \mathbf{g}_{3}^{T} &= (0, 0, 1, 0, 0, 0, -I, 0, 0) \\ \mathbf{g}_{3}^{T} &= (0, 0, 0, D_{4}, 0, 0, 0, -Y, 0) \\ \mathbf{g}_{5}^{T} &= (0, 0, 0, 0, D_{4}, 0, 0, -Z, 0) \\ \mathbf{g}_{5}^{T} &= (0, 0, 0, 0, 0, D_{4}, 0, 0, -Z, 0) \\ \mathbf{g}_{7}^{T} &= (0, 0, 0, 0, 0, D_{4}, 0, -1, 0) \\ \mathbf{g}_{7}^{T} &= (0, 0, 0, 0, D_{5}, 0, 0, 0, 0, -Y) \\ \mathbf{g}_{8}^{T} &= (0, 0, 0, 0, 0, D_{5}, 0, 0, 0, -Z) \\ \mathbf{g}_{9}^{T} &= (0, 0, 0, 0, 0, 0, D_{5}, 0, 0, 0, -1) \end{aligned}$$

If measured image coordinates  $(0, y_a)$  have random errors, the system of linearized observation equations can be described as

$$\mathbf{v} = \mathbf{A} \, \Delta \hat{\mathbf{x}} - \mathbf{L} \tag{10}$$

where v is a vector of residuals to the observations. The free network adjustment is then carried out in a following manner (Ebner (1974))

$$\mathbf{v}^T \mathbf{P} \mathbf{v} \rightarrow \min$$
under the condition  $\mathbf{G}^T \Delta \hat{\mathbf{x}} = 0$ 

where **P** is a weight matrix of the observations.

## TESTS WITH SIMULATED EXAMPLES

ITERATIVE ORIENTATION APPROACH OF SAT-ELLITE CCD LINE-SCANNER IMAGERY

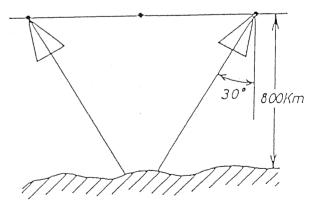


Figure-5: stereo CCD line-scanner images taken in a convergent manner

The proposed iterative orientation method was tested

with simulated satellite line-scanner imagery. In the construction of the simulation models stereo line-scanner images were assumed to be employed, which were taken in a convergent manner as is shown in Figure-5. The image point coordinates of 65 object points were calculated by means of the collinearity equations under the following conditions:

flying height: H = 800km

the principal point coordinate.

field angle of the line-scanner:  $\alpha = 4$  degrees

focal length of the scanner: c = 1000 mmconvergent angle:  $\omega = \pm 30$  degrees

Then, the perturbed image coordinates were provided in which the perturbation consisted of random normal deviates having a standard deviation of 3.3 micrometers. In addition, maximum errors of the orientation parameters of the scanner along the flight path were assumed to be as follows: ±15 minutes regarding the rotation parameters  $(\omega, \phi, \kappa)$  and  $\pm 1.0$ km regarding the translation parameters  $(X_0, Y_0, Z_0)$ . The flying course (60km) of the platform is divided into three sections and the exterior orientation parameters are assumed to vary linearly in each section (See Figure-6). Errors of the interior orientation elements are 1.0mm for the principal distance of the scanner and 0.5mm for

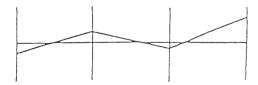
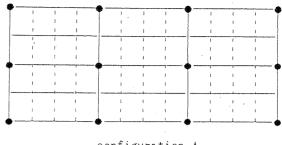


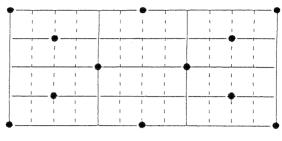
Figure-6: changes of orientation parameters along the flight path

The proposed iterative orientation approach was checked with three different configurations of ground control points(See Figure-7). The obtained results regarding the standard error of unit weight, the average internal error at the check points, and the average external error were given in Table-1. We can find in Table-1 the following characteristics of the orientation problem of satellite CCD line-scanner imagery using affine transformation:

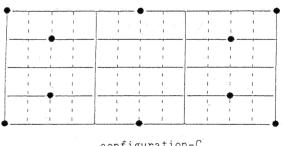
- 1) When the control points are arranged on the lines including the planes of calculating the orientation parameters, higher accuracy can be attained in comparison with the other control point configuration.
- 2) Two iteration steps are quite enough in order to remove the image transformation errors, even though the maximum height difference exceeds 4.000 meters.



configuration-A



configuration-B



configuration-C

Figure-7: congigurations of control and check points in the analysis of satellite CCD line-scannerimagery

# FREE NETWORK ANALYSIS OF SATELLITE CCD LINE-SCANNER IMAGERY

In order to explore the practical characteristics of the proposed free network theory for affine line-scanner images, the same simulation models were employed as those in the previous section. The free network theory is essentially a linear theory. Thus, we must have fairly good approximations for unknowns, because the basic equations (Equation 1) are non-linear with respect to the orientation parameters and object space coordinates. Further, the free network analysis needs an iterative approach in order to obtain a very high external accuracy. This comes from the fact that the solutions obtained in the free network adjustment are not unbiased estimates of both the orientation parameters at the exposure instants and the real object space coordinates

maximum height difference : 500m				
iteration step	T T	II		
standard error of	3.4	3.5		
unit weight(µm)				
average internal	5.6	5.8		
error(m)	J.0			
average external	5.9	4.6		
error(m)				
maximum height difference : 1000m				
iteration step	I	II		
standard error of	3.4	3.5		
unit weight(µm)				
average internal	6.1	6.5		
error(m)				
average external	9.1	6.0		
error(m)				
maximum height difference : 4000m				
iteration step	I	II		
standard error of	3.4	3.9		
unit weight(μm)				
average internal	5.4	6.4		
error(m)				
average external	20.0	7.3		
error(m)				

Table-1a: the obtained results for the iterative orientation approach (control point configuration-A)

Remonstrate the second industrial and the se				
maximum height difference : 500m				
iteration step	I	II		
standard error of	3.5	3.5		
unit weight(μm)				
average internal	7.5	7.5		
error(m)	7.5			
average external	9.4	8.9		
error(m)				
maximum height difference : 1000m				
iteration step	I	II		
standard error of	3.5	3.6		
unit weight(μm)	3.3			
average internal	11.5	11.7		
error(m)	11.5			
average external	15.1	10.8		
error(m)				
maximum height difference : 4000m				
iteration step	I	II		
standard error of	3.6	4.1		
unit weight(µm)				
average internal	7.4	9.2		
error(m)				
average external	41.2	6.2		
error(m)				

Table-1b: the obtained results for the iterative orientation approach (control point configuration-B)

P		TOTAL CO. C.	
maximum height difference : 500m			
	PS	FS	
standard error of	2 -	2 -	
unit weight(µm)	3.5	3.5	
average internal	7.5	5.1	
error(m)	1.5	3.1	
average external	0.0	8.2	
error(m)	8.9		
maximum height difference : 1000m			
	PS	FS	
standard error of	3.6	3.6	
unit weight( $\mu$ m)	3.0		
average internal	11 7	5.3	
error(m)	11.7	5.5	
average external	10.8	10.2	
error(m)			
maximum height diff	erence: 4		
rs e	PS	FS	
standard error of	4.1	3.9	
unit weight(µm)			
average internal	9.2	6.0	
error(m)	3.4	0.0	
average external	6.2	*7.3	
error(m)	0.2	*1.0	

Table-2a: the obtained results in the free network adjustment (control point configuration-B)
PS: particular solutions
FS: free netwok solutions

maximum height difference : 500m					
	. PS	FS			
standard error of unit weight( $\mu$ m)	3.6	3.6			
average internal error(m)	10.0	. 8.2			
average external error(m)	11.7	9.9			
maximum height difference : 1000m					
	PS	FS			
standard error of unit weight(μm)	3.5	3.5			
average internal error(m)	17.1	12.0			
average external error(m)	17.0	15.0			
maximum height diff	maximum height difference : 4000m				
	PS	FS			
standard error of unit weight( $\mu$ m)	3.9	4.1			
average internal error(m)	48.0	21.3			
average external error(m)	43.2	*98.5			

Table-2b: the obtained results in the free network adjustment (control point configuration-C)
PS: particular solutions
FS: free netwok solutions

depend mathematically on approximations. In this method, the first approximations were calculated by contaminating the control point coordinates with random errors having a standard deviation of 15 meters. Then, the iterative calculation of the free network adjustment was performed by regarding the solutions obtained in the (i-1)th step as the approximations in the i-th step and replacing only the approximations of the control point coordinates by the true values. Weights assigned to the control point coordinates must be very loose in the conventional free network adjustment. However, the nine free network constraints (Equation 11) correspond to three control points in the orientation problem of satellite CCD line-scanner imagery, and they are too weak to obtain stable solutions. Thus, rather tight weights(one tenth of the unit weight) were given to the control points.

The obtained results regarding the standard error of unit weight, the average internal error at the check points and the average external error are shown in Table-2. From these tables the following characteristics may be extracted for the free network adjustment of satellite CCD line-scanner imagery:

- Unlike the conventional free network adjustment, the improvements in the internal accuracy are not very great, because comparatively tight weights were assigned to the control point coordinates.
- The improvements in the external precision are almost 10 percents in the case where the terrain is hilly.
- The solution sometimes diverges when the terrain is mountainous and the number of control points is small.

## CONCLUDING REMARKS

In order to employ the orientation theory based on affine transformation for the analysis of satellite CCD line-scanner imagery, we must transform the central-perspective line images into affine ones. However, this image transformation cannot be performed without errors due to height differences in the terrain. Therefore, this paper presented an orientation approach of removing the image transformation errors by employing an iterative calculation. Further, the free network theory of affine line-scanner images has been constructed by finding nine linearly independent vectors. The proposed theories have been tested with simulated examples and have proved to be very effective for the analysis of satellite CCD line scanner imagery.

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