Basis of the Orthoimage Generation Method

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ABSTRACT

The generation of digital orthoimage is a process whose full formal description is very difficult. In this paper we attempted to do it by the use of terms coming from the image processing. The transformations between various geometrical coordinate systems also need a formal description starting from the digital image coordinate system through image space and model system to local ground or object coordinate system. We outline the way of treating the aerial photographs and also the orthoimages as functions.

We aim is to provide precise conditions of existence of the point in the orthoimage. The analysis of the reasons of the existing errors in computing the ground coordinates in DTM is made. After the orthoimage generation the analysis of the errors of the pixel position in the various parts of the image is discussed.

1 INTRODUCTION

The image of the terrain obtained after the scanning process of the aerial photograph has the features which we usually want to change; it can be the position of the observer, the size of the cartometric features and other geometric or radiometric characteristics of the image. It means that we want to have an image of the object in other projection than this in which its registration was done. The changes of the type of the transformation i.e. parameters of the projection should be made not only by changing the parameters of the camera or its position but also by suitable calculations done in the digital process of the image.

When we consider an orthoimage it means we have to obtain an image not only in the orthogonal projection. We require also it to be an orthogonal projection on the plane parallel to X0Y plane in the ground coordinate system and at the given (determined) scale. To generate this image we should first recreate the situation of the moment the image was taken. The central projection in which the photo was taken is determined explicitly by the positions of the projection plane and the projection centre. The connections between the position of the point (X,Y,Z) in the terrain and its projection position (x,y) in the image can be expressed analytically by the projective transformation and in the particular case by the collinearity equations of the form:

$$x-x_0=-c_k\frac{a_{11}(X-X_0)+a_{12}(Y-Y_0)+a_{13}(Z-Z_0)}{a_{13}(X-X_0)+a_{23}(Y-Y_0)+a_{33}(Z-Z_0)}$$
$$y-y_0=-c_k\frac{a_{11}(X-X_0)+a_{22}(Y-Y_0)+a_{23}(Z-Z_0)}{a_{13}(X-X_0)+a_{23}(Y-Y_0)+a_{33}(Z-Z_0)} \qquad (1)$$

where:

- X,Y,Z ground coordinates of the point,
- $x,y,-c_k$, image coordinates
- X_0, Y_0, Z_0 coordinates of the projection centre
- x_0, y_0 coordinates of the principle point of the image,
- c_k focal distance of the camera,
- $a_{11}, a_{12}, \ldots, a_{33}$ elements of the matrix of the cosine angles between axes of the ground coordinate system.

We will not consider the particular case when two different points of terrain lie on the same projection ray (on the same line to which belongs the projection centre). Then to calculate position of the point in the object space (on the ground) on the basis of its image coordinates one coordinate in the object (ground) coordinate system is missing. If we know the function Z=Z(X,Y) i.e. the digital model of terrain (DTM), then from the image coordinates x,y we can calculate the object coordinates X,Y and then the position of the point X,Y,Z in any other projection can be also calculated. To generate orthoimages we should know not only the elements of the interior and absolute orientations of the photo but also the DTM.

2 ORTHOIMAGE AS A FUNCTION

In order to determine the position of the image point on the plane it is enough to give its two coordinates. Raster im s of the terrain can be described as functions from Euclidean R^2 space to colour space C. In the space R^2 the coordinate image system x,y is defined. For the tonal photograph the section [0,1] can be assumed as a space C. For the colour photograph in the RGB palette, the cubic

$$C=[0,1]x[0,1]x[0,1] \qquad (2)$$

can be assumed as a space C.

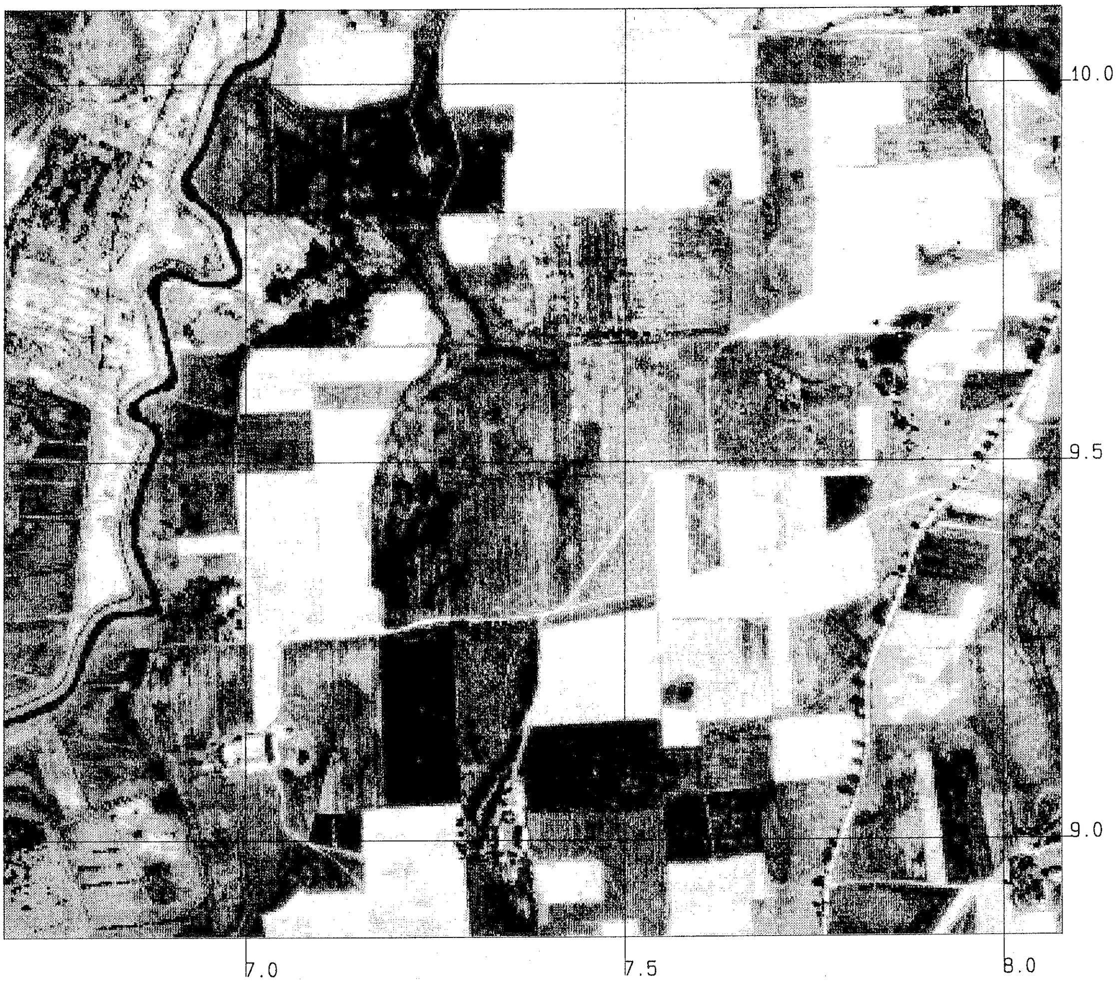

Figure 1. Computed orthoimage.

As the result of scanning process we obtain the digital image, i. e. the function from the pixel space to colour space C (Skarbek, 1993).
In general the colour space can be the space of the multispectral images in the following form

$$C = [0,L_1)\mathrm{x}\ldots\mathrm{x}[L_k) \tag{3}$$

where $[0,L_i)=\{0,\ldots,L_i-1\}\subset Z$ is the section of the integer numbers, to which digital levels of i–th component of the image belong.

The space C can be one of the following:

a) binary images, then $k = 1$, $L_1 = 1$;
b) images with grey values, then $k = 1$, $L_1 = h$; where h is generally equal to 4,16,64 or 256.
c) three–spectral images, then k=3; i.e. 24–bit images in the RGB palette for which $L_1=L_2=L_3=256$.

If we have the aerial photograph which corresponds to the function

$$c:\ (x,y)\rightarrow c(x,y) \tag{4}$$

we can assume that it is a tonal photograph for which $c(x,y)\in[0,1]$. The generalisation of the following considerations for the colour photographs will be based on the duplication of the formulas for successive spectral bands.

In the domain $D_c \subset R^2$ of the c function we can describe a set P of pixels in the following way:

$$P=\{P_{ij}:(i,j)\in IP(i_1,j_1)\} \tag{5}$$

where $IP(i_1,j_1)=\{(i,j)\in Z^2 : 0\le i\le i_1, 0\le j\le j_1\}$

is the set of the pixel indices.
If we assume that the pixel is a square with the side b

$$P_{ij} = \{(x,y): |x - x_{ij}| < b, |y - y_{ij}| < b\} \quad (6)$$

then the centre of the pixel of (i,j) has the coordinates

$$x_{ij} = x_1 + jb, \; y_{ij} = y_1 + ib \quad (7)$$

where x_1, y_1 are the coordinates of the pixel centre with indices (0,0).

The image (4) which has h grey levels is then in the digital form the function

$$f: \; P_{ij} \rightarrow c_{ij} = INT(\frac{h}{b^2} \iint_{P_{ij}} c(x,y) \; dxdy) \quad (8)$$

For the given point (x,y) the pixel indices are determined by the equations

$$j = INT(x - x_1)/b, \; i = INT((y_1 + b/2)/b) \quad (9)$$

which we write then as the function

$$(i,j) = e(x,y) \quad (10)$$

From the equation (6) it arises that for given x_1 y_1 b pixels and their indices are explicitly assigned. The digital image f, which was made as the result of the scanning process of the photograph, corresponds then to the function

$$c_{ij} = f_1(i,j) \quad (11)$$

We will consider now building the terrain image in the ground coordinate system. The composition of the functions f_1 and e gives the ordering dependence between the image coordinates of the photo and the grey levels

$$c_{ij} = f_1(e(x,y)j) \quad (12)$$

If we know the transformation of the form (1) from the X,Y,Z coordinate system to the x,y coordinate system, i.e. the transformation

$$(x,y) = g(X,Y,Z) \quad (13)$$

then for the given surface , described in the form of DTM (digital terrain model), we have the image

$$(X,Y) \rightarrow f_1(e(g(X,Y,H(X,Y)))) \quad (14)$$

which we will write as the function

$$(X,Y) \rightarrow F(X,Y) \quad (15)$$

This is already the image in the ground coordinate system obtained from the image $f(P_{ij})$. If would be possible to generate all points we will have a cartometric image of the terrain in the local ground coordinate system at the scale 1:1 i.e. an orthoimage.

It results that from the equations (14) and (15) the following conditions are fulfilled :

- for the chosen ground area we have DTM, i.e. $D_F \subset D_H$,
- the chosen ground area after the transformation (13) is inside the photo area, i.e. $D_g^{-1} \subset D_e$.

Because all the photo points lie on the opposite side of the plane to the projection centre, the condition $(X,Y,H(X,Y)) \in D_g$ is always satisfied.

The problem of the existence of the image point in the photograph for each point of the terrain which belongs to D_F must be considered

We notice that from two points lying on the same line passing through the project centre, one can see (in the photo) point which is closer to the project centre. In this way it is not possible to reconstruct the distant point in the orthoimage .

The digital orthoimage (Fig. 1) is the image composed of pixels in the different coordinate system than that introduced earlier and assigned by O'x'y'. The connections between coordinates X,Y of the point in the ground coordinate system and its x',y' coordinates are

$$X = a_1 + x'/\lambda, \; Y = a_2 + y'/\lambda \quad (16)$$

where: a_1 ,a_2 are the coordinates of the O' in the system OXY and λ is the scale coefficient of the creating image. Using the equations (16) the image (15) can be written as the function

$$(x',y') \rightarrow F((a_1 + x'/\lambda),(a_2 + y'/\lambda)) \quad (17)$$

which can be finally written as

$$(x',y') \rightarrow F_1(x',y') \quad (18)$$

If we make a visualization with the d side of a pixel it means that we build the square grid in the terrain with the side

$$B = \frac{d}{\lambda} \quad (19)$$

Let's assume that we are building the image of a rectangular terrain area. The domain of the function F(X,Y) is a rectangle. The domain of the function F_1 (x',y') is also a rectangle. The set of indices of the pixels P' of the image F_1 and of the pixels P" of the terrain grid is the same and can be written as

$$IP'(i'_1,j_1') = \{(i,j) \in Z^2 : 0 \le i \le i_1^{'}, 0 \le j \le j_1^{'}\} \qquad (20)$$

If we know the indices of the pixels the coordinates of the centre of the pixel P_{ij}' can be calculated in the coordinate system O'x'y'. They will be equal:

$$x_i^{'} = \frac{d}{2} + id, \quad y_j^{'} = \frac{d}{2} + jd \qquad (21)$$

According to (10) we have

$$(x_i^{'}, y_j^{'}) = e_1^{-1}(i,j) \qquad (22)$$

Building the orthoimage we fill the grid of the pixels P' of the side d with corresponding grey levels changing successively the indices (20) in the lines and columns.

Then using the previous associations we can indicate possibly precisely from which point in the photo the grey value should be taken or which points should be chosen to calculate it .

If we are building the digital othoimage

$$G: \; P_{ij}^{'} \to c_{ij}^{'} \qquad (23)$$

means that we are building the image in the ground coordinate system

$$G_1: \; P_{ij}^{"} \to c_{ij}^{'} \qquad (24)$$

because moving along the pixels P'_{ij} corresponds to moving in the terrain along squares of B sides and centres determined by the formulas (16) and (21).
To determine c_{ij}' we will compare G_1 and F images assuming that the distance between the images should be the smallest i.e.

$$\iint_{D_F} (F(X,Y) - G_1(X,Y))^2 \, dXY = \sum_{i=0}^{i_1^{'}} \sum_{j=0}^{j_1^{'}} \iint_{P_{ij}^{"}} (F(X,Y) - c)^2 = \min \qquad (25)$$

It results that

$$\iint_{P_{ij}^{"}} (F(X,Y)dXY - B^2 c'_{ij} = 0 \qquad (26)$$

The pixel P'_{ij} of the orthoimage should then obtain the grey level

$$c'_{ij} = INT(\frac{1}{B^2} \iint_{P_{ij}^{"}} F(X,Y)\, dXY) \qquad (27)$$

During the orthoimage generation the values of finite sums

$$c'_{ij} = INT(\frac{1}{(2l+1)(2k+1)} \sum_{n=-l}^{l} \sum_{m=-k}^{k} F((X_i + \frac{n}{2l+2}),(Y_j + \frac{m}{2k+2}))) \qquad (28)$$

are calculated more quickly than the integrals (28).
For k=l=0 it is obtained

$$c'_{ij} = F(X_i, Y_j) \qquad (29)$$

and then the orthoimage is the function

$$(i,j) \to F(F_1(e_1^{-1}(i,j))) \qquad (30)$$

The reconstruction (resampling) of the image by the averaging formula (29) causes that each average decrease the variance what can be observed as the smearing of the contours details. On the other hand using an interpolation by the duplication (30) we obtain the image more distant from the original (scanned photograph).

3 Construction of DTM

In the previous paragraph it was assumed that for the given area the function Z=H(X,Y) is known, i.e. the digital terrain model (DTM).

This model could be prior determined and written in an appropriate data base. If DTM doesn't exist then it can be obtained by one of the following ways:

- digitalization of the contours of the existing maps,
- the direct terrain measurement,
- the photogrammetric methods from at least two air photos (stereo pairs).

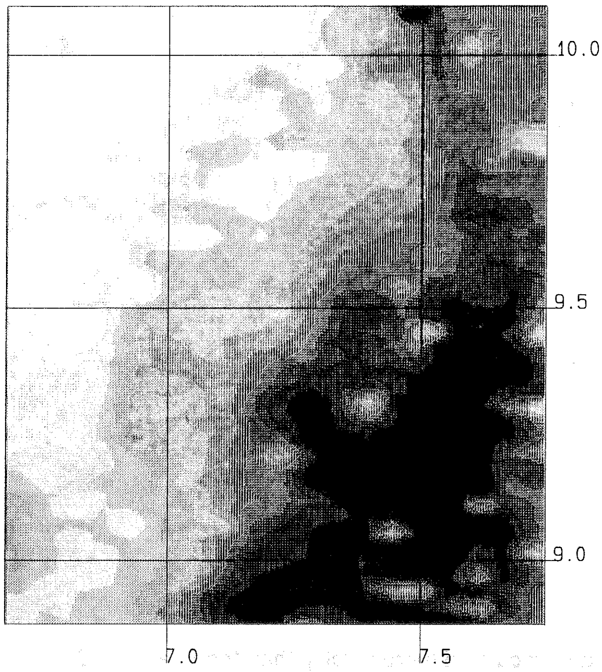

Figure 2. The result of the DTM generation

The basic parameters of DTM are the distance between centres of the nodes of the grid in the ground coordinates system and the accuracy of the determination of their positions. The cost and the time of the coordinates (X,Y,Z) creation is also important.

Building DTM from existing maps needs the contours extraction from the raster image of the maps registered by a scanner. The contours are pointed by the operator on the display screen. The same operation can be done directly on the map using a digitizer. In this two cases

we receive the vector image of the contours in the ground coordinate system. To obtain DTM, the heights of the terrain, which are written as contours, should be interpolate into heights in the nodes of the given grid.

Each cell of this grid that does not have a corresponding positioned vector element will have an interpolated Z-value. The Z-value may be determined as the result of the linear interpolation. For example the Z-value can be determined from the line between the closest known values on both sides of the cell. The Z-value that corresponds to one of the eight linear solutions with the greatest slope is assigned as the final cell value. The resulting surface may be irregular. To obviate such problems e.g. an automatic smoothing routine should be done, but this will not be described in this paper.

DTM building by photogrammetric measurements needs as input material a stereo pairs (at least one).

DTM can be obtained by two ways:

- calculating the elements of the relative and absolute orientations,
- direct calculating X,Y,Z coordinates in the ground coordinate system using the project transformation.

In the first point it is necessary to know the parameters of a camera and the function from pixel to image coordinate system.

In the second point it is very important to have the high precision of the measurements of the control points coordinates in the pixel coordinate system as well as their accurate identification in both images. The algorithm of DTM building can be obtained e.g. on the basis of the correlation method. It matches area from two images taking into account the correlation coefficient. The algorithm calculate it many times for surroundings of the pixel (node in a grid of the left image). The area is matched for the maximum value of this coefficient. The algorithm searches the one (left) image along the nodes of the given grid. These nodes are determined a priori.

The information from one iteration (image pyramid) about matching is used in the next one. Because of this we can make shorter the time of the calculation.

The Figure 2 is the result of the DTM, generated by correlation method, after interpolation.

4 ERRORS IN DETERMINATION OF X,T,Z COORDINATES IN DTM

To calculate the errors in determination of the X,Y,Z coordinates in the process of DTM generation is very important because they affect the errors of the orthoimage. We will consider the case when we use the interior and absolute orientations in computation of X,Y,Z coordinates.

We use the formulas:

$$\begin{aligned} X-X_o &= -Z\left(\frac{x_1^o}{c_k}\right) = B\left(\frac{x_1^o}{p^o}\right) \\ Y-Y_o &= -Z\left(\frac{y_1^o}{c_k}\right) = B\left(\frac{y_1^o}{p^o}\right) \\ Z-Z_o &= B\left(\frac{c_{k1}}{p^o}\right) \end{aligned} \tag{31}$$

where
X,Y,Z the ground coordinates,
x_1^o, y_1^o the image coordinates of the horizontal left photo
X_o, Y_o, Z_o the translation vector
$p^o = x_1^o - x_2^o$ the difference of the abscissas of the corresponding points on the horizontal left and right photos
c_k focal distance
B the base of the photo in the ground coordinate system

The mean square errors m_x, m_y, m_z we obtain first calculating the differentials of (31), e.g. the formula of d_z is as follows

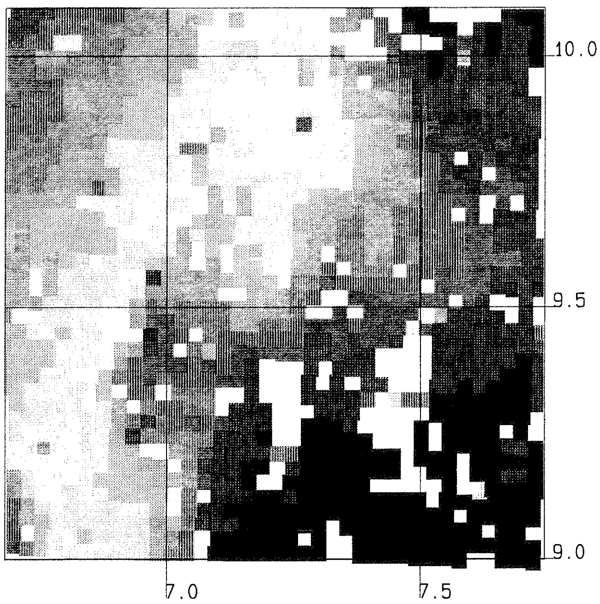

Figure 3. The errors of the DTM.

$$d_Z = Z(\frac{1}{\lambda}d\lambda + \frac{b_y}{b^2\lambda}db_y + (\frac{b_z}{b^2\lambda} + \frac{x_1 - b}{c_k bx})db_z + \frac{dc_k}{c_k} - \frac{dx_1}{p} + \frac{dx_2}{p} + \frac{x_1 y_1}{c_k p}d\omega + (\frac{c_k}{p} + \frac{x_1^2}{c_k p})d\phi + \frac{y_1}{p}d\kappa) + d_{Z_0} \quad (32)$$

where λ is the scale denominator. (Lobanow,1984)
We take into account that in differentials dx_1^o and dx_2^o are included the errors of the absolute orientation and in the dp^o we consider only influences of the elements of the interior orientation.
Figure 3 shows the distribution of the Z coordinate error. The grey levels coorespond with the error from 0,4m to 0.5m.

5 ERRORS OF ORTHOIMAGE

The correctness of obtained image depends not only on errors of the calculation of grey levels in the resampling process. We should, first of all, take into account all variables in transformation which was used in the generation of the orthoimage. Looking at the formula (31) and (14) we can see that we should analyse errors of the determination of the point position in the photo which results from the errors of the parameter estimation of the exterior orientation and appropriate determination of the Z coordinate from DTM.

After we had made the relative and absolute orientations of stereo pair we received the project transformation introducing the formulas of model coordinates

$$\mathbf{R}_p = \mathbf{R}_p^o + \lambda_p \mathbf{A}(\mathbf{R} - \mathbf{R}_s) \quad (33)$$

into formulas

$$x = -\frac{X_p}{Z_p}c_k, \quad y = -\frac{Y_p}{Z_p}c_k \quad (34)$$

where
$\mathbf{R}_p = [X_p, Y_p, Z_p]^T$ the model ccordinates of the point
$\mathbf{R}_p^o = [X_p^o, Y_p^o, Z_p^o]^T$ the translation vector
λ_p the scale denominator
$\mathbf{A}$ rotation matrix
$\mathbf{R} = [X, Y, Z]^T$ the ground coordinates of the point
$\mathbf{R}_s = [X_s, Y_s, Z_s]^T$ the mean vector

The mean square errors of the image coordinates m_x, m_y we receive calculating differentials of (34). We assume (as usual) that rotations angles are small (Wang,1990). The results of the calculation of the error $m = \sqrt{m_x^2 + m_y^2}$ are shown in Figure. 4.
The grey levels correspond with the error from 0.15mm (white) to 0.21mm (black) when the image pixel size was 0.06mm. We can see that this errors depend on the distant from $\mathbf{R}_s$. The influence of the errors of the estimation $\mathbf{R}_p^o$ is big and constant in the whole area.
The effect of the error of the point position in the photo will influence on the error of the grey level determination of the pixel in the orthoimage.
Finally using the function (4) we have

$$m_{c(x,y)}^2 = a_x^2 m_x^2 + a_y^2 m_y^2 \quad (35)$$

where a_x, a_y are the values of the partial differentials of the function (4) at the point (x,y).

6 CONCLUSIONS

We want to inform that all tests we performed using our programme on the PC computer. We have written it in Borland C++ ver. 4.5 language in Windows 95 environment. Programme is working with Paradox Database for Windows. The coordinates of control points are inserted into tables of this database. The data computed during the DTM generation process by correlation method are stored in special files. Figure 3 shows that not all parts of the area have Z coordinates so the needed additional measurements are stored in

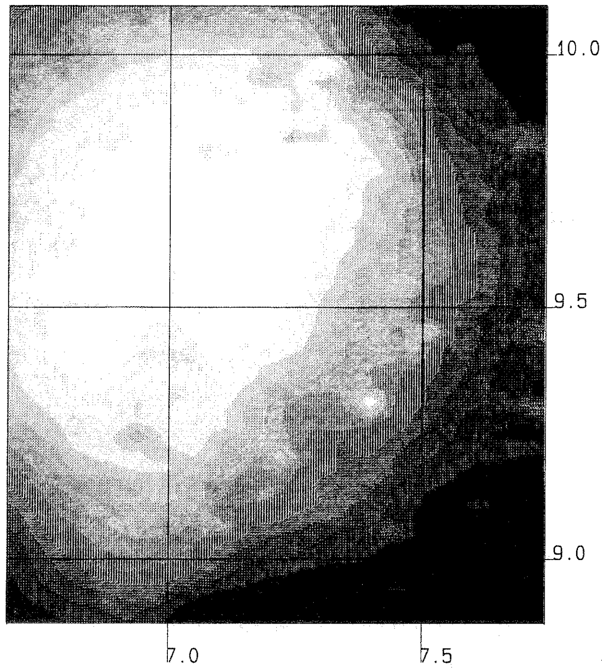

Figure 4. The errors m of points position.

special Paradox tables. To interpolate DTM the data from various resources are join (Loodts,1993).

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