ON CRITICAL CONFIGURATIONS OF PROJECTIVE STEREO CORRELATION

Gerhard Brandstätter
Professor, Institute of Applied Geodesy and Photogrammetry
Technical University Graz, Austria

Commission III

KEY WORDS: Projective photogrammetry, stereo correlation, critical configurations

ABSTRACT:

Stereocorrelation in projective photogrammetry is performed by means of *eight* irregularly distributed points and yields eight linear equations for the determination of the components of the correlation matrix. Critical configurations are characterized by vanishing of the determinant of those eight equations, wherefrom critical point distributions may be derived. Of course, the well-known critical configuration of five orientation points will belong to it but with respect to eight points, additional conditions of singularity will arise. The following considerations contain investigations on this subject using projective relations in homogeneous vector notation.

KURZFASSUNG:

Stereokorrelation der projektiven Photogrammetrie erfolgt mittels *acht* unregelmäßig verteilter Punkte und beruht auf acht linearen Gleichungen für die Bestimmung der Komponenten der Korrelationsmatrix. Kritische Anordnungen sind charakterisiert durch das Verschwinden der Determinante jener acht Gleichungen, woraus die kritischen Punktanordnungen abzuleiten sind. Natürlich wird auch die wohlbekannte kritische Konfiguration der fünf Orientierungspunkte dazugehören, bei acht Punkten müssen aber noch weitere Singularitätsbedingenen auftreten. Die nachstehenden Ausführungen enthalten einige Entwicklungen zu diesem Problem unter Verwendung projektiver Beziehungen in homogener Schreibweise.

0. INTRODUCTION

Algebroprojective photogrammetry deals with projective images of completely unknown interior orientation. The coordinate system of such images cannot be referred to a more or less orthogonal and isometric external coordinate system, but to an (affine) internal system defined by suitable object points which are projected to homologous points in the images. In this case, stereo correlation results in the determination of *eight* parameters, consisting of the usual set of relative orientation and an additional set of three parameters related to an interior affine coordinate system defined by three non-collinear homologous image points (*Brandstätter 1991*). These two affine systems are corresponding images of the first coordinate plane of a tridimensional affine coordinate system in the object space and produce most uncomplicated projective transformations.

The parameters of stereo correlation are the eight significant components of a 3x3-matrix **C,** the core of the homogeneous coplanarity condition. Applying this condition to eight pairs of homologous points, a system of 8 linear equations results wherefrom the components mentioned above may be computed. The solution depends on the regularity of its matrix of coefficients exclusively composed of the homologous affine image coordinates and hence on the spatial distribution of the corresponding object points in the tridimensional model space. The matrix will become singular if the positions of those points of correlation correspond with a critical configuration. The intention of the following treatise is, to discover this critical situation from the geometric relations and to find out a strategy to avoid it. For this aim the following symbols in homogeneous notation are used:

$\mathbf{u}^T = (1, u_1, u_2)$	homogeneouse affine image coordinates
$\mathbf{y}^T = (1, y_1, y_2, y_3)$	affine coordinates of the object space
$\mathbf{y}_0$	center of projection
$\mathbf{M} = \{m_{jk}\},\ j = 0,1,2;\ k = 0,1,2,3$	singular projective matrix
$\mathbf{P} = \{p_{jk}\},\ j = k = 0,1,2,3$	regular projective matrix
$\mathbf{C} = \{c_{ij}\},\ i = j = 0,1,2$	matrix of correlation
$\mathbf{A} = \{a_{lm}\},\ l = m = 3...7$	matrix of coefficients
$\mathbf{Q} = \{q_{jk}\},\ j = k = 0,1,2,3$	matrix of a quadric surface
μ............................	local stretching coefficient

1. BASIC RELATIONS

Composing the matrix of projection by its row-vectors $\mathbf{m}_j^T = (m_{j0}\ m_{j1}\ m_{j2}\ m_{j3})$, an image point results from

$$\mu \mathbf{u} = \mathbf{M}\,\mathbf{y} \quad \text{or} \quad u_j = \frac{\mathbf{m}_j^T \mathbf{y}}{\mathbf{m}_0^T \mathbf{y}} \tag{1.1}$$

after eliminating the tiresome factor μ. From **M** result the coordinates of the center of projection by means of

$$\mathbf{M}\,\mathbf{y}_0 = \mathbf{0} \quad \text{or} \quad \mathbf{m}_j^T \mathbf{y}_0 = 0 \tag{1.2}$$

(*Brandstätter 1996*). Referring to the interior coordinate system mentioned above, **M** itself reads simply

$$\mathbf{M} = \begin{bmatrix} 1 & \mu_1 - 1 & \mu_2 - 1 & \mu_3 - 1 \\ 0 & \mu_1 & 0 & u_{31}\mu_3 \\ 0 & 0 & \mu_2 & u_{32}\mu_3 \end{bmatrix} \tag{1.3}$$

(*Brandstätter 1993*). It depends on the four projective parameters $\mu_0 = 1$, μ_1, μ_2, μ_3, and on the image coordinates u_{31}, u_{32} of a point $\mathbf{u}_3$, that is the image of the third affine unit point of the object space (Figure 1). By means of these elements the image coordinates result from

$$u_j = \frac{y_j\,\mu_j + y_3\,u_{3j}\,\mu_3}{1 + \sum_{k=1}^{3}(\mu_k - 1)y_k} = \frac{n_j}{d}, \quad j = 1,2 \tag{1.4}$$

representing the pure projective transformation whereas (1.1) also contains elements of affine transformations.

Stereoscopic image correlation between two images $P'(\mathbf{u}')$ and $P''(\mathbf{u}'')$ is defined by the coplanarity condition

$$\mathbf{u}'^T \mathbf{C}\,\mathbf{u}'' = c_{10}\mathbf{u}'^T \mathbf{Z}\,\mathbf{u}'' = 0 \quad (z_{ij} = c_{ij} / c_{10}) \tag{1.5}$$

(*Thompson 1968*, *Fuchs 1988*) with a matrix **Z** containing besides of z_{10}=1 the significant eight components z_{ij} of correlation. Using now, out of the set of eight points of correlation, three *non-collinear* points for the definition of the interior affine coordinate system, their homogeneous point vectors will contain the simple components

$$\mathbf{u}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

and the corresponding system $\mathbf{A}_8$ will arise with coefficients as listed in Tab. 1. Therefrom follows directly that z_{00}=0 and hence det($\mathbf{A}_8$)=det($\mathbf{A}_7$). Furthermore, from the equations 1 and 2 of $\mathbf{A}_7$ follow two additional simple relations:

$$z_{01} = -1 - z_{11} \quad \text{and} \quad z_{02} = -z_{20} - z_{22}$$

By their means, projective correlation is reduced to a 5x5-system $\mathbf{A}_5\mathbf{z}=\mathbf{a}$ consisting of five rows according to

$$u_1''(u_1' - 1)z_{11} + u_1'u_2''z_{12} + (u_2' - u_2'')z_{20} + u_2'u_1''z_{21} + u_2''(u_2' - 1)z_{22} = u_1'' - u_1' \tag{1.5}$$

with a vector of unknowns $\mathbf{z}^T = (z_{11}\ z_{12}\ z_{20}\ z_{21}\ z_{22})$.

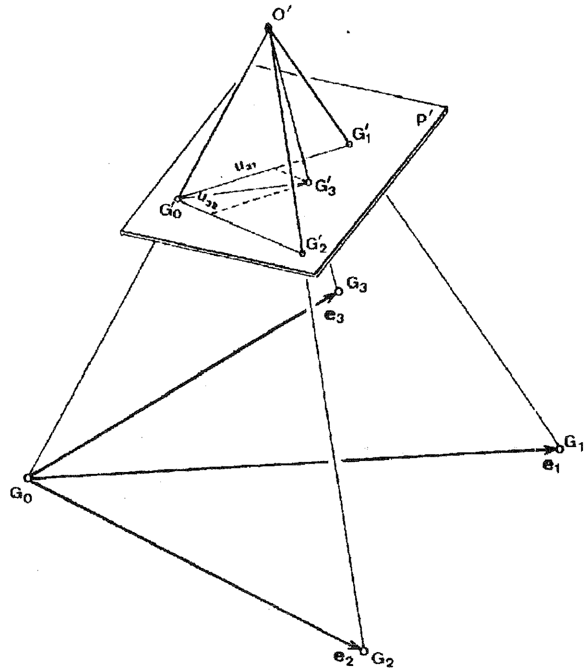

Figure 1: Definition of the affine coordinate systems

2. THE CONDITION OF CRITICAL CONFIGURATIONS

Critical configurations referring to projective image correlation will be indicated by vanishing of the system determinant, that is

$$\det(\mathbf{A}_8) = \det(\mathbf{A}_7) = 0. \tag{2.1}$$

Hence the coefficients of matrix $\mathbf{A}_8$ (Tab. 1) will be responsible for critical situations caused by all point distributions satisfying this condition. In order to recognize their *spatial* distribution, the components of $\mathbf{A}_8$ in Tab. 1, which are composed originally by plane image coordinates, must be transformed to the object space by means of (1.1) or (1.4). Using (1.1), the transformation will contain the typical *dyadic* product

$$(\mathbf{m}_j'^T \mathbf{y})(\mathbf{m}_k''^T \mathbf{y}) = \mathbf{y}^T(\mathbf{m}_j'\mathbf{m}_k''^T)\mathbf{y} = \mathbf{y}^T \mathbf{Q}_{jk}''' \mathbf{y}. \tag{2.2}$$

But expressing at first the components of $\mathbf{A}_8$ by means of the abbreviated terms according to (1.4), the condition (2.1) will read

Tab.1: Coefficients of the four functional matrices $\mathbf{A}_i$ (i=5,6,7,8)

A	row	z_{00}	z_{01}	z_{02}	z_{11}	z_{12}	z_{20}	z_{21}	z_{22}	a
$\mathbf{A}_8$	0	1	0	0	0	0	0	0	0	0
	1	1	1	0	1	0	0	0	0	-1
	2	1	0	1	0	0	1	0	1	0
	3 ⋮ 7	1	u_1''	u_2''	$u_1'u_1''$	$u_1'u_2''$	u_2'	$u_2'u_1''$	$u_2'u_2''$	$-u_1'$
$\mathbf{A}_7$	1	—	1	0	1	0	0	0	0	-1
	2	—	0	1	0	0	1	0	1	0
	3 ⋮ 7	—	u_1''	u_2''	$u_1'u_1''$	$u_1'u_2''$	u_2'	$u_2'u_1''$	$u_2'u_2''$	$-u_1'$
$\mathbf{A}_6$	2	—	—	1	0	0	1	0	1	0
	3 ⋮ 7	—	—	u_2''	$(u_1'-1)u_1''$	$u_1'u_2''$	u_2'	$u_2'u_1''$	$u_2'u_2''$	$u_1''-u_1'$
$\mathbf{A}_5$	3 ⋮ 7	—	—	—	$(u_1'-1)u_1''$	$u_1'u_2''$	$u_2'-u_2''$	$u_2'u_1''$	$(u_2'-1)u_2''$	$u_1''-u_1'$

$\det(\mathbf{A}_8)=\det(\mathbf{A}_7)=$

$$=\begin{vmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{n''_{1k}}{d''_k} & \frac{n''_{2k}}{d''_k} & \frac{n'_{1k}n''_{1k}}{d'_kd''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}}{d'_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} & \frac{n'_{2k}n''_{2k}}{d'_kd''_k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{vmatrix} = 0$$

and its expansion into six subdeterminants with respect to all components equal to 1

$\det(\mathbf{A}_7)=$

$$=\begin{vmatrix} \frac{n'_{1k}n''_{1k}}{d'_kd''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}}{d'_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} & \frac{n'_{2k}n''_{2k}}{d'_kd''_k} \end{vmatrix} - \begin{vmatrix} \frac{n''_{2k}}{d''_k} & \frac{n'_{1k}n''_{1k}}{d'_kd''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} & \frac{n'_{2k}n''_{2k}}{d'_kd''_k} \end{vmatrix} -$$

$$- \begin{vmatrix} \frac{n''_{2k}}{d''_k} & \frac{n'_{1k}n''_{1k}}{d'_kd''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}}{d'_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} \end{vmatrix} - \begin{vmatrix} \frac{n''_{1k}}{d''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}}{d'_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} & \frac{n'_{2k}n''_{2k}}{d'_kd''_k} \end{vmatrix} - \quad (2.3)$$

$$- \begin{vmatrix} \frac{n''_{1k}}{d''_k} & \frac{n''_{2k}}{d''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} & \frac{n'_{2k}n''_{2k}}{d'_kd''_k} \end{vmatrix} - \begin{vmatrix} \frac{n''_{1k}}{d''_k} & \frac{n''_{2k}}{d''_k} & \frac{n'_{1k}n''_{2k}}{d'_kd''_k} & \frac{n'_{2k}}{d'_k} & \frac{n'_{2k}n''_{1k}}{d'_kd''_k} \end{vmatrix}$$

The subdeterminants (symbolized by one row k, k=3....7) are composed exclusively by the spatial coordinates of the respective object points from which the geometry of the critical configuration may be derived.

Because of the fact that $\det(\mathbf{A}_6)$ results from $\det(\mathbf{A}_7)$ by subtracting the first column from the third one, and $\det(\mathbf{A}_5)$ results from $\det(\mathbf{A}_6)$ by subtracting its first column from its fourth and from its sixth one, all determinants are equivalent or in other words, the condition (2.1) indicates singularity also for $\mathbf{A}_5$ and $\mathbf{A}_6$ (Tab. 1) and vice versa. Therefore the relation

$\det(\mathbf{A}_5)=$

$$= \begin{vmatrix} \frac{\mathbf{y}_k^T(\mathbf{Q}'''_{11}-\mathbf{Q}'''_{01})\mathbf{y}_k}{\mathbf{y}_k^T\mathbf{Q}'''_{00}\mathbf{y}_k} & \frac{\mathbf{y}_k^T\mathbf{Q}'''_{21}\mathbf{y}_k}{\mathbf{y}_k^T\mathbf{Q}'''_{00}\mathbf{y}_k} & \frac{\mathbf{y}_k^T(\mathbf{Q}'''_{02}-\mathbf{Q}'''_{02})\mathbf{y}_k}{\mathbf{y}_k^T\mathbf{Q}'''_{00}\mathbf{y}_k} & \frac{\mathbf{y}_k^T\mathbf{Q}'''_{12}\mathbf{y}_k}{\mathbf{y}_k^T\mathbf{Q}'''_{00}\mathbf{y}_k} & \frac{\mathbf{y}_k^T(\mathbf{Q}'''_{22}-\mathbf{Q}'''_{02})\mathbf{y}_k}{\mathbf{y}_k^T\mathbf{Q}'''_{00}\mathbf{y}_k} \end{vmatrix} = 0 \quad (2.4)$$

is equivalent to the previous condition (2.1). As the identic denominators of one row may be cancelled out, (2.4) converts to

$$\begin{vmatrix} \mathbf{y}_k^T\mathbf{Q}_{11}\mathbf{y}_k & \mathbf{y}_k^T\mathbf{Q}_{12}\mathbf{y}_k & \mathbf{y}_k^T\mathbf{Q}_{20}\mathbf{y}_k & \mathbf{y}_k^T\mathbf{Q}_{21}\mathbf{y}_k & \mathbf{y}_k^T\mathbf{Q}_{22}\mathbf{y}_k \end{vmatrix} = 0 \quad (2.5)$$

in which the indices of the $\mathbf{Q}_{jk}$ refer simply to the respective unknowns z_{jk} of Tab. 1. It is obvious that each component of this final formulation of the initial condition $\det(\mathbf{A}_8)=0$ represents a quadratic form. This fact will be of particular interest in the following discussion.

3. DISCUSSION OF CRITICAL SITUATIONS

A first inspection of the subdeterminants of (2.3) shows that in the case of complete coplanarity, that is by means of (1.4)

$y_{k3}=0,\ d_k=1+(\mu_1-1)y_{k1}+(\mu_2-1)y_{k2},\ n_{jk}=\mu_j y_{kj}$

and hence

$$n'_{1k}n''_{2k} = \mu'_1\mu''_2 y_{k1}y_{k2}; \quad n'_{2k}n''_{1k} = \mu'_2\mu''_1 y_{k1}y_{k2};$$

all of them will vanish simultaneously because of containing *proportional columns* with $n'_{1k}n''_{2k}$ and $n'_{2k}n''_{1k}$. Therefrom follows that at least one selected point P_x out of the subset {3...7} has to be situated outside the $[\mathbf{e}_1,\mathbf{e}_2]$-plane G_0-G_1-G_2 in Fig. 1 and that all together two points (P_x and G_3) out of the whole set are not allowed to be co-planar with the rest. Otherwise, the projective relations refer to regular projectivities between bidimensional spaces. This statement agrees with the need for *five* non-coplanar homologous points in connection with the determination of a regular transformation between tridimensional projective spaces (*Brandstätter 1991*).

If one point out of the set {3...7}, for example P_7, is considered to be a variable point, an expansion of the determinant (2.5) into subdeterminants D_{jk} may be performed with respect to its row k=7. Therefrom follows in analogy to (*Rinner 1972*) the quadratic function

$$\begin{aligned} &\mathbf{y}^T\mathbf{Q}_{11}\mathbf{y}D_{11} + \mathbf{y}^T\mathbf{Q}_{12}\mathbf{y}D_{12} + \mathbf{y}^T\mathbf{Q}_{20}\mathbf{y}D_{20} + \\ &\quad + \mathbf{y}^T\mathbf{Q}_{21}\mathbf{y}D_{21} + \mathbf{y}^T\mathbf{Q}_{22}\mathbf{y}D_{22} = 0 \end{aligned} \tag{3.1}$$

in the projective object space. It may be expressed by means of the short homogeneous matrix formula

$$\mathbf{y}^T\mathbf{Q}\,\mathbf{y} = 0$$

in which $\mathbf{Q}$ represents the sum of all $\mathbf{Q}_{jk}D_{jk}$ in (3.1). Therefore $\mathbf{A}$ becomes singular if all points of the subset {3...7} belong to a general *quadric surface.* The centers of projection $\mathbf{y}_0$, however, must be points of this surface as well, because, according to (1.2), all products of the kind $\mathbf{m}^T\mathbf{y}_0$ equal zero and hence the condition (3.1) (=formula of the quadric surface) is satisfied identically. This statements are well-known from literature, especially from the famous "*Vienna School of Geometry*" represented mainly by Josef Krames (*Krames 1942*) and Walter Wunderlich (*Wunderlich 1941*). Their results were achieved by admirable synthetic considerations based on a profound knowledge of constructive and projective geometry. But in regard to analytical or digital photogrammetry, calculational methods are essential and hence, in order to detect and avoid critical situations of projective image correlation, an algebraic analysis of this problem should be prefered.

A detailed evaluation of the partial $\mathbf{Q}_{jk}$'s in equation (3.1) by means of relation (2.2), and using the $\mathbf{m}_i$ of (1.3) results in the contents of Tab. 2. They show that in general, the resultant quadric

$$\mathbf{Q} = \mathbf{Q}_{11}D_{11} + \mathbf{Q}_{12}D_{12} + \mathbf{Q}_{20}D_{20} + \mathbf{Q}_{21}D_{21} + \mathbf{Q}_{22}D_{22}$$

Tab. 2: The $\mathbf{Q}_{jk}$ in detail

$\mathbf{Q}_{jk}$	row	0	1	2	3
$\mathbf{Q}_{11}$	0	0	$-\mu''_1$	0	$-u''_{31}\mu''_3$
	1	0	μ''_1	0	$u''_{31}\mu''_3$
	2	0	$-\mu''_1(\mu'_2-1)$	0	$-u''_{31}(\mu'_2-1)\mu''_3$
	3	0	$\mu''_1[\mu'_3(u'_{31}-1)+1]$	0	$u''_{31}\mu''_3[\mu'_3(u'_{31}-1)+1]$
$\mathbf{Q}_{12}$	0	0	0	0	0
	1	0	0	0	0
	2	0	$\mu'_1\mu''_2$	0	$u'_{31}\mu'_3\mu''_2$
	3	0	$u''_{32}\mu'_1\mu''_3$	0	$u'_{31}u''_{32}\mu'_3\mu''_3$
$\mathbf{Q}_{20}$	0	0	0	$\mu'_2-\mu''_2$	$u'_{32}\mu'_3-u''_{32}\mu''_3$
	1	0	0	$\mu'_2(\mu''_1-1)-\mu''_2(\mu'_1-1)$	$u'_{32}\mu'_3(\mu''_1-1)-u''_{32}\mu''_3(\mu'_1-1)$
	2	0	0	$\mu''_2-\mu'_2$	$u'_{32}\mu'_3(\mu''_2-1)-u''_{32}\mu''_3(\mu'_2-1)$
	3	0	0	$\mu'_2(\mu''_3-1)-\mu''_2(\mu'_3-1)$	$u'_{32}\mu'_3(\mu''_3-1)-u''_{32}\mu''_3(\mu'_3-1)$
$\mathbf{Q}_{21}$	0	0	0	0	0
	1	0	0	$\mu''_1\mu'_2$	$u'_{32}\mu''_1\mu'_3$
	2	0	0	0	0
	3	0	0	$u'_{31}\mu'_2\mu''_3$	$u''_{31}u'_{32}\mu'_3\mu''_3$
$\mathbf{Q}_{22}$	0	0	0	$-\mu''_2$	$-u''_{32}\mu''_3$
	1	0	0	$-\mu''_2(\mu'_1-1)$	$-u''_{32}\mu''_3(\mu'_2-1)$
	2	0	0	μ''_2	$u''_{32}\mu''_3$
	3	0	0	$\mu''_2[\mu'_3(u'_{32}-1)+1]$	$u''_{32}\mu''_3[\mu'_3(u'_{32}-1)+1]$

contains one significant component =0, that is q_{00}. It represents the constant part of the tridimensional equation of second order. Hence, apart of the centers of projection the surface must pass the origine G_0 of the spatial affine coordinate system as shown in Fig. 2. Therefrom follows that all quadrics which can be seen from the base in the mode of closed *concave* surfaces represent *critical loci* of projective stereo correlation. The well-known traditional critical quadrics cylinder, cone and hyperbolic paraboloid form a subset of this group.

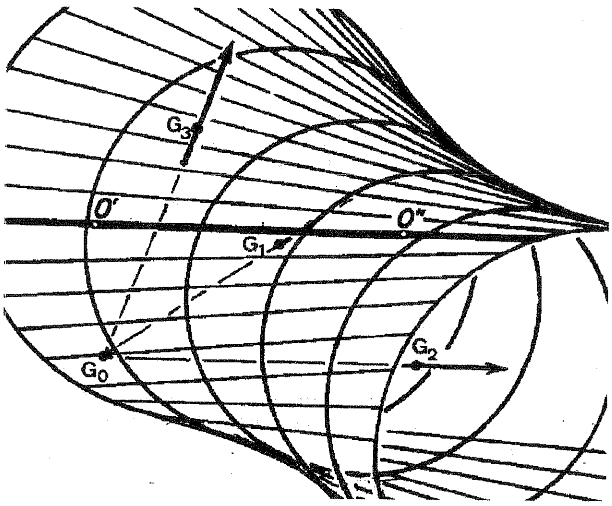

Figure 2: Quadric surface and coordinate system

The coefficients in Tab. 2 depend on the parameters of relative orientation. They can be substituted by expressions depending on the coordinates $\mathbf{y}_0$ of the centers of projection if a predetermination of critical situations is required. Regarding the structure (1.3) of **M**, equation (1.2) can be solved with respect to the μ_i. The results read by means of the abbreviations $u_{30}=1-u_{31}-u_{32}$ and $y_{00}=1-y_{01}-y_{02}-y_{03}$

$$\mu_1 = \frac{u_{31}y_{00}}{u_{30}y_{01}},\ \mu_2 = \frac{u_{32}y_{00}}{u_{30}y_{02}},\ \mu_3 = -\frac{y_{00}}{u_{30}y_{03}}$$

(Brandstätter 1996). By their use, for any arrangement base-to-object the critical locus of stereo correlation is predictable.

For each column additionally exist individual critical quadric surfaces which are caused by the possibility that all its components become simultaneously zero. This case occurs if all points out of the subject {3...7} satisfy one of the equations $\mathbf{y}^T\mathbf{Q}_{jk}\mathbf{y} = 0$. These surfaces must also pass the origine and the centers of projection. Critical surfaces *not* passing the origine will be defined by proportional columns caused by dyadic products resulting in constants c_{jk}, that is $\mathbf{y}^T\mathbf{Q}_{jk}\mathbf{y} = c_{jk}$. If two columns show that behaviour, critical loci exist as well.

4. Final Remarks

Because of the above mentioned statements it will not be difficult, to avoid critical situations of projective stereo correlation. First of all, there must exist *two* points clearly outside the subset of the others. Further, the internal coordinate system of the model space must be defined by one of those two and three non-collineatory points of the subset. At last and as usual, all points of the subset should not be located on surfaces similar to quadrics passing origine and centers of projection. Finally it should be pointed out that all methods of projective relative orientation based on the matrix of correlation (*Rinner 1963, Haggren & Niini 1990, Brandstätter 1992*) also must obey these guidelines in order to guarantee stable numerical conditions.

References:

Brandstätter G. Zur relativen Orientierung projektiver Bündel. Zeitschr. f. Photogrammetrie und Fernerkundung, 59. Jg. Heft 6, Karlsruhe 1991, pp. 199-212

Brandstätter G. Notes on the Direct Projective Transformation of General Stereo Pairs into the Rigorous Normal Case by Image Correlation. XVII ISPRS Congress, Comm. III, Washington 1992, pp. 701-706

Brandstätter G. The Inverse Problem of Algebroprojective Photogrammetry. Proceedings of the International Conference on Inverse Problems, Potsdam August/September 1993,pp. 87-99

Brandstätter G. Fundamentals of Algebroprojective Photogrammetry. Session Report of the Austrian Academy of Sciences, May 1996, in edition

Fuchs H. Projektive Geometrie; Anwendungen in Photogram- metrie und Robotik. Mittlgn. d. geodätischen Institute d. TU Graz, Folge 63, Graz 1988

Haggren H. & I. Niini Relative Orientation Using 2-D Projective Transformation. The Photogrammetric Journal of Finland, Vol. 12, No. 1, 1990, pp-22-33

Krames J. Über die bei der Hauptaufgabe der Luftphotogrammetrie auftretenden "gefährlichen Flächen". Bildmessung und Luftbildwesen, 1/2, 1942, pp.1-18

Rinner K. Studien über eine allgemeine, voraussetzungslose Lösung des Folgebildanschlusses. ÖZfV, Sonderheft 23, Wien 1963

Rinner K. in Handbuch der Vermessungskunde J/F/K, Band IIIa/1 Photogrammetrie. J. B. Metzler, Stuttgart 1972, pp. 422-427

Thompson E.H. The Projective Theory of Relative Orientation. Photogrammetria, Vol. 23, 1968, pp. 67-75

Wunderlich W. Zur Eindeutigkeitsfrage der Hauptaufgabe der Photogrammetrie. Monatshefte für Mathematik und Physik, 50. Bd. 1941, pp. 151-184