# IMPLEMENTATION OF SEQUENTIAL ESTIMATION FOR SINGLE-SENSOR VISION METROLOGY

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## ABSTRACT

Investigations into on-line triangulation, robot vision, image sequence analysis, and autonomous vehicle navigation have established the merits of sequential estimation methods in aerial and non-topographic photogrammetry utilising both film-based cameras and digital sensors. These merits generally focus on enhancement of the speed and efficiency of the triangulation procedure through the incorporation of quality control and observational error detection into the measurement procedure. The on-line quality control of industrial measurement with vision systems utilising a single sensor such as a CCD camera is a natural extension for sequential techniques. This paper examines how the sequential estimation process may be incorporated into single-sensor vision metrology for typical industrial photogrammetric inspection. Issues investigated in the context of the industrial application include the sequential nature of data collection and adjustment, the influence of normal equation structure on system response, generation of approximate values, additional parameters for systematic error compensation, blunder detection procedures, and datum establishment. With regard to datum establishment, a factorisation method for recursively updating the equation system obtained in a free-net adjustment by inner constraints is suggested.

## 1.0 INTRODUCTION

Multi-camera, stereo configurations have, up to now, been the focus of the bulk of the research effort in close-range vision metrology (VM). The recent commercial availability of large-area, high-resolution CCD cameras, coupled with the proven advantages of a single metric camera for high accuracy measurement, has heightened the potential for the single-sensor VM system in industrial inspection. The performance of industrial measurement tasks such as localised inspection, re-work, and fit checking is additionally enhanced through the use of VM systems containing an on-line link between the camera and an external computer. While real-time three dimensional measurements are not achievable with the single camera system, the near real-time image measurement capabilities associated with digital imagery, in combination with sequential estimation techniques such as on-line triangulation (OLT) can provide rapid data turnaround.

The acceptance of CCD cameras for industrial photogrammetry continues at a pace which is constrained primarily by questions of accuracy related to the typically reduced format and resolution of CCD sensors as compared to medium and large format film-based metric cameras. Studies presented in *Fraser & Shortis (1995)* and *Maas & Kersten (1994)* have indicated that CCD and still video cameras such as the Kodak DCS200 (and DCS420) can yield acceptable accuracies for many industrial measurement tasks. One consequence of the lower resolution afforded by CCD sensors is that significantly more images may be necessary to achieve a precision comparable to a network obtained with a metric film camera. The potential of the VM system eases previous limitations in the number of images that may be readily processed and allows their incorporation into the network with minimal time expenditure. After an optimal convergent camera station network is in place, the use of multiple exposures is the principal means of improving object space precision. Here, OLT can serve as a mechanism for recursively monitoring object point accuracy. The photogrammetrist, while still on site, can interactively strengthen the network geometry until the desired level of accuracy is obtained.

OLT methods have typically focused on the detection and removal of gross errors. By incorporating quality control and observational error detection into the measurement process, the speed and efficiency of the overall triangulation is enhanced. A comprehensive historical background of sequential estimation as applied to OLT can be found in *Gruen (1985)*. Several recent studies apply these methods in non-topographic applications. These include robot vision (*Gruen & Kersten, 1992*), image sequence analysis (*Kersten & Baltsavias, 1994*), and autonomous vehicle navigation (*Edmundson & Novak, 1992*). The suggested application of sequential estimation in OLT to industrial quality control (*Kersten et al, 1992*) has for the most part remained unexamined.

This paper, which builds upon work reported in *Edmundson & Fraser (1995)*, explores the utilisation of sequential estimation for OLT in single-sensor vision metrology. We begin with a re-examination of the mathematics behind the general sequential estimation problem focusing on the computational algorithm, an orthogonal transformation technique known as Givens

Transformations (*Blais, 1983*). Emphasis is placed on those areas of concern in general OLT relating to close-range, industrial photogrammetry. These include the issues of system response time, generation of approximate values, the compensation for systematic errors, blunder detection methodology and datum establishment. A procedure is suggested for sequentially updating the system obtained via a free-net adjustment.

## 2.0 SEQUENTIAL ESTIMATION FOR ON-LINE TRIANGULATION

The process of modifying least squares computations by updating either the normal equations matrix or its inverse has been used in control and signal processing for some time in the context of linear sequential filtering. Taking into consideration the sequential nature of the photogrammetric data collection process, the utilisation of sequential algorithms for OLT follows naturally.

The appropriate simultaneous solution for photo-triangulation is the well known bundle adjustment. All data must be available prior to the adjustment. Conversely, the sequential procedure builds the object in a stepwise fashion, proceeding image by image (or point by point) and incorporating data into the system as it is collected.

The primary goal of OLT for aerial triangulation is to provide a clean data set for a final, rigorous simultaneous adjustment. This is achieved by accommodating blunder detection and re-measurement quickly within the measurement process. Observations are added to the system as they become available and deleted or replaced if found to be unacceptable. Sequential algorithms enhance this process by updating the system with new information without starting from scratch with the entire data set. In the aerial case, blunder detection takes precedence over the solution vector while in the VM application, the monitoring of object point precision throughout the measurement process assumes the highest priority.

Notable sequential algorithms which have been examined for OLT include the Kalman filter, which updates the inverse of the normal equations matrix (*Mikhail & Helmering, 1973*), the "Triangular Factor Update" (*Gruen, 1982*) which updates the factorised normals directly, and Givens Transformations which can be used to update either the factorised normal equation system or its inverse. With respect to general least-squares, Givens Transformations possess certain advantages over other orthogonalisation techniques such as the Householder and Gram-Schmidt methods. (*Gentlemen, 1973; George & Heath, 1980*). As applied to OLT, several studies have demonstrated the superiority of the Givens method over the Kalman filter and "Triangular Factor Update" algorithms (*Wyatt, 1982; Runge, 1987; Holm, 1989*).

Givens Transformations are based on the use of plane rotations to annihilate matrix elements. This approach, compatible with the Cholesky method, provides a direct method for solving linear least-squares problems without forming the normal equations. Because all updating is done in the factorised normals, numerical instabilities associated with forming and solving the normal matrix are avoided. Only one row of the design matrix is processed at a time, making it ideal for sequentially adding or deleting observations in an on-line environment. If necessary, the solution vector can be obtained at any stage of the process by back substitution. The method can easily accomodate weighted observations and parameters. A version of Givens Transformations presented in *Gentleman (1973)* avoids the computation of square roots, reduces the number of required multiplications, and facilitates weighted least-squares. This and similar "fast" recursive algorithms are gaining favour in the area of parallel processing due to the absence of the square root operation (*Hsieh et al, 1993*). Additionally, the ability to yield a solution in the absence of a positive definite system may prove to be advantageous for the update of systems encountered in the free-net adjustment of close-range photogrammetric networks. This square root free version of Givens is stressed in this study.

### 2.1 Least-Squares with Orthogonal Transformations

First, the use of orthogonal transformations for standard least-squares estimation with the familiar Gauss-Markov model is illustrated. Given an $n$ x 1 observation vector $l$ and an $m$ x $n$ design matrix $A$ such that $m \geq n$, the goal is to determine the $n$ x 1 parameter vector $x$ in such a way as to minimise the sum of the squares of the elements of the $m$ x 1 residual vector $v$ which is defined by

$$v = Ax - l. \tag{1}$$

Initially considering only unweighted observations, the solution is given by

$$\hat{x} = \left(A^T A\right)^{-1} A^T l \tag{2}$$

This solution may be obtained with the Cholesky factorisation $A^TA = U^TU$, where $U$ is an upper triangular matrix, or with the related factorisation $A^TA = U^TDU$, where $U$ is unit upper triangular and $D$ is diagonal. The only significant difference between the two is that the former uses square roots and the latter does not.

Applying Cholesky to the normals and to the right hand side results in the system

$$U^T U\hat{x} = b \tag{3}$$

With $d = \left(U^T\right)^{-1} b$, the system reduces to

$$U\hat{x} = d \tag{4}$$

which is solved by back substitution.

If the decomposition $A = QR$ is available, where $Q$ is an $m$ x $n$ matrix with orthonormal columns and $R$ is an $n$ x $n$ upper triangular matrix, the normal equations may be written as

$$R^T Q^T QR\hat{x} = R^T Q^T l. \quad (5)$$

$Q$ is orthogonal hence $Q^T Q$ = I. Because $R$ is nonsingular if $A^T A$ is, Eq. 5 reduces to

$$R\hat{x} = Q^T l. \quad (6)$$

Equations 4 and 6 are then equivalent with $U = R$ and $d = Q^T l$.

$R$ and $d$ are obtained by applying a series of orthogonal transformations to $A$ and $l$. Thus the solution to this system may be determined without forming the normal equation matrix directly.

Extending the system with an observational weight matrix $P$ (assuming uncorrelated observations) the least-squares solution is given by

$$\hat{x} = (A^T PA)^{-1} A^T Pl. \quad (7)$$

Because $P$ is diagonal, $A$ can be simply premultiplied by $P^{1/2}$ and the $QR$ decomposition then applied to this modified design matrix.

Now assume a sequential process where Eq. 4 represents the reduced system at stage $k$ - $1$. The addition, deletion, or replacement of observations via orthogonal transformations is shown below and follows closely that of *Gruen (1985)*. The addition of one observation equation to stage $k$ - $1$, including a set of new unknown parameters, results in the following form at stage $k$

$$\begin{bmatrix} U \\ a^T_{(k)} \end{bmatrix} \begin{bmatrix} x \\ -- \\ x' \end{bmatrix} = \begin{bmatrix} d \\ -- \\ l_k \end{bmatrix} \quad (8)$$

Here, $a^T_{(k)}$ represents the coefficient vector of the added observation, $x'$ is the new $p$ x 1 parameter vector, and $l_k$ is the right hand side of the new observation equation. The total number of system parameters is $n$. Applying a series of $n$ orthogonal transformations (in our case Givens Transformations)

$$Q = Q_n Q_{n-1} \cdots Q_1, \quad (9)$$

to Eq. 8 yields

$$Q \begin{bmatrix} U & 0 \\ 0 & 0 \\ a^T_{(k)} & \end{bmatrix} \begin{matrix} \}n-p \\ \}p \\ \}1 \end{matrix} = \begin{bmatrix} \dot{U} \\ \\ 0 \end{bmatrix} \begin{matrix} \}n \\ , \\ \}1 \end{matrix} \quad (10)$$

and for the right hand side,

$$Q \begin{bmatrix} d \\ 0 \\ l_{(k)} \end{bmatrix} \begin{matrix} \}n-p \\ \}p \\ \}1 \end{matrix} = \begin{bmatrix} \dot{d} \\ \\ \tilde{l}_{(k)} \end{bmatrix} \begin{matrix} \}n \\ , \\ \}1 \end{matrix} \quad (11)$$

Zeros in Eqs. 10 and 11 show that when new parameters are introduced, the rows and columns of the existing $U$ matrix and the existing $d$ vector must be extended with zeros.

Finally, the solution vector for the updated system is obtained by back substitution into

$$\dot{U} \begin{bmatrix} \dot{x} \\ \dot{x}' \end{bmatrix} = \dot{d}. \quad (12)$$

### 2.2 Givens Transformations

To illustrate the use of Givens Transformations for the addition of one observation into an existing system, an expanded form of Eq. 8 is shown in Figure 1.

| | | | | | |
|---|---|---|---|---|---|
| $u_{11}$ | $u_{12}$ | $u_{13}$ | $\cdots$ | $u_{1n}$ | $d_1$ |
| | $u_{22}$ | $u_{23}$ | $\cdots$ | $u_{2n}$ | $d_2$ |
| | | $u_{33}$ | $\cdots$ | $u_{3n}$ | $d_3$ |
| | | | $\cdots$ | $\cdots$ | $\cdots$ |
| | | | | $u_{nn}$ | $d_n$ |
| | | | | | $\Omega$ |
| $a_1$ | $a_2$ | $a_3$ | $\cdots$ | $a_n$ | $l$ |

**Figure 1: U matrix augmented by new coefficient vector**

Consider one row vector from the system $Ux = d$ and a coefficient vector from the system $Ax = l$,

$$\begin{matrix} 0 \cdots 0 \; u_i \; u_{i+1} \cdots u_k \cdots \\ 0 \cdots 0 \; a_i \; a_{i+1} \cdots a_k \cdots \end{matrix} \quad (13)$$

One Givens Transformation replaces these two vectors with

$$\begin{matrix} 0 \cdots 0 \; u'_i \; u'_{i+1} \cdots u'_k \cdots \\ 0 \cdots 0 \; 0 \; a'_{i+1} \cdots a'_k \cdots \end{matrix} \quad (14)$$

where

$$\begin{aligned} u'_k &= cu_k + sa_k \\ a'_k &= -su_k + ca_k \\ c^2 + s^2 &= 1 \end{aligned} \quad (15)$$

To annihilate $a_i$ to zero, the rotation parameters are computed from the diagonal elements of $U$ and the corresponding elements of the coefficient vector as follows:

$$u_i' = \sqrt{u_i^{\,2} + a_i^{\,2}} \; ; \; c = \frac{u_i}{u_i'} \quad s = \frac{a_i}{u_i'} \tag{16}$$

Each observation vector is rotated through $U$, row by row until each of its elements is transformed to zero. The additional element $\Omega$ of the right hand side $d$ vector maintains the root residual sum of squares and is updated with Givens Transformations along with $U$ and $d$.

The alternative square root free implementation of Givens Transformations used in this study involves finding a diagonal matrix $D$ and a unit upper triangular matrix $\overline{U}$ such that

$$U = D^{\frac{1}{2}}\overline{U} \tag{17}$$

A row of the product $D^{\frac{1}{2}}\overline{U}$ is rotated with a scaled row of $A$ (*Gentleman, 1973*),

$$\begin{matrix} 0 \cdots 0 \, \sqrt{d} & \cdots & \sqrt{d}\,\overline{u}_k \cdots \\ 0 \cdots 0 \, \sqrt{\delta}\, a_i & \cdots & \sqrt{\delta}\, a_k \cdots \end{matrix} \tag{18}$$

where $d$ is the diagonal element of the matrix $D$ and $\delta$ is the scale factor for the coefficient vector, initially set to one. After one rotation, the newly transformed rows are

$$\begin{matrix} 0 \cdots 0 \, \sqrt{d'} & \cdots & \sqrt{d'}\,\overline{u}_k' \cdots \\ 0 \cdots 0 \quad 0 & \cdots & \sqrt{\delta'}\, a_k' \cdots \end{matrix} \tag{19}$$

where

$$\begin{aligned} d' &= d + \delta a_i^2 \\ \delta' &= d\delta / d' \\ \overline{c} &= d / d' \\ \overline{s} &= \delta\, a_i / d' \\ a_k' &= a_k - a_i \overline{u}_k \\ \overline{u}_k' &= \overline{c}\,\overline{u}_k + \overline{s}\,\overline{a}_k \end{aligned}$$

Weighted least-squares is simplified with this method by initialising the scale factor $\delta$ to the weight instead of to one. Introducing an observation several times with various positive and negative weights is equivalent to introducing it once with the sum of the weights. Thus an observation can be removed by reintroducing it with the negative of its original weight.

## 3.0 OLT FOR SINGLE-SENSOR VISION METROLOGY

In this section important concerns in OLT with respect to close-range, convergent photogrammetry are highlighted. These include system response time, approximate values, compensation for systematic errors, blunder detection, and appropriate datum.

### 3.1 System Response Time

Response time is critical in on-line VM applications and particularly so in the industrial environment where inspection costs are directly influenced by the extent of site disruption. Although significant, improvements in computer hardware should not curb the search for efficient algorithmic solutions. Sequential techniques such as Givens Transformations improve response time but efficiency is also affected by the size of the system which is in turn dependent on the number of active parameters.

Ignoring self-calibration, phototriangulation involves six exterior orientation parameters for each photo and three coordinate parameters for each object point. Consider a system involving *m* photos and *n* object points. In the standard formulation of the bundle adjustment, object point parameters are eliminated, leaving a *6m x 6m* system of orientation parameters. In aerial photogrammetry, this system is still too large to yield permissible OLT response times. The normal case geometry of the aerial network permits the use of sub-blocks of photos in the on-line procedure. The sub-block must be of sufficient size to provide reliability and yet be small enough to yield adequate response times. *Gruen (1981)* recommended the use of a 3 *x* 3 sub-block of photos with both 60% overlap and sidelap.

The irregularity of convergent, close-range networks does not offer such a straightforward answer to effectively deal with system size. While image sensor parameters are less than point parameters (*6m ≤ 3n)*, the elimination of point parameters is the optimum solution. The typical inspection for a single-sensor VM system will likely involve only 50-100 points. As previously mentioned however, a sizeable number of exposures may be needed to achieve a desired level of accuracy. Using a 50 point inspection as an example, as we collect in excess of 25 exposures, the number of sensor parameters begins to exceed the number of point parameters. From this point on, a reversal of the standard bundle in which photo parameters are eliminated will certainly provide a faster response. This approach would also be useful for the standard simultaneous adjustment. The incorporation of inner constraints for a free-net adjustment and the use of additional parameters for self-calibration may also be simplified. The most efficient solution is to incorporate both elimination techniques into the OLT procedure. It is little effort to compare the current number of sensor and point parameters to determine which to eliminate.

Efficiency is also enhanced through the exploitation of the sparsity patterns of the reduced normal equation system. A special matrix storage technique described in *Gruen (1982)* for the Triangular Factor Update and also utilised by *Runge (1987)* for standard Givens Transformations is modified here to accommodate the elimination of image sensor parameters as discussed above. This technique, when combined with Givens, facilitates the direct updating of the *reduced* normal equations. A representative example for six object points is shown in Figure 2.

The lighter shaded portions of the structure correspond to $U$ and $d$ of Eq. 4. This structure is expanded by six rows and columns (dark shaded portions) to provide storage for the submatrices associated with a particular image. Assuming a consistent system at any particular stage of the process, $U$ and $d$ are fully occupied. The introduction of a new image into the system begins by setting all matrix elements of the image submatrices to zero. All observation coefficient vectors for a given image are rotated through the entire structure via Givens Transformations. All subsequent images are treated in the same manner. If observation deletions or re-measurements are required in a previously introduced image, its existing associated submatrices must be re-positioned in the dark shaded areas. The necessary observation vectors are then rotated through the system with weighting appropriate for either insertion or deletion. Back substitution into the $U,d$ system at any time yields the current solution vector for the object point parameters.

### 3.2 Approximate Values

Providing optimum initial parameter values for OLT is a major concern associated with sequential processing in a non-linear model such as the collinearity equations. The costly re-linearization of the system is avoided by using the same set of initial values throughout the sequential process. Coarse values may eventually cause drift in the solution vector sufficient enough to produce a detrimental effect upon efficient blunder detection and precision evaluation. The solution is two-fold. The most obvious answer is to provide good approximate values. This is not always possible. However, for the assumptions presented here, namely highly convergent imagery with measurement restricted to signalised targets, this is reasonable. Secondly, the performance of a periodic simultaneous solution provides a "clean" version of the parameter vector which may be used as the basis for continuing sequential updating. The procedure is straightforward. A minimum of four convergent images is needed to obtain a consistent, reliable system. Four well-distributed rays per object point are necessary for blunder detection with data snooping. All object points with four rays which meet a pre-established geometric criteria are included in a simultaneous adjustment. From

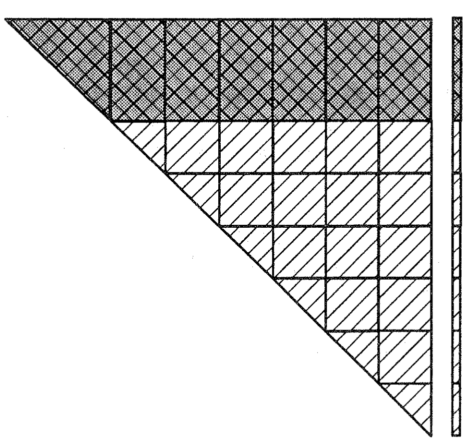

**Figure 2: Reduced normal equation matrix structure for sequential estimation**

this consistent system the sequential procedure begins. The parameters of newly introduced images are determined by space resection and the coordinates of new object points with sufficient observations are determined by spatial intersection.

### 3.3 Compensation for Systematic Errors

Extending the mathematical derivation above to accommodate additional parameters for the compensation of systematic errors is a simple matter. For a full bundle adjustment with self-calibration these may include interior orientation parameters of focal length and principal point coordinates, plus those of radial and decentring distortion. Among researchers reporting experiences in OLT there is a general agreement as to the importance of additional parameters in the sequential process. However, to the authors' knowledge, there are no published findings in which the effects of additional parameters in OLT are examined. The capability of recovering these additional parameters is enhanced in a convergent network and their presence has a direct influence on object point precision. With the primary objective of monitoring object point accuracy, the inclusion of these additional parameters must be addressed. During OLT, changes in interior orientation will likely occur which will in turn influence overall precision. Two approaches should be studied with respect to their effect on the variance of object coordinates. The first involves the utilisation of additional parameters from a prior calibration throughout the procedure and the second is based on updating the additional parameters periodically with a simultaneous adjustment and proceeding with a fixed interior orientation.

### 3.4 Blunder Detection

Baarda's strict data snooping technique, based upon the examination of standardised image coordinate residuals, is one method which has been utilised for blunder detection in the bundle adjustment procedure. There have been more efficient modifications to data snooping such as the "unit observation vector" method (*Gruen, 1982*), but the technique remains computationally intensive and has proven to be the most time consuming aspect of previous implementations. Investigations into less rigorous, approximate techniques are needed. Graphical procedures which are simpler and less expensive to implement, hold great potential for the detection of gross errors in OLT. Ongoing research will compare the efficiency and accuracy of both the "unit observation vector" method and graphical techniques.

### 3.5 Appropriate Datum

It is necessary to establish an optimal, consistent system prior to the start of the sequential procedure. In industrial photogrammetry the preferred means of accomplishing this is through the implementation of a free-net adjustment with inner constraints (*Fraser, 1982*). An important problem to be considered is that of countering the datum defect throughout the sequential procedure. Two basic options are outlined below.

Employing minimal constraints, the datum defect can be removed by fixing seven appropriate object point parameters. The cofactor matrix of the object points is an inestimable quantity and changes in the minimum datum alter object coordinate precision (*Fraser, 1982*). Seven coordinates from three well-distributed object points must be fixed. Two in XYZ and a third, non-collinear point fixed in the coordinate axis most nearly normal to the plane containing the three points. Choosing those points to fix may be accomplished in an automated manner.

Clearly, the capability of directly updating the system produced by the free-network adjustment is desirable. The potentially detrimental influence from the imposition of explicit control would be avoided. The inner constraint method of computing a free network solution is accomplished by applying a similarity transformation to the network in question. One means of achieving this is to border the singular normal equations matrix with a transformation matrix $G$ subject to the condition that $AG = 0$. This is suggested in *Fraser (1982)* and detailed in *Blaha (1971)*.

If the "border" is eliminated along with point parameters in the standard formulation of the bundle adjustment, the resulting upper triangular matrix can have negative values on the diagonal. For the reverse procedure in which photo parameters are eliminated, the border may be most conveniently applied after the elimination. In either case the resulting matrix is clearly not positive definite which precludes the use of Cholesky and standard Givens Transformations for the simultaneous and sequential solutions.

Here, the potential advantage of the $U^TDU$ factorisation coupled with the square-root free Givens method becomes evident. This method can be completed even in the presence of negative diagonal elements *(Martin et al, 1965)*. It should be noted that numerical stability can only be guaranteed in the positive definite case. Potential instabilities can however be detected and compensated for.

## 4.0 CONCLUDING REMARKS

This paper has focused on the application of sequential estimation techniques in OLT to single-sensor VM systems. The potential for these systems is largely dependent upon the refinement of techniques such as sequential estimation, emphasising increasing levels of automation in data acquisition and analysis. Further studies into the suitability of algorithms such as Givens Transformations for various applications of OLT are required. Important issues in OLT as they relate to single-sensor VM have been highlighted here and warrant additional examination. These include system response time, approximate values, additional parameters, blunder detection, and appropriate datum. Practical evaluation of these and other issues is ongoing.

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